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Split Litact Domination in Graphs

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Abstract

Let $G = (V, E)$ be a graph and $m(G) = C(G) \cup E(G)$ be a litact graph of G. Let $D \subseteq V(m(G))$ be a litact dominating set is said to be a split litact dominating set, if the sub graph $\langle V(m(G)) - D\rangle$ is disconnected. The minimum cardinality of split litact dominating set is called split litact domination number and it is denoted by γ_{sm}. In this paper we investigate different variants of $\gamma_{sm}(G)$ with other domination variants of G for some established graphs like Cp, Wp , Kp, Km, n, Trees, etc. Further the Nordhaus- Gaddum types of results are also established.

Keywords: $m(G)$, $\gamma_m(G)$, $\gamma_{sm}(G)$

Mathematics *subject classification number*: AMS-05C69,70.

1. INTRODUCTION

We refer F.Harary [5] and V.R.Kulli [10] for basic notations and definitions. Here we considered all graphs as finite, connected, simple, undirected and non-trivial. The concept of split domination number was introduced by *Kulli* and *Janakiram* in [9]. We are interested in finding the split domination in a litact graph which was introduced and described by *M.H.Muddebihal* [11]. Many authors explored the variant in their research. *K.V.Suryanarayana Rao* and *V. Sreenivansan* discussed " *The Split Domination in Arithmetic Graphs* " in [15]. A study on "*Split block subdivision domination in graphs*" was given by *M.H.Muddebihal,P.Shekanna* and *Shabbir Ahmed* in [12]. *"Split and Strong Split Steiner Domination Number of graphs"* was studied by *K. Ramalakshmi*, & *K. Palani* in [14]. A brief study on "*Split Domination, Independence, and Irredundance in Graphs*" described by *Stephen Hedetniemi, Fiona Knoll, Renu Laskar* can be found in [16] and a discussion of "*Split domination number of k - Duplication of outer planar graphs*" is given by *M. Priyadharshini* and *N. Parvathi* in [13]. *A. Alwardi, K. Ebadi, M. Manrique* And *N. Soner* discussed the "*Semi-strong split domination in graphs*" in [1]. *Chelvam T.T., Chellathurai S.R* [3] has given many bounds for the "*A note on split domination number of a graph*". *S. Delbin Prema , C. Jayasekaran* has given a detailed study on *"The Split Domination and Irredundant* Number of a Graph" in [4]. A discussion on "*The Split Domination, Inverse Domination and Equitable Domination in the Middle and the Central graphs of the Path and the Cycle graphs*" is provided by *K. Ameenal Bibi* & *P.Rajakumari* in [2]. *B. Janakiraman, N.D. Soner* and *B. Chaluvaraju* discussed the "*Total Split Domination in Graphs*" in [8].

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2. DEFINITIONS

Definition 2.1: An ***induced subgraph*** of a graph is another graph, formed from a subset of the vertices of the graph and all of the edges connecting pairs of vertices in that subset. It is denoted by $\langle x \rangle$.

Definition 2.2 The neighbourhood does not include v itself is called ***open neighbourhood*** is denoted by $N_G(V)$. Neighbourhood which v is included called the ***closed neighbourhood*** and is denoted by $N_G[V]$.

Defintion 2.3: A vertex C is called a ***Cut vertex*** if $G - C$ has more connected components of G.

Definition 2.4: Let $G = (V, E)$ be a graph of order n. The ***Complement*** graph $\bar{G}$ is the graph with $V(\bar{G}) = V(G)$

and $E(\bar{G}) = E(K_n) \setminus E(G)$.

Definition 2.5: A set D is said to be ***Dominating set*** if each vertex $v \in V$ is either an vertex of D or is adjacent to an vertex of D and the number is denoted by $\gamma(G)$ and $\gamma(G) = min|D|$.

Definition 2.6: A set D is said to be a ***Total Dominating set*** if it must be dominating set and each vertex $v \in V$ must be adjacent with at least one vertex $u \in V, u \neq v$ in D and it is denoted by $\gamma_t(G)$ and $\gamma_t(G) = min|D|$.

Definition 2.7: A set D is said to be a **C*onnected Dominating set***, if it must be dominating set and the subgraph induced by $\langle D \rangle$ has no isolated vertices. Connected domination number of G, is denoted by $\gamma_c(G)$ and $\gamma_c(G) = min|D|$.

Definition 2.8: The ***Litact Graph*** $m(G)$ of a graph G is the graph whose vertex set is the union of the

set of edges and the set of cut vertices of G in which two vertices are adjacent if and only if the corresponding edges and cut vertices are adjacent or incident in G.

Definition 2.9: A dominating set $D \subseteq V(m(G))$ is called ***Litact Dominating set*** of G, if each vertex in $V - D$ is adjacent to a vertex v in D. Litact domination number of G, is denoted by $\gamma_m(G)$ and $\gamma_m(G) = min|D|$.

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Definition 2.10: A litact dominating set $D \subseteq V(m(G))$ is a ***Split Litact Dominating set***, if the sub graph $\langle V(m(G)) - D \rangle$ is disconnected. Split litact domination number in $m(G)$, is denoted by $\gamma_{sm}(G)$ and $\gamma_{sm}(G) = min|D|$.

3. OUTCOMES

For furthermore outcomes we need the following statements.

Theorem 3.A [10]: For every graph G, $\gamma(G) \leq p - \Delta(G)$.
Theorem 3.B [10]: For every graph G, $\left\lceil \frac{p}{1+\Delta(G)} \right\rceil \leq \gamma(G)$.
Theorem 3.C [10]: For every graph G, $\gamma(G) \leq \beta_0(G)$.
Theorem 3.D[7]: For every graph G, $\alpha_0(G) + \beta_0(G) = p$ and if G has no disconnected vertices, then $\alpha_1(G) + \beta_1(G) = p$.

4. THEOREMS*:*

The split litact domination number for a few well-known graphs is given beneath:

Theorem 4.1 Particular values:

(i) For each cycle graph C_p, with $p \geq 3$vertices, $\gamma_{sm}(C_p) = 0$ if $p = 3$ and $\gamma_{sm}(C_p) \geq 2$ if $p > 3$.

(ii) For each wheel graph W_p $\gamma_{sm}(W_p) = p,\ p \geq 4$.

(iii) For each graph $K_p$with $p \geq 3$ vertices, $\gamma_{sm}(K_p) = 0$ if $p = 3$ and
$\leq 2p - 3$ if $p > 3$.

(iv) For each star graph $K_{1,p}$, $\gamma_{sm}(K_{1,p}) = 0$.

We get the relation between $\gamma_{sm}(G)$ and $V(G)$ in the next theorem.

Theorem 4.2: For every graph G, $\gamma_{sm}(G) < 2p - 1$ except $K_p, p \geq 6$.

Proof: Let edge set be $E = \{e_1, e_2, \dots\dots, e_j\}$ in G and cut vertex set be $C = \{c_1, c_2, \dots\dots, c_k\}$ in G so that $E \cup C \subseteq V(m(G))$. Let split dominating set be $D_1 \subseteq V(m(G))$ is minimal in $m(G)$, $D_2 \subseteq V(m(G)) - D_1$ and $D_2 \in N(D_1)$ then $|D_1 \cup D_2| = V(m(G))$ *and* $|D_1 \cup D_2| < 2p$.

$$\text{Since } 1 < |D_2|$$
$$\Rightarrow 1 + |D_1| < |D_2| + |D_1|$$
$$\Rightarrow |D_1| < |D_1 \cup D_2| - 1$$
$$\Rightarrow |D_1| < 2p - 1$$
$$\text{Therefore, } \gamma_{sm}(G) < 2p - 1.$$

The following corollary relates split litact domination number in graph G, vertices of G and maximum degree of G.

Corollary 4.1: For each graph G except K_p, $\gamma(G) + \gamma_{sm}(G) < 3p - \Delta(G) - 1$.

Proof: From Theorem 3.A, $\gamma(G) \leq p - \Delta(G)$ and from Theorem 4.2, $\gamma_{sm}(G) < 2p - 1$ we get, $\gamma(G) + \gamma_{sm}(G) < 3p - \Delta(G) - 1$.

We relate with the below theorem $\gamma_{sm}(G)$ & $\gamma(G)$.

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Theorem 4.3: For each graph G, $\gamma_{sm}(G) \geq \gamma(G) - 1$. Equality holds when $\gamma_{sm}(G) = 0$.

Proof: Let D be a minimal set which is dominating in G so that $\gamma(G) = |D|$. Consider edge set be E and cut vertex set be C in G. Let $D_1 \in V(m(G))$ be a minimal set which is dominating in $m(G)$. Additionally if $D_2 \in V\big(m(G)\big) - D_1$ and $D_2 \in N(D_1)$ then recollect a set $D_2{}' \subset D_2$ so that $D_2{}' \cup D_1$ forms a split minimal set that is dominating in $m(G)$, that is $\gamma_{sm}(G) = |D_1 \cup D_2'|$. Openly $|D| \leq |D_1 \cup D_2{}'| + 1$. Thus $|D| - 1 \leq |D_1 \cup D_2{}'|$. It follows that $\gamma_{sm}(G) \geq \gamma(G) - 1$.

In the succeeding corollary, we relates $\gamma_{sm}(G), p$ and $\Delta(G)$.

Corollary 4.2: For each graph G, $\left\lceil \frac{p}{1+\Delta(G)} \right\rceil \leq \gamma_{sm}(G) + 1$.

Proof: From Theorem 3. B & Theorem 4.3 we are able to obtain $\left\lceil \frac{p}{1+\Delta(G)} \right\rceil \leq \gamma_{sm}(G) + 1$.

Within the following theorem, we get a relation among $\gamma_{sm}(G)$, $\alpha_0(G)$, $\gamma(G)$ *and* $diam(G)$.

Theorem 4.4: For every graph G, $\gamma_{sm}(G) \leq \alpha_0(G) + \gamma(G) + diam(G)$. Equality holds for some standard graphs.

Proof: Let all edges covered by a minimal vertex set be $A \subseteq V(G)$ of G then $|A| = \alpha_0(G)$. Further there exists an edge set $E' \subseteq E$, are incident with the vertices of A constituting the longest path of G so that $|E'| = diam(G)$. Let D be a minimal set which is dominating and cut vertex set be C in G. Let D be a $\gamma_{sm} -$ set of G so that $\gamma_{sm}(G) = |D|$. Since D has minimum no of vertices, $|D| \leq |A| \cup |D| \cup |E'|$. Hence, $\gamma_{sm}(G) \leq \alpha_0(G) + \gamma(G) + diam(G)$.

The following corollary gives relationship between split litact domination number, vertices and diameter of graph G.

Corollary 3.3: For each graph G, $\gamma_{sm}(G) \leq p + diam(G)$.

Proof: From Theorem 4.4, Theorem 3.C and Theorem 3.D, we have

$$\gamma_{sm}(G) \leq \alpha_0(G) + \gamma(G) + diam(G)$$
$$\Rightarrow \gamma_{sm}(G) \leq \alpha_0(G) + \beta_0(G) + diam(G)$$
$$\Rightarrow \gamma_{sm}(G) \leq p + diam(G).$$

The succeeding theorem relates with split litact domination number, $\gamma_t(G)$ & $\gamma(G)$.

Theorem 4.5: For each graph G, $\gamma_{sm}(G) + \gamma_t(G) \geq 2\gamma(G) - 1$.

Proof: We have $\gamma_t(G) \geq \gamma(G)$ and from Theorem 4.3, we get the above result.

In the next corollary, we relates $\gamma_{sm}(G), \gamma_t(G), V(G)$ and $\Delta(G)$.

Corollary 4.4: For each graph G, $\gamma_{sm}(G) + \gamma_t(G) > \left\lceil \frac{p}{1+\Delta(G)} \right\rceil$.

Proof: From Theorem 4.5 and Theorem 3.B we get $\gamma_{sm}(G) + \gamma_t(G) > \left\lceil \frac{p}{1+\Delta(G)} \right\rceil$.

Within the following theorem, we obtain a relation among $\gamma_{sm}(G)$ & $E(G)$.

Theorem 4.6: For any graph G, $\gamma_{sm}(G) < q$.

Proof: Let dominating minimal set be D and cut vertex set be C in G. Let D be a $\gamma_{sm} -$ set of G so that $\gamma_{sm}(G) = |D|$.Let edge set be $E(G)$ in G that is $|E(G)| = q$. Thus $|D| < q$. Hence $\gamma_{sm}(G) < q$.

In the succeeding theorem we obtain the $\gamma_{sm}(G)$ with regard to $\alpha_0(G)$.

Theorem 4.7: For each graph G, $\left\lfloor \frac{\gamma_{sm}(G)}{2} \right\rfloor < \alpha_0(G)$.

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Proof: Let all edges covered by a minimal vertex set be $A \subseteq V(G)$ in G then $|A| = \alpha_0(G)$. Let $|D|$ be forms a split minimal set and is dominating in $m(G)$, that is $|D| < 2|A|$. Thus $\gamma_{sm}(G) < 2\alpha_0(G)$. Hence $\left\lfloor \frac{\gamma_{sm}(G)}{2} \right\rfloor < \frac{\gamma_{sm}(G)}{2} < \alpha_0(G)$.

In succeeding theorem we obtain a relation among diameter in G and split domination number of $m(G)$.

Theorem 4.8: In a graph G, $\left\lfloor \frac{diam(G)+1}{2} \right\rfloor \leq \gamma_{sm}(G)$ except some standard graphs.

Proof: Let edge set be E in G which forms the greatest distance between two vertices in G. Then $|E| = diam(G)$ and cut vertex set be C in G. Then $E \cup C \subseteq V(m(G))$. Let D forms a split minimal set and is dominating in $m(G)$. Since $E \subseteq V(m(G))$ and D is a γ_{sm} − set, $diam(G) \leq 2\,\gamma_{sm}(G) - 1$ which gives $\left\lfloor \frac{diam(G)+1}{2} \right\rfloor \leq \gamma_{sm}(G)$.

The next corollary we relates $\gamma_{sm}(G), \gamma(G)$ and $V(G)$.

Corollary 4.5: For each graph G, $\left\lceil \frac{\gamma_{sm}(G)}{2} \right\rceil + \gamma(G) < p$.

Proof: It follows from Theorem 4.7, Theorem 3.C and Theorem 3.D.

In the next theorem we get a relationship between vertex set of G, diameter, domination number of graph G and split domination number in $m(G)$.

Theorem 4.9: For each graph G, $\gamma_{sm}(G) + p \geq diam(G) + \gamma(G)$. Equality holds for P_2.

Proof: Let edge set be $A \subseteq E(G)$ has a path which is longest among any two apparent vertices of graph G so that $|A| = diam(G)$. Furthermore, Let D be any dominating set which is minimal in G and D' forms a split minimal set which is dominating in $m(G)$ so that $|D'| + P > |A| + |D|$. This gives $\gamma_{sm}(G) + p > diam(G) + \gamma(G)$.

In the next theorem we obtain a relationship between $E(G), \Delta(G)$& $\gamma_{sm}(G)$.

Theorem 4.10: For each graph G, $\gamma_{sm}(G) < \frac{q\Delta(G)}{\Delta(G)+1}$.

Proof: Let a dominating set be $D = \{v_1, v_2, v_3, \ldots\ldots, v_n\}$ in G and D be a dominating set in G there exists at least one vertex $v \in D$ of maximum degree $\Delta(G)$ in G. Let the edge number be $q = |E|$ in G and cut vertex set be $C \in V(G)$ in G then the set $D_1 \in V(m(G))$ where $D_1 \subseteq E \cup C$ such that $N(D_1) = V(m(G)) - D_1$ is disconnected. Then D_1forms a split minimal set which is dominating in $m(G)$ so that $|D_1| < \frac{q\Delta(G)}{\Delta(G)+1}$. Hence $\gamma_{sm}(G) < \frac{q\Delta(G)}{\Delta(G)+1}$.

In the next theorem we establish a relationship between $\gamma(G), \delta(G)$ and $\gamma_{sm}(G)$.

Theorem 4.11: In G, $\left\lfloor \frac{\gamma_{sm}(G)}{2} \right\rfloor < \gamma(G) + \delta(G)$.

Proof: Let minimal dominating set be $D' = \{v_1, v_2, v_3, \ldots\ldots, v_n\}$ in G and $V' = V - D'$. Now there exists at least a vertex v of minimum degree $\delta(G) \in V'$ in G. Let edge set be $E = \{e_1, e_2, \ldots\ldots e_n\}$ in G and set of cut vertices be $C = \{c_1, c_2 \ldots\ldots c_n\}$ in G. Then $E \cup C \subseteq V(m(G))$. Let dominating minimal set be $D_1 \subseteq E \cup C$ in $m(G)$ & $N(D_1) \in V(m(G)) - D_1$ is disconnected. Then D_1 forms a split minimal dominating set in $m(G)$ and thereexists as a minimum one vertex v of D_1 is adjacent to a vertex of minimum degree such that $\left\lfloor \frac{D_1}{2} \right\rfloor < |D'| + \delta(G)$. Hence $\left\lfloor \frac{\gamma_{sm}}{2} \right\rfloor < \gamma(G) + \delta(G)$.

In the succeeding proof, we obtain a relation for $\gamma_t(G), \gamma_{sm}(G)$& $\Delta(G)$.

Theorem 4.12: For every graph G, $\gamma_{sm}(G) \leq \gamma_t(G) + \Delta(G) + 1$. Equality holds for W_n & K_6.

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Proof: Let minimal dominating set be D in$G$. Let $V_1 = V(G) - D$ and $H \subseteq V_1$ such that $H \subseteq N(D)$ in G, then dominating set $|D \cup H|$ is a total in G. Let $A = \{v_1, v_2, v_3, \ldots\ldots, v_n\}$ be the vertex set in G, thereexists atleast a vertex $v \in A$ utmost $deg(G)$ in G $(D \subseteq A)$. Consider a edge set $E \subseteq E(G)$ in G and $C \in V(G)$ be a cut vertex in G. Let minimal dominating set be $D_1 \subseteq E \cup C$ in $m(G)$ and $N(D_1) \in V(m(G)) - D_1$ is disconnected. Then dominating set D_1 constitute a minimal set in $m(G)$. Thus $|D_1| \subseteq |D \cup H| + \Delta(G) + 1$. Thus $\gamma_{sm}(G) \leq \gamma_t(G) + \Delta(G) + 1$.

5. NORDHAUS-GADDUM TYPE OUTCOMES:

Theorem 5.1: For every graph G,

i)$\gamma_{sm}(G) + \gamma_{sm}(\bar{G}) < 2p$ ii)$\gamma_{sm}(G) \,.\, \gamma_{sm}(\bar{G}) < 2p$

6. CONCLUSION:

Graph properties have a significant role in encryption. A dominating set of a graph is used to break the code easily. The concept of split litact domination is a variant of usual domination. In the information retrieval system, the dominating set and the elements of split litact dominating sets can stand alone to make the process of communication more easy. Also it has wide applications in coding theory, computer science, switching circuits, electrical networks etc.,

In this paper we extend litact dominating set to split litact dominating set of a graph. The accurate values of this new variant is calculated for different graphs and obtained the upper and lower bounds of it with different parameters of a graph. One can extend this work by studying their applications in a wider sense.

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