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Analysis Numerical Solution of VSCIR Pneumonia Model by using Laplace Decomposition Method

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Abstract

In this paper, model analysis is presented using analytic and numerical methods and the analytic series solution of the pneumonia model is approximated. Laplace – Adomian Decomposition method (LADM) is applied to pneumonia model. It is analysed with Euler's method and is found to be in agreement with it.

Keywords*: *Differential equation, Laplace – Adomian Decomposition method (LADM),Pneumonia model, Euler's method, solution.

1. Introduction

Pneumonia is a high – incidence respiratory disease known by an inflammatory condition of the lungs. The micro organismswhich caused it namely: bacteria, viruses, and parasites and fungi. The susceptible causing pneumonia bacteria is said to be the leading cause [1], [2] especially Streptococcus Pneumoniae [3], [4], [5]. The bacteria mutiply in numbers after going into the lungs where they settle down inside the alveoli and passages. The region is surrounded with fluid and pus [6]. This inhibit the supply of oxygen and creates problematic condition in breathing.

Despite the increasing focus on the Millennium Development Goal 4 of United Nation – MDG [7] " to reduce child mortality" almost 1.9 million children still die from pneumonia each year in the developing countries, accounting for 20% deaths globally [8]. Within three days death can occur if untreated [9].

The notion of the present theme is to find a simple series solution of the VSCIR pneumonia model. to acquire this goal we deduce the basic equation of the VSCIR

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model, which is expressed by a first order nonlinear differential equation . We find the solution of this model by Laplace Adomian Decomposition method (LADM), which is an continuation of the standard Adomian Decomposition Method (ADM) [10 – 12]. The ADM is a strong mathematical method that allows to find solutions of ordinary, partial and integral differential nonlinear equations in the form of a series, with the terms of the series determined recursively with the help of the Adomian polynomials [10, 11]. The Laplace Adomian Decomposition method is put on to VSCIR pneumonia model and its solution can be represented as a series that gives an efficient provision of the numerical results.

Formulation and Description of the model.

Model Equations :

---------(1)

-----(2)

--------(3)

... (4)

--------(5)

With initial condition $S(0)=S_{0}$, $(V_0)=V_0$, $C(0)=C_{0}$, $I(0)=I_{0}$, $R(0)=R_{0}$...(6)

Table I : Description of Variable and parameters of the model

Variable	**Description**
V(t)	No. of vaccinated individuals at time t
S(t)	No. of susceptible individuals at time t
C(t)	No. of Carrier individuals at time t
I(t)	No. of infected individuals at time t
R(t)	No. of recovered individuals at time t

Parameters	**Interpretation**
π	Recruitment rate
μ	Natural death rate

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δ	Disease induced death rate for I class
λ	Force of infection
a	Probability that newly infected individuals are asymptomatic / carrier.
α	Waning rate of vaccine
β	Rate of vaccination from S to V
$\in \lambda$	Rate of vaccinated getting carrier and infected
γ	Rate at which carrier transform to susceptible class
ω	Rate at which carrier transform in infected class
η	Rate of infected getting carrier and infected.
ξ	Recovery rate due to prompt treatment
$\emptyset$	Recovery rate due to infected class
v	Rate at which infected transferring to susceptible class.
σ	Rate at which recovered person getting susceptible.

The model divides the total population into five subclasses namely susceptible S(t), Vaccinated V(t), Carrier C(t), Infected I (t) and recovered R(t). The individuals are recruited into the vaccinated and susceptible class either by immigration or by birth rate

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be the natural death at any compartment. Let p be the number of vaccinated persons. Let (1-p)susceptible number of people. Since vaccines wanes with time the protected individuals after its expiry return backs to susceptible compartment at the rate . Individuals move from susceptible class to vaccinated class with vaccination rate of . The susceptible class is infected either by carrier or symptomatically infected individuals with a force of infection

$$\lambda_1 = x\left(\frac{I(t) + pc(t)}{N}\right)$$

where K is constant rate, is the probability that contact is effective to cause infection and is transmission coefficient for the carrier. If >1 then, the carriers infect susceptible more highly than infective. If =1, then both carriers and infective have good chance to infect susceptible than carriers. It is assumed that the model is not 100% effective, so vaccinated classes (V) also have a chance of being infection or carrier with small proportion and the force of infection for the vaccinated class be = where 0 and is the proportion of the serotype not covered by the vaccine newly infected individuals by the force of infection become either carrier with a probability of a to join the carrier class C or move to the infected class I with probably of 1-a. The carrier class can develop and join the infected class with a rate of or recover by gaining natural immunity at rate. Individuals in the infected class move to recovered compartment at a per capita rate of by treatment, with treatment efficiency of q proportion of individuals join the recovered class or join the carrier class with (1-q) proportion by adapting the treatment, or die from the disease at the rate . Individuals from recovered class lose their temporary immunity by rate . Recovery rate from infected class be $\emptyset$. Let be the rate at which carriers gets back to susceptible class.

Laplace Adomian Decomposition Method:

Consider the VSCIR Pneumonia model (1 -5) subject to the initial condition (6). For simplicity, we will change the variables for the system of equations (1-5) becomes

= p …….(7)

= (1-p)(8)

= ……(9)

= ………(10)

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= …….(11)

where at time t,
x(t) represents a proportion of people with vaccinated population
y(t) represents a proportion of people with susceptible population
c(t) represents a proportion of people with carrier population
i(t) represents a proportion of people with infected population
r(t) represents a proportion of people with recovered population

We know that, laplace transform of (t) are defined by,

Lx'=sLx-x(o) for i=1,2,3,………n

Taking laplace transform to both sides of (7-11) and satisfying yields,

Lx(t)= + +Ly(t)x(t)-Lx(t).

y(t)-Lx(t)c(t)i(t)-μLx(t)

Ly(t)= + Ly(t)+Lx(t)y(t)

+ Lc(t)y(t) + Li(t)y(t)+ Lr(t)

y(t)-Ly(t)x(t)- y(t)c(t)i(t)- Ly(t)

Lc(t)= + Lx(t)c(t)+ Ly(t)c(t)

+ Li(t)c(t)- Lc(t)y(t)

- Lc(t)i(t)- Lc(t)r(t)- Lc(t)

Li(t)= + Ly(t)c(t)+ Lx(t)c(t) + Lc(t)i(t)

+ Lr(t) c(t)-i(t)y(t)- Li(t) c(t) r(t) – Li(t)

Lr(t)= + Lc(t)r(t) + Li(t)r(t) - Lr(t)i(t)- Lr(t)y(t) - Lr(t)…………..(12).

Let F(t)=x(t) y(t), G(t)=c(t) y(t)

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H(t)=I (t) y(t),J(t)=x(t) c(t)

K(t)=i(t)c(t), M=C(t) r(t) ,U(t)=y(t) r(t)

N(t)=x(t)c(t) i(t),Q (t)=y(t)c(t) i(t)

W(t)= i(t)c(t) r(t), X(t)= r(t)i(t)…….(13).

Using the Adomian Decomposition Method [10-12] and the Adomian polynomials for (12) and (13) .We represent the solutions as infinite series,

$x=_k$,$y=_k$, $c = _k$,

$i= _k$, $r=_k$ ……………(14)

Where the components x_k are to be recursively found. Moreover, the (13) will be represented by an infinite series of Adomian polynomials [10-12].

F(t,x,y) = $_k$, G(t,c,y)= $_k$,

H(t,i,y) = $_k$, J(t,x,c)= $_k$,

K(t,i,c) = $_k$, M(t,c,r)= $_k$,

U(t,y,r) = $_k$, N(t,x,c,i)= $_k$,

Q(t,y,c,i) = $_k$, W(t,i,c,r)= $_k$,

X(t,r,i) = $_k$ ……………..(15)

Where F_k, G_k, H_k, J_k, K_k, M_k, U_k, N_k, Q_k, W_k, X_k,$k\geq0$, are defined by

$F_k =$ [F(t,x_j, y_j)], k=0,1,2,….

$G_k =$ [G(t,c_j, y_j)], k=0,1,2,….

$H_k =$ [H(t,I_j, y_j)], k=0,1,2,….

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$J_k =$ [J(t,$_j$, $_j$)], k=0,1,2,….

$K_k =$ [K(t,i_j, c_j)], k=0,1,2,….

$M_k =$ [M(t,c_j, $_j$)], k=0,1,2,….

$U_k =$ [U(t,$_j$, r_j)], k=0,1,2,….

$N_k =$ [N(t,$_j$, c_j,i_j)], k=0,1,2,….

$Q_k =$ [Q(t,$_j$, c_j,i_j)], k=0,1,2,….

$W_k =$ [W(t,$_j$, $c_{j,j}$)], k=0,1,2,….

$X_k =$ [X(t,$_j$, i_j)], k=0,1,2,….

F_k,G_k, H_k,J_k,K_k,M_k,U_k,N_k,Q_k,W_k, and X_k, are called Adomian polynomials and can be evaluated for all forms of non-linear functions.
Substituting (14) and (15) into (12) gives,

L= = + L + L - - L

L= = + L+ +
L + L + L - - L- L=

L= = + L+ + L + L - L - - L=

L= = + L+ + L + L - L - - L=

L= = + L+ - L + L - L………(17)

Comparing both sides of (17) we get the following iterative form.

L {x_o}= , {x_k+1}= L{x_k} + L{F_k} - L{F_k}- L{N_k} - {x_k}
L{y}=,

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$L\{y_k+1\} = L\{y_k\} + L\{F_k\} + L\{G_k\} + L\{H_k\} + L\{U_k\} - L\{F_k\} - L\{Q_k\} - L\{y_k\}$

$L\{c_o\} =$

$L\{c_{k+1}\} = L\{J_k\} + L\{G_k\} + L\{K_k\} - L\{G_k\} - L\{K_k\} - L\{M_k\} - L\{c_k\}$

$L\{i_o\} =$

$L\{i_{k+1}\} = L\{G_k\} + L\{J_k\} + L\{K_k\} + \quad L\{M_k\} - L\{H_k\} - L\{W_k\} - L\{i_k\}$

$L\{r_o\} = .$

$L\{r_k+1\} = L\{M_k\} + L\{X_k\} - L\{X_k\} - L\{U_k\} - L\{r_k\}$

Taking the inverse laplace transform to the first part of (18) gives
$x_0,y_0,C_0,G_0,H_0,J_0,K_0,M_0,U_0,N_0,Q_0,W_0,X_0$ as follows:
$F_o=F(t,x_0,y_0)=x_0y_0$, $G_0=(t,c_0,y_0)=C_0y_0$,
$H_o=H(t,i_0,y_0)=i_0y_0$, $J_0=(t,x_0,c_0)=x_0y_0$,
$K_o= (t,i_0,c_0)=i_0c_0$, $M_0=(t,c_0,r_0)=C_0r_0$,
$U_o= (t,y_0,r_0)=y_0r_0$, $N_0=(t,x_0,c_0,i_0)= x_0c_0i_0$,
$Q_o= (t,y_0,c_0,i_0)=y_0c_0i_0$, $W_0=(t,i_0,c_0,r_0)= i_0c_0r_0$,
$X_o= (t,r_0,i_0)= r_0i_0.$

These values will give us to find x_1,y_1,c_1,i_1, & r_1, as follows:

$L\{x_1\} = L\{x_0\} + L\{F_0\} - L\{F_0\} - L\{N_0\} - L\{x_0\}$

$=x_0+ x_0y_0 - x_0y_0 - x_0c_0i_0 - x_0.$

$x_1 =L^{-1}\{ x_0+ x_0y_0- x_0y_0 - x_0c_0i_0- x_0 \}$

$=p\pi x_0+ x_0y_0- x_0y_0 - x_0c_0i_0.$

Similarly,

$y_1=(1-p)\pi y_0 + x_0y_0+ \gamma c_0y_0+V i_0y_0+ \sigma y_0r_0 -\beta x_0y_0 -\alpha y_0c_0i_0-\mu y_0$

$c_1= \epsilon\alpha x_0c_0+ \alpha c_0 y_0+ (1-q)ni_0c_0 -\gamma c_0y_0- \omega i_0c_0-\xi c_0r_0- \mu c_0.$

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$i_1=(1-\alpha)c_0y_0 +(1-\alpha) \in x_0c_0 + \omega i_0c_0+\phi i_0y_0+ \sigma c_0r_0 -v\, i_0y_0 -qn\, i_0c_0r_0-(\mu+\delta)i_0$

$r_1=\xi\, c_o\, r_0 +qnr_0c_0 - \phi\, r_0i_0- \sigma\, y_0r_0 – \mu r_0.$

Now using (16) we obtain,

$F_1=x_0y_1 + x_1y_0 +2\lambda x_1y_1$

$G_1=c_0y_1 + c_1y_0 +2\lambda c_1y_1$

$H_1=i_0y_1 + i_1y_0 +2\lambda i_1y_1$

$J_1=x_0c_1 + x_1c_0 +2\lambda x_1c_1$

$K_1=i_0c_1 + i_1c_0 +2\lambda i_1c_1$

$M_1=c_0r_1 + c_1r_0 +2\lambda c_1r_1$

$U_1=y_0r_1 + y_1r_0 +2\lambda y_1r_1$

$N_1=x_0c_1 + x_1c_0 + x_0i_1+x_1i_0+c_0i_1 +c_1i_0+2\lambda x_1c_1+ 2\lambda x_1i_1+2\lambda c_1i_1$

$Q_1=y_0c_1 + y_1c_0 + y_0i_1+y_1i_0+c_0i_1 + c_1i_0 +2\lambda y_1c_1+ 2\lambda y_1i_1+2\lambda c_1i_1$

$W_1=i_0c_1 + i_1c_0 + i_0r_1+ i_1r_0 +c_0r_1+c_1r_0 +2\lambda i_1c_1+ 2\lambda i_1r_1+2\lambda c_1r_1$

$X_1 =r_0i_1 + r_1i_0 +2\lambda r_1i_1$, so that

$x_2= p\pi x_1 +\beta(x_0y_1 +x_1y_0+2\lambda x_1y_1) - \propto(x_0y_1 +x_1y_0+2\lambda x_1y_1) -\in\lambda_1 (x_0c_1+x_0i_1+x_1i_0+c_0i_1+c_1i_0+2\lambda x_1c_1 +2 \lambda x_1i_1+2 \lambda c_1i_1$

$y_2=(1-p)\pi y_1 + (x_0y_1+ x_1y_0 + 2\lambda x_1y_1)+ \gamma(c_0y_1+ c_1y_0 +2 \lambda c_1y_1)+v(i_0y_1+ i_1y_0+2 \lambda i_1y_1)+ \sigma(y_0r_1+ y_0r_1+2\lambda y_1r_1) -\beta (x_0y_1+ x_1y_0+2 \lambda x_1y_1) -\alpha (y_0c_1+ y_1c_0+2\lambda y_1c_1+y_1i_0+ y_0i_1+ 2\lambda y_1i_1+c_0i_1+ c_1i_0+2\lambda c_1i_1)-\mu y_1$

$c_2= \in\alpha(x_0c_1+ x_1c_0+2\lambda x_1c_1)+ \alpha(c_0\, y_1+ c_1\, y_0+2 \lambda c_1y_1)+ (1-q)n(i_0c_1+ i_1c_0+2\lambda i_1c_1) –\gamma(c_0y_1+ c_1y_0+2 \lambda c_1y_1)- \omega(i_0c_1+ i_1c_0+2\lambda i_1c_1)-\xi (c_0r_1+ c_1r_0+2\lambda c_1r_1)-\mu c_1.$

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$i_2 = (1\text{-}a)(c_0y_1+c_1y_0+2\lambda c_1y_1)+(1\text{-}a) \in(x_0c_1 + x_1c_0+2\lambda x_1c_1)+ \omega(i_0c_1+ i_1c_0+2\lambda i_1c_1)+ \phi(i_0y_1+ i_1y_0+2\lambda i_1y_1) -v(i_0y_1+ i_1y_0+2 \lambda i_1y_1) -qn(i_0c_1+i_1c_0+2\lambda i_1c_1+ i_0r_1+ i_1r_0+2\lambda i_1r_1+ c_0r_1+ c_1r_0+2\lambda c_1r_1)-(\mu+\delta)i_1$

$r_2 = \xi(c_0r_1 + c_1r_0+2\lambda c1r1)+qn(r_0i_1+ r_1i_0+2\lambda r_1i_1) - \phi(r_0i_1+ r_1i_0+2\lambda r_1i_1)- \sigma (y_0r_1+ y_1r_0+2\lambda y_1r_1) - \mu r_1$

This successively will lead to the complete determination of the components of x_k, y_k, c_k, i_k, r_k, $k \geq 0$ upon using (18). The series solution follows immediately after using equation (14). Solution can be written as

$x(t) = x_0+x_1+x_2+\ldots.$

$y(t) = y_0+y_1+y_2+\ldots..$

$c(t) = c_0+c_1+c_2+\ldots..$

$i(t) = i_0 +i_1+i_1+i_2+\ldots..$

$r(t) = r_0+r_1+r_2+\ldots..$ (19)

Let $x_0 = 600$, $y_0 = 400$, $c_0 = 250$, $i_0 = 100$, $r_0 = 50$

For the computation, we used the followings table 1 values of parameters.

Parameters	**Values**
λ_1	0.3
∈	0.4
p	0.6
α	0.001
β	0.9
ω	1
q	0.5
n	0.0238
ξ	0.0115
v	0.2
a	0.338
μ	0.002
δ	0.33
ϕ	0.01
γ	0.1
σ	0.05
π	1000

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By doing computation work , we get the following series

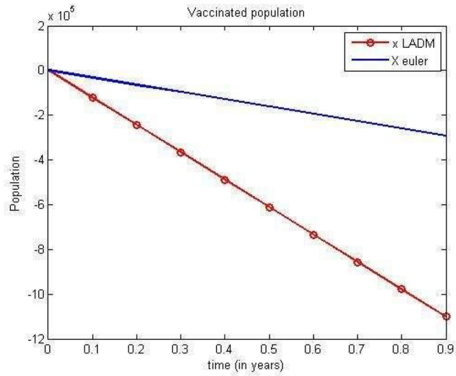
x(t) = 600 – 1224240t + ……

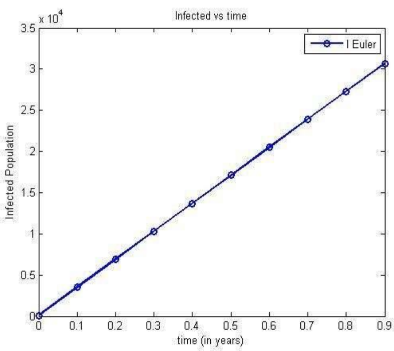
y(t) = 400 – 36760101014000.8t + …..

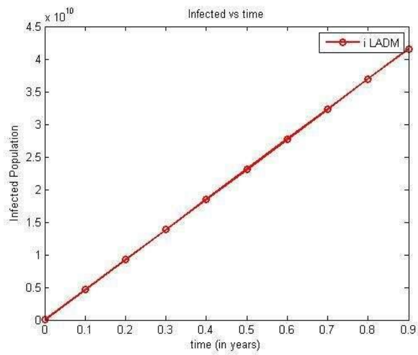
c(t) = 250 – 18622.75t +…….

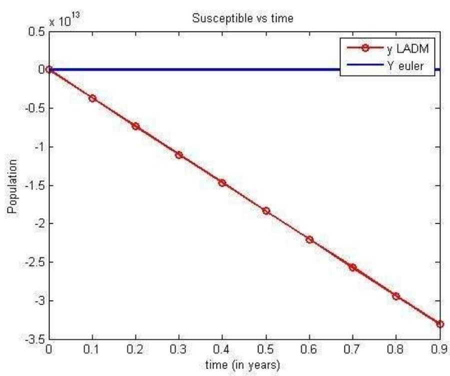
i(t) = 100 + 46126235091.8t+…..

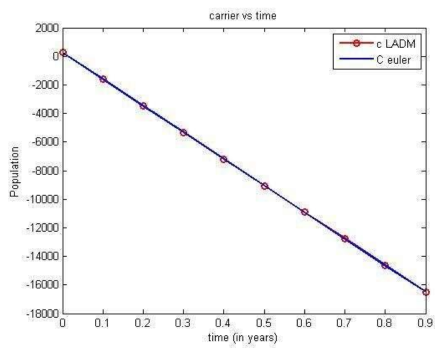
r(t) = 50 – 846.85t+…..











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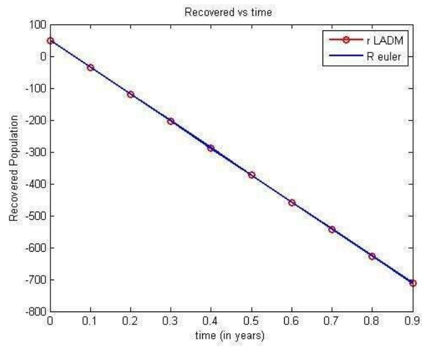

Conclusions

From the above graphs of vaccinated x(t), susceptible y(t), carrier c(t), infected i(t), recovered r(t) versus time . It shows that x(t), y(t), c(t), and r(t) are at a steady state whereas i(t) increases with time . The analysis of the model has been done by employing Laplace Adomian Decomposition method [10]. The analysis reflects that the series solution of the system (1) can be approximated by a powerful Laplace Adomian Decomposition method. For x(t), y(t), c(t), and r(t) there is no difference in the graph when compared with Euler's method. There is computation difference for i(t) in LADM and Euler's method but the nature shows the increase with time. So, we can conclude that it is in agreement with Euler's method.

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