Static Flexure analysis of Cantilever beam Subjected to UDL using HSDT

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Abstract

In this paper, utilizing another higher request shear distortion hypothesis cantilever beam exposed to udl is examined. The axial displacement, transverse removal, axial bending stress and transverse shear pressure are resolved for given cantilever bar for various boundary conditions. The outcomes are gotten by understanding numerical for different length to thickness proportions of the beams and that acquired outcomes are contrasted and other shear deformation theory.

Keywords: Higher order shear deformation theory, Isotropic beam, virtual work, Shear deformation, thick beam, static flexure, transverse shear stress etc.

I. INTRODUCTION

The isotropic thick beams are broadly utilized in rapid vehicle, aviation structure car engineering and so forth. The examinations of thick beam are finished by utilizing shear distortion hypotheses. The bar hypotheses are delegated ETB, FSDT and HSDT. Detail audit of uprooting based shear miss happening speculations for beams is introduced. ETB is followed for the investigation thick pillars, avoidances are thought little of and common frequencies and clasping loads are overestimated. The first right limit conditions for the Timoshenko bar were inferred by Kruszewski E. T. [1] and Dengler M. A. also, Goland M. [2] and further it was very much talked about by Dym and Shames [3]. In Timoshenko bar hypothesis transverse shear strain circulation is steady through the pillar thickness sand in this way requires shear amendment factor to address the strain vitality of disfigurement. Mindlin R.D. what's more, Deresiewicz

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Vol. 13, No. 3s, (2020), pp. 304–313

H. [4] calculated this factor for variety of cross sections of beams. Cowper G. R. [5, 6] and Murty A. V. K. [7,8]have given new expressions for this coefficient for different cross-sections of the beam. History of shear coefficient is given by Kanek T. [9] and fundamentally analyzed by Hutchinson J. R. furthermore, Zillmer S. D. [10], Hutchinson [11]. Further conversation on the shear coefficients in shaft twisting is introduced by Rychter Z. [12]. Stephen N. G. what's more, Levinson M. [13] have presented a refined hypothesis consolidating shear arch, transverse direct pressure and rotatory latency impacts. The overseeing differential condition is comparative in structure to the Timoshenko beam equation. However, the hypothesis requires two coefficients, one for cross sectional twisting and the second subject to the transverse direct anxieties. These coefficients for different cross sections are assessed.

II. MATHEMATICAL FORMULATION

A cantilever light emission "L", width "b", and profundity "h" are exposed to udl stacking as appeared in fig 1. Flexural investigation of the given shaft is done by utilizing HSDT. Results for hub removals, hub twisting pressure, and transverse shear pressure are acquired and spoken to graphically.

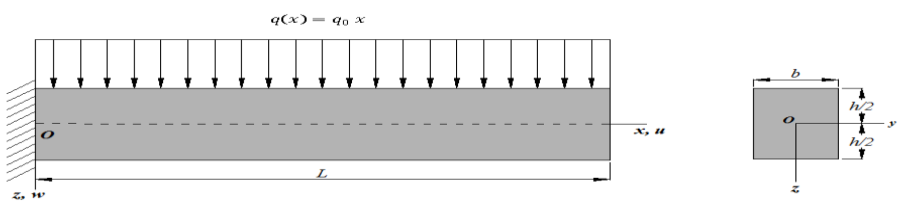

Fig. 1Cantilever beam with uniformly distributed load

The Displacement field-

In light of the previously mentioned presumptions, the uprooting field of the current bar hypothesis can be communicated as follows.

$$u(x,z) = -z\frac{\partial w}{\partial x}(x) + \left[\frac{z}{2}\left(\frac{h^2}{4} - \frac{z^2}{3}\right)\right]\phi(x) \quad (1)$$

$$w(x,z) = w(x) \quad (2)$$

Where,

u = Axial relocation in x bearing which is capacity of x and z.

w = Transverse removal in z course which is capacity of x.

ϕ = Rotation of cross area of beam at impartial pivot which is capacity of x.

Normal Strain:

$$\in_x = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} + \left(\frac{zh^2}{8} - \frac{z^3}{6}\right)\frac{\partial \phi}{\partial x} \quad (3)$$

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Shear strain

$$\gamma_{xz} = \left[\frac{h^2}{8} - \frac{3z^2}{6} \right] \phi(x) \quad (4)$$

Stresses:

$$\sigma_x = E \in_x = -zE \frac{\partial^2 w}{\partial x^2} + E \left[\frac{zh^2}{8} - \frac{z^3}{6} \right] \frac{\partial \phi}{\partial x} \quad (5)$$

$$\tau_{xz} = G\gamma_{xz} = G \left[\frac{h^2}{8} - \frac{3z^2}{6} \right] \phi \quad (6)$$

Where E and G after elastic constant of the beam material.

C. Governing differential equation

Governing differential conditions and limit conditions are acquired from Principle of virtual work. Utilizing conditions for stresses, strains and guideline of virtual work, variationally steady differential conditions for shaft viable are acquired. The rule of virtual work when applied to shaft prompts:

$$b \int_{x=0}^{x=L} \int_{z=-h/2}^{z=h/2} \left(\sigma_x . \delta \in_x + \tau_{xz} . \delta \gamma_{xz} \right) dxdz - \int_{x=0}^{x=L} q \delta w dx = 0 \quad (7)$$

Where δ = variational operator

Utilizing Green's hypothesis in condition (7) progressively we acquire the coupled Euler Lagrange's conditions which are the overseeing differential conditions and related limit states of the shaft. The overseeing differential conditions got are as per the followin

$$EI \left[\frac{\partial^4 w}{\partial x^4} - A_0 \frac{\partial^3 \phi}{\partial x^3} \right] = q(x) \quad (8)$$

$$EI \left[A_0 \frac{\partial^3 w}{\partial x^3} - B_0 \frac{\partial^2 \phi}{\partial x^2} \right] + GAC_0 \phi = 0 \quad (9)$$

Where A_0, B_0 and C_0 are the firmness coefficients in overseeing conditions. The related steady regular limit conditions got are of following structure along the edges x = 0 and x = L.

$$V_x = EI \left[\frac{\partial^3 w}{\partial x^3} - A_0 \frac{\partial^2 \phi}{\partial x^2} \right] = 0 \quad (10)$$

Where w is prescribed

$$M_x = EI \left[\frac{\partial^2 w}{\partial x^2} - A_0 \frac{\partial \phi}{\partial x} \right] = 0 \quad (11)$$

Where $\frac{dw}{dx}$ is prescribed.

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Vol. 13, No. 3s, (2020), pp. 304–313

$$M_x = EI\left[A_0\frac{\partial^2 w}{\partial x^2} - B_0\frac{\partial \phi}{\partial x}\right] = 0 \quad (12)$$

Where ϕ is Prescribed.

D. The General solution of Governing equilibrium equations of beam:

The general answer for transverse uprooting w(x) and (x) can be acquired from Eqn. (8) and (9) by disposing of the terms containing time (t) subsidiaries. Coordinating and adjusting the Eqn. (9), we got the accompanying condition,

$$\frac{\partial^3 w}{\partial x^3} = A_0\frac{\partial^2 \phi}{\partial x^2} + \frac{Q(x)}{D} \quad (13)$$

where, Q(x) is summed up shear power for bar and it is given by

$$Q(x) = \int_0^x q\,dx + k1 \quad (14)$$

Also, by revising second overseeing Eqn. (9) the accompanying condition is gotten.

$$\frac{\partial^3 w}{\partial x^3} = \frac{B_0}{A_0}\frac{\partial^2 \phi}{\partial x^2} - B_0 \quad (15)$$

Presently a solitary condition regarding is gotten, by putting the Eqn. (6) in second overseeing Eqn. (15)

$$\alpha\left(\frac{\partial^2 \phi}{\partial x^2}\right) - \beta(\phi) = \frac{Q(x)}{EI} \quad (16)$$

$$\phi = k_2\cosh(\lambda x) + k_3\sinh(\lambda x) - \left(\frac{Q(x)}{\beta EI}\right) \quad (17)$$

The condition of transverse relocation w(x) is gotten by subbing the statement of ϕ(x) in Eqn. (15) and

$$EIw(x) = \int\int\int\int\int q\,dxdxdxdx + \frac{D}{\lambda^3}\left(\frac{B_0}{C_0}\lambda^2 - \beta\right)\left(k_2\sinh\lambda x + k_3\cosh\lambda x\right) + \frac{k_1 x^3}{6} + k_4\frac{x^2}{2} + k_5 x + k_6 \quad (18)$$

incorporating it threefold as for x. The general answer for w(x) is acquired as follows:

where k_1, k_2, k_3, k_4, $k_5$and k_6 are the constants of mix and can be acquired by forcing regular (constrained) and kinematic limit states of beams.

Limit conditions related with this issue are as per the following:

At free end: x=L

$$\mathrm{EI}\frac{\partial^2 w}{\partial x^2} = \mathrm{EI}\frac{\partial \phi}{\partial x} = \mathrm{EI}\frac{\partial^3 w}{\partial x^3} = \mathrm{EI}\frac{\partial^2 \phi}{\partial x^2} = 0$$

At fixed end: x=0

$$\mathrm{EI}\frac{\partial w}{\partial x} = \mathrm{EI}\phi == \mathrm{EI}w = 0$$

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Utilizing general answers for w(x) and (x) from Eqn. (9) and (10) the total answer for a shaft is gotten by forcing common (constrained) and geometric or kinematical end states of pillar as referenced in Eqn. (11) through Eqn. (13).The last articulations for transverse removal w(x) and ϕ(x) acquired from this arrangement are as per the following:

General Expression for $\phi_{(x)}$ and w (x)

$$\phi(x)=\frac{A_0}{C_0}\frac{q_0 L}{Gbh}\left[\sinh\lambda x-\cosh\lambda x+1-\frac{x}{L}\right] \tag{19}$$

$$w(x)=\frac{q_0L^4}{10Ebh^3}\left\{\left[\frac{5x^4}{L^4}-\frac{20x^3}{L^3}+\frac{30x^2}{L^2}\right]+10\frac{E}{G}\frac{h^2}{L^2}\frac{A_0^2}{C_0}\left(\frac{\cosh\lambda x-\sinh\lambda x-1}{\lambda L}+\frac{x}{L}\right)\right\} \tag{20}$$

$$\bar{w}(x)=\left\{\left[\frac{5x^4}{L^4}-\frac{20x^3}{L^3}+\frac{30x^2}{L^2}\right]+10\frac{E}{G}\frac{h^2}{L^2}\frac{A_0^2}{C_0}\left(\frac{\cosh\lambda x-\sinh\lambda x-1}{\lambda L}+\frac{x}{L}\right)\right\} \tag{21}$$

Expression for axial displacement, (u)

$$u=-z\frac{q_0L^3}{10Ebh^3}\left\{\begin{matrix}\left[\frac{2x^3}{L^3}-\frac{6x^2}{L^2}+\frac{6x}{L}\right]+\\ \frac{E}{G}\frac{h^2}{L^2}\frac{A_0^2}{C_0}\left(\sinh\lambda x-\cosh\lambda x+1\right)\end{matrix}\right\}+\left[\begin{matrix}\frac{A_0}{C_0}\frac{E}{G}\frac{L}{h}\left(\frac{zh}{8}-\frac{z^3}{6h^3}\right)\\ \left(\sinh\lambda x-\cosh\lambda x+1-\frac{x}{L}\right)\end{matrix}\right] \tag{22}$$

$$\bar{u}=-\frac{z}{h}\frac{L^3}{h^3}\left\{\begin{matrix}\left[\frac{2x^3}{L^3}-\frac{6x^2}{L^2}+\frac{6x}{L}\right]+\\ \frac{E}{G}\frac{h^2}{L^2}\frac{A_0^2}{C_0}\left(\sinh\lambda x-\cosh\lambda x+1\right)\end{matrix}\right\}+\left[\begin{matrix}\frac{A_0}{C_0}\frac{E}{G}\frac{L}{h}\left(\frac{zh}{8}-\frac{z^3}{6h^3}\right)\\ \left(\sinh\lambda x-\cosh\lambda x+1-\frac{x}{L}\right)\end{matrix}\right] \tag{23}$$

Expression for axial stress, ($\bar{\sigma}_x$)

$$\sigma_x=-\frac{z}{h}\frac{q_0}{b}\frac{L^2}{h^2}\left\{\left[\begin{matrix}\left(\frac{6x^2}{L^2}-\frac{12x}{L}+6\right)+\frac{E}{G}\frac{h^2}{L^2}\frac{A_0^2}{C_0}\\ \left(\lambda\mathrm{L}\cosh\lambda x-\lambda L\sinh\lambda x\right)\end{matrix}\right]+\right\}+\left[\begin{matrix}\frac{A_0}{C_0}\frac{E}{G}\left(\frac{zh}{8}-\frac{z^3}{6h}\right)\\ \left(\lambda\mathrm{L}\cosh\lambda x-\lambda L\sinh\lambda x-1\right)\end{matrix}\right] \tag{24}$$

$$\bar{\sigma}_x=-\frac{z}{h}\frac{L^2}{h^2}\left\{\left[\begin{matrix}\left(\frac{6x^2}{L^2}-\frac{12x}{L}+6\right)+\frac{E}{G}\frac{h^2}{L^2}\frac{A_0^2}{C_0}\\ \left(\lambda\mathrm{L}\cosh\lambda x-\lambda L\sinh\lambda x\right)\end{matrix}\right]+\right\}+\left[\begin{matrix}\frac{A_0}{C_0}\frac{E}{G}\left(\frac{zh}{8}-\frac{z^3}{6h}\right)\\ \left(\lambda\mathrm{L}\cosh\lambda x-\lambda L\sinh\lambda x-1\right)\end{matrix}\right] \tag{25}$$

Expression for transverse shear stress using constitutive relationship ($\bar{\tau}_{zx}^{CR}$)

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$$\tau_{zx}^{CR} = \frac{A_0}{C_0}\frac{q_0}{b}\frac{L}{h}\left(\sinh\lambda x - \cosh\lambda x + 1 + \frac{x}{L}\right)\left[\left(\frac{h^2}{8}\right) - \frac{3z^2}{6}\right] \quad (26)$$

$$\bar{\tau}_{zx}^{CR} = \frac{A_0}{C_0}\frac{L}{h}\left(\sinh\lambda x - \cosh\lambda x + 1 + \frac{x}{L}\right)\left[\left(\frac{h^2}{8}\right) - \frac{3z^2}{6}\right] \quad (27)$$

Expression for transverse shear stress ($\bar{\tau}_{zx}^{EE}$) obtained from equilibrium equation ,

$$\tau_{zx}^{EE} = \frac{1}{8}\frac{q_0}{b}\frac{L}{h}\left(4\frac{z^2}{h^2} - 1\right)\left[\begin{array}{l}\left(\frac{12x}{L} - 12\right) + \frac{E}{G}\frac{h^2}{L^2}\frac{A_0{}^2}{C_0} \\ \left(\lambda^2 L^2 \sinh\lambda x - \lambda^2 L^2 \cosh\lambda x\right)\end{array}\right] + \left\{\frac{A_0}{C_0}\frac{E}{G}\left[\left(\frac{z^2h^2}{16}\right) - \left(\frac{z^4}{24}\right) + \left(\frac{5h^4}{384}\right)\right]\left(\lambda^2 L^2 \sinh\lambda x - \lambda^2 L^2 \cosh\lambda x\right)\right\} \quad (28)$$

$$\overline{\tau_{zx}^{EE}} = \frac{1}{8}\frac{L}{h}\left(4\frac{z^2}{h^2} - 1\right)\left[\begin{array}{l}\left(\frac{12x}{L} - 12\right) + \frac{E}{G}\frac{h^2}{L^2}\frac{A_0{}^2}{C_0} \\ \left(\lambda^2 L^2 \sinh\lambda x - \lambda^2 L^2 \cosh\lambda x\right)\end{array}\right] + \left\{\frac{A_0}{C_0}\frac{E}{G}\left[\left(\frac{z^2h^2}{16}\right) - \left(\frac{z^4}{24}\right) + \left(\frac{5h^4}{384}\right)\right]\left(\lambda^2 L^2 \sinh\lambda x - \lambda^2 L^2 \cosh\lambda x\right)\right\} \quad (29)$$

3. Illustrative Example

So as to demonstrate the proficiency of the current hypothesis, the accompanying numerical models are thought of. The accompanying material properties for beams are utilized.

Material properties:

Modulus of Elasticity E = 210 GPa

Poisson's ratio μ = 0.30

Density = 7800 kg/m^3

Table 1: Non-Dimensional Axial Displacement ($\bar{u}$) at (x = L, z = h/2), Transverse Deflection ($\bar{w}$) at (x = L, z = 0.0), Axial Stress ($\bar{\sigma}_x$) at (x = 0:0, z = h/2), Maximum Transverse Shear Stresses $\bar{\tau}_{zx}^{CR}$ () and ($\bar{\tau}_{zx}^{EE}$) at (x = 0:01L, z = h/2) of Cantilever Beam Subjected to Uniformly Distributed Load for Aspect Ratio 4.

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Model	$\overline{w}$	$\overline{u}$	$\overline{\sigma_x}$	$\overline{\tau_{zx}^{CR}}$	$\overline{\tau_{zx}^{EE}}$
PRESENT HSDT	21.24	70.24	64.17	0.188	165.38
HPSDT	22.16	70.49	64.24	3.20	808.74
TSDT	21.20	70.2	64.17	2.88	163.38
FSDT	18.9	64	48	1.51	5.94
ETB	15	64	48	-	5.94

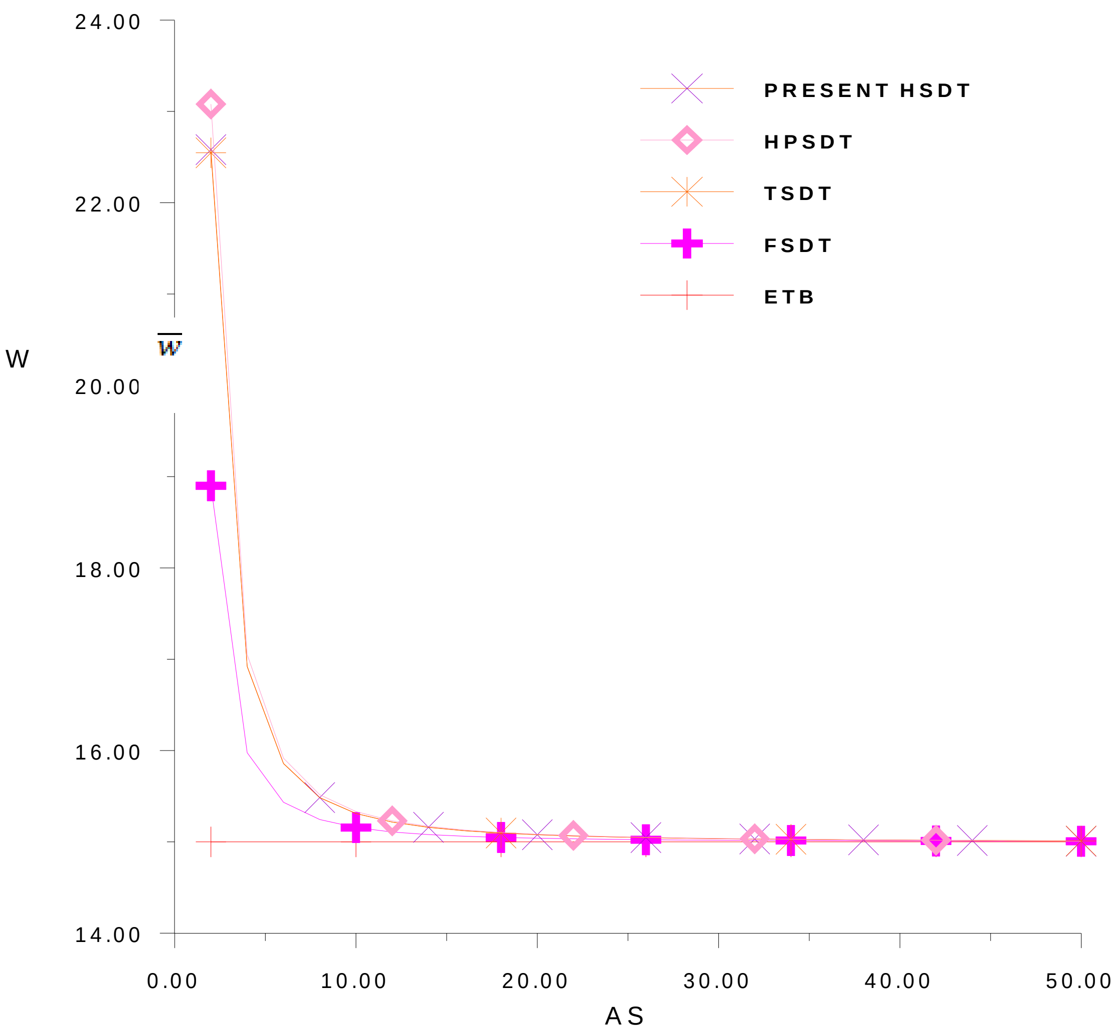

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Fig. No. 2 Assortment of most outrageous transverse evacuation $(\overline{w})$ of cantilever shaft at(x = L, z = 0) when presented to reliably passed on load with perspective extent (AR).

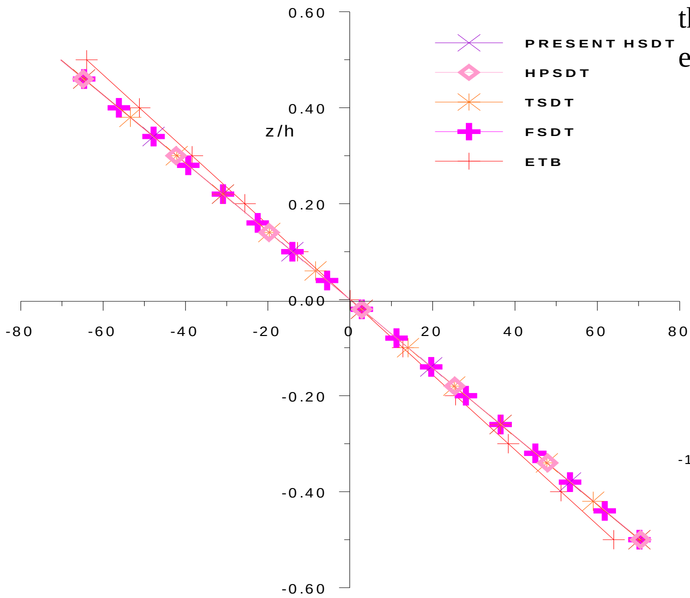

Fig. No. 3 Assortment of center point $\overline{u}$ evacuation through the thickness of cantilever pillar at (x = L, z) when mistreated reliably dispersed burden for AR 4

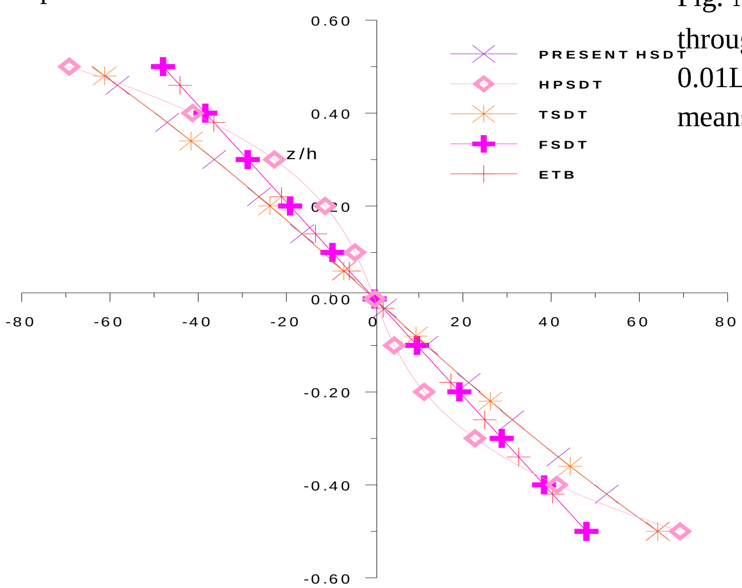

Fig. No. 4 Variety of hub stress $(\overline{\sigma}_x)$ through the thickness of cantilever pillar at (x =0, z) w exposed to UDL for AR 4

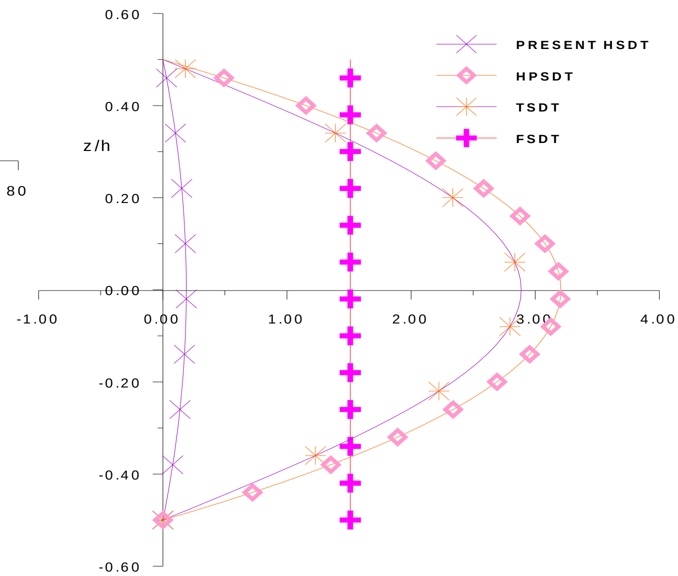

Fig. No. 5 Variety of transverse shear $\overline{\tau}_{ZX}^{CR}$ worry through the thickness of cantilever bar at (x = 0.01L, z) when exposed to udl and acquire by means of constitutive connection for AR

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4.

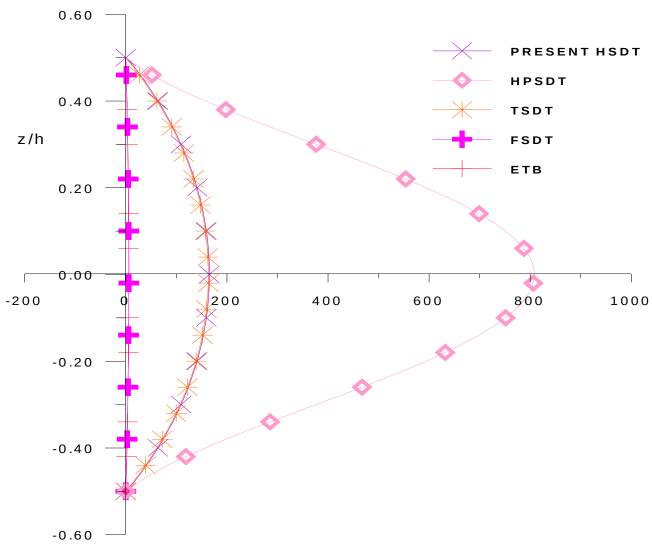

Fig. No. 6 Variety of transverse shear $\bar{\tau}_{zx}^{EE}$ worry through the thickness of cantilever bar at (x = 0.01L, z) when exposed to udl and acquire by means of constitutive connection for AR 4.

IV.CONCLUSIONS

From the static flexural analysis of Cantilever beam following conclusion are drawn:

1. The after effect of most extreme transverse dislodging acquired by present hypothesis is in great concurrence with those of other comparable refined and hyperbolic speculations. The variety of transverse removal with viewpoint proportion is shown in fig.- 2.

2. From Fig. 3 it tends to be seen that, the pivotal dislodging changes straightly through thickness of bar for angle proportion 4.

3. The most extreme pivotal worries for perspective ratio 4 changes straightly through the thickness of bar as appeared in Fig 4.

4. The transverse shear stresses is acquired straightforwardly by balance condition and is gotten legitimately by constitutive connection.

5. From fig.5 and fig.6. Shows the through thickness variety of transverse shear worry for thick beam for viewpoint proportion 4 and it is seen that transverse shear pressure fulfill the zero condition at top and base surface of the shaft.

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