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Co-Even Domination In Graphs

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Abstract

The aim of this paper is to introduce a new domination parameter in the graphs it is called co-even domination number denoted by $\gamma_{coe}\ (G)$. We will touch only a few aspects of the theory to this definition. Some properties and boundaries of this definition are introduced. Furthermore, some properties for some standard graphs and complement graphs are discussed, such as path, cycle, complete, complete bipartite, star, regular, and wheel.
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1. Introduction

Through this work, all graphs are considered finite, simple, and undirected. On a graph $G = (V, E)$ with vertex set $V\ (G)$ and edge set $E(G)$. For each vertex $v \in V(G)$, the set $N_G(\mathrm{v}) = \{u \in V(G): uv \in E(G)\}$ refers to the open neighborhood of v and the set $N_G[v] = N_G(\mathrm{v}) \cup \{v\}$ refers to the closed neighborhood of v in G. The degree of v, denoted by $deg(v)$, is the cardinality of $N_G(v)$. An isolated, a pendant vertex is a vertex of degree zero and one respectively. The minimum and maximum degree (G) and $\Delta(G)$, respectively. In case where $\Delta(G) = \delta(G)$, G is called a regular graph [4], [6]. For new types of parameter of graph domination and refer to [1-3,8-10].

A subset D of V is called a dominating set of G if every vertex not belong to the set D is adjacent to at least one vertex u: u ∈ D. A dominating set D of G is minimal if has no proper a dominating set in G. MDS(G)refers to all minimal dominating sets of a graph G. If $\mathrm{D} = \min\{|\mathrm{D_i}|, \mathrm{D_i} \in \mathrm{MDS(G)}\ \}$ where $|\mathrm{D_i}|$is the cardinal of the set $\mathrm{D_i}$, then D is called the domination number of G and is denoted by $\gamma(\mathrm{G})$ [5], [7].

Now, a new domination parameter in the graphs it is called co-even domination number denoted by $\gamma_{coe}\ (G)$ is definition below. Some properties for some standard graphs of co-even domination are been discussed. Also, some bounds for $\gamma_{\mathrm{coe}}(\mathrm{G})$ are obtained and characterized the graphs obtaining those bounds.

2. Co-even Domination number

Definition 2.1. Let G be a graph and D is a dominating set, the set D is called co-even dominating set if, $\deg(v)$ is even number for all $v \in V - D$.

Definition 2.2. Consider G be a graph and D is a co-even dominating set ,then D is called a minimal co-even dominating set if has no proper subset $\acute{D} \subseteq$ D is a co-even dominating of G . Take us MCEDS(G) refers to all minimal co- even dominating sets of a graph G.

Definition 2.3. The set D is called the co-even domination number if

$D = min\{|D_i|, D_i \in MCEDS(G)\ \}$, and is denoted by $\gamma_{\mathrm{coe}}\ (G)$.

Proposition 2.4. Let $G = (n, m)$ be a graph and D is a co-even dominating set, then

1. All vertices of odd or zero degrees belong to every co-even dominating set.

2. $deg(v) \geq 2, forall\ v \in V - D$

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3. If G is r- regular graph then $\gamma_{coe}(G) = \begin{Bmatrix} n, if\ r\ is\ odd \\ \gamma(G), if\ r \text{ is even} \end{Bmatrix}$.

4. $\gamma\ (G)\ \leq \gamma_{coe}\ (G)$.

Proof. It is clear that from the definition of co-even dominating set. □

Proposition 2.5. If G be a graph of order n, then $1 \leq \gamma_{coe}(G) \leq n$.

Proof. If graph G has a vertex of degree $n-1$ and vertices of even degree to each other then, in this case, the lower bound is obtained. The upper bound has appeared when the degrees of all vertices are odd or zero, according to proposition 2.4(1).

Proposition 2.6. If $G = (n, m)$is a graph has co-even domination number γ_{coe}, then $(n-\gamma_{coe}) + \left\lceil \frac{n-\gamma_{coe}}{2} \right\rceil \leq m \leq \frac{n(n-1)}{2}$.

Proof. Let D be a γ_{coe}- set, then all the vertices in $V - D$ have at least one edge that joins each of them with D. Therefore, there are at least $n - \gamma_{coe}$ edges. Furthermore, these vertices must be of even degree, so the minimum number of edges to add to $n - \gamma_{coe}$ is $\left\lceil \frac{n-\gamma_{coe}}{2} \right\rceil$. This case occurs when the induced subgraph generated by $V - D$ ($\langle V - D \rangle$) is isomorphic to the match graph. This means that the edges are independent in $\langle V - D \rangle$ when $n - \gamma_{coe}$ is even. Now, if $n - \gamma_{coe}$ is odd, similarly, the minimum number of edges occurs when every two vertices are joined pairwise. One vertex remains, therefore, the minimum number of edges will be obtained when this vertex joins with another vertex in D, since if we join this vertex with another vertex, let's say w in $V - D$ the degree of w becomes odd. Therefore, that we must add a new edge. Thus, the lower bound of edges is $(n - \gamma_{coe}) + \left\lceil \frac{n-\gamma_{coe}}{2} \right\rceil$.

It is clear that the upper limit is obtained when the graph is completed. Thus, the result is obtained.

Observation 2.7.

1. $\gamma_{coe}(C_n) = \left\lceil \frac{n}{3} \right\rceil, where\ C_n\ is\ a\ cycle\ of\ order\ n.$

2. $\gamma_{coe}(K_n) = \begin{Bmatrix} 1, if\ n \equiv 1\ (mod\ 2) \\ n, if\ n \equiv 0\ (mod\ 2) \end{Bmatrix}.,where\ K_n\ is\ a\ complete\ of\ order\ n;\ n \geq 3$

Proof. By proposition 2.4(3). □

Proposition 2.8. If G is a star graph or wheel graph of order n,($n \geq 3, n \geq 4$ respectively) , then $\gamma_{coe}(G) = \begin{Bmatrix} n-1, if\ n \equiv 1\ (mod\ 2) \\ n, \quad if\ n \equiv 0\ (mod\ 2) \end{Bmatrix}$.

Proof. Let G be a star or wheel graph of order n, then by proposition 2.4(2), all pendant vertices in star and all vertices that lie on the cycle of the wheel belong to every co-even dominating set D, since that the degree of all these vertices is odd. Therefore, the remaining vertex of each graph is the center. So, two cases are discussed as follows.

Case 1. If $n \equiv 1\ (mod\ 2)$, then the degree of center vertex is even. Therefore, we place this vertex in set $V - D$. Thus, $\gamma_{coe}(G) = n - 1$.

Case 2. If $n \equiv 0\ (mod\ 2)$, then the degree of center vertex is odd. Again, by Proposition 2.4(2), this vertex belongs to every co-even dominating set. Thus, $\gamma_{coe}(G) = n$. □

Proposition 2.9. If G is a complete bipartite graph $K_{m,n}$, then $\gamma_{coe}(G) = \begin{Bmatrix} 2, \quad if\ n\ and\ m\ are\ even \\ n,\ \ if\ n\ \ is\ even\ and\ m\ is\ odd \\ n+m,\ \ if\ both\ n\ and\ m\ are\ odd \end{Bmatrix}$.

Proof. Suppose that V_1 and V_2 are the bipartite sets of the graph G of order n and m, respectively. Then there are three cases that depend on n and m as follows.

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Case1. If n *and* m are even, then let $u \in V_1$ and $v \in V_2$. It is clear that u *and* v dominate all other vertices in the graph as well, all other vertices with even degree. We cannot dominate this graph by a vertex. Therefore, $\gamma_{coe}(G) = 2$.

Case 2. If m is odd and n is even, then, according to Proposition 2.4(1) all the vertices in set V_1 belong to every co-even dominating set. Therefore,$\gamma_{coe}(G) = n$, since all vertices in the set V_2 have even degree.

Case 3. If n and m are odd, then by proposition 2.4(1) all vertices in the sets V_1 and V_2 belong to each co-even dominating set. Thus, $\gamma_{coe}(G) = m + n$.

Therefore, from all the previous cases, the result is obtained. □

Proposition 2.10. If G is a path graph of order n, then

$$\gamma_{coe}(G) = 2 + \left\lceil \frac{n-4}{3} \right\rceil.$$

Proof. Let $v_1, v_2, \dots, v_n$ be the vertices of the path. By proposition 2.4(1) the two pendant vertices v_1 and v_n lie in each co-even dominating set. These vertices dominate the support vertices v_2 and v_{n-1}. Now, let $D_1 = \left\{ v_{4+3k}, k = 0,1, \dots, \left\lceil \frac{n-4}{3} \right\rceil - 1 \right\}$. It is clear that D_1 is a minimum co-even dominating set to the induced subgraph $\langle v_3, v_4, \dots, v_{n-2} \rangle$, Thus, $\gamma_{coe}(G) = 2 + \left\lceil \frac{n-4}{3} \right\rceil$.

3. Co-even dominating set in the complement of a graph.

Proposition 3.1. If G is r- regular graph then $\gamma_{coe}(\bar{G}) = \begin{cases} n, if\ r\ is\ odd(even) and\ n\ is\ odd(even) \\ \gamma(G), if\ r\ is\ odd(even) and\ n\ is\ even(odd) \end{cases}$

Proof. There are four cases that depend on r and n as follows.

Case 1. If r and n are odd or even together, then the degree of each vertex in $\bar{G}$ is odd. Thus, all vertices belong to co-even dominating set according to Proposition 2.4(1), so $\gamma_{coe}(\bar{G}) = n$.

Case 2. If r is odd (even) and n is even (odd), respectively, then the degree of each vertex in $\bar{G}$ is even. Therefore, $\gamma_{coe}(\bar{G}) = \gamma(G)$. Therefore, from the two previous cases, the result is obtained.

Proposition 3.2. if G is a path graph of order n, then

$$\gamma_{coe}(\overline{P_n}) = \begin{cases} n-2, & if\ n\ is\ even \\ 2, & if\ n\ is\ odd \end{cases}.$$

Proof. Two cases are discussed as follows.

Case 1. If n is even, then the degree of all vertices in $\overline{P_n}$ are odd, except for the pendant two vertices. Therefore, according to proposition 2.4(1) $\gamma_{coe}(\overline{P_n}) = n - 2$.

Case 2. If n is odd, then the degree of all vertices in $\overline{P_n}$ are even except for the pendant two vertices. Also, the pendant vertices in P_n have dominated all vertices in $\overline{P_n}$. Therefore, $\gamma_{coe}(\overline{P_n}) = 2$.

Therefore, from the two previous cases, the result is obtained. □

Proposition 3.3. . If G is a cycle graph of order n, then

$$\gamma_{coe}(\overline{C_n}) = \begin{cases} n, & if\ n\ is\ even\ or\ n = 3 \\ 2, & if\ n\ is\ odd; n \neq 3 \end{cases}.$$

Proof. Three cases are discussed as follows.

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Case 1. If $n = 3$, then $\overline{C_3} \equiv \overline{K_3}$, therefore $\gamma_{coe}(\overline{C_n}) = 3$, according to proposition 2.4(1).

Case 2. If n is even, then the degree of all vertices in $\overline{C_n}$ are odd. Therefore, according to proposition 2.4(1) $\gamma_{coe}(\overline{C_n}) = n$.

Case 3. If n is odd; $n \neq 3$, then the degree of all vertices in $\overline{C_n}$ are even. Also, any adjacent vertices in C_n have dominated all vertices in $\overline{C_n}$. Therefore, $\gamma_{coe}(\overline{C_n}) = 2$.

Therefore, from the three previous cases, the result is obtained. □

Proposition 3.4. If G is a wheel graph of order n, then

$$\gamma_{coe}(\overline{W_n}) = \begin{Bmatrix} n, if\ n\ is\ odd\ or\ n = 4 \\ 3,\ \ if\ n\ is\ even; n \neq 4 \end{Bmatrix}.$$

Proof. Three cases are discussed as follows.

Case 1. If $n = 4$, then $\overline{W_4} \equiv \overline{K_4}$, therefore $\gamma_{coe}(\overline{W_n}) = 4$, according to proposition 2.4(1).

Case 2. If n is odd, then the degree of all vertices in $\overline{W_n}$ are odd except the center vertex which degree is zero. Therefore, according to proposition 2.4(1) $\gamma_{coe}(\overline{W_n}) = n$.

Case 3. If n is even; $n \neq 4$, then the degree of all vertices in $\overline{W_n}$ are even except the center vertex which degree is zero. Therefore, by using Proposition 3.3. we get $\gamma_{coe}(\overline{C_{n-1}}) = 2$, so, $\gamma_{coe}(\overline{W_n}) = 3$.

Therefore, from the three previous cases, the result is obtained. □

Proposition 3 .5. If G is a star graph of order n, then

$$\gamma_{coe}(\overline{S_n}) = \begin{Bmatrix} 2, & if\ n\ is\ even \\ n, & if\ n\ is\ odd \end{Bmatrix}.$$

Proof. If $G \equiv S_n$, then $\overline{S_n} \equiv K_{n-1} \cup K_1$. Therefore, by using Proposition 2.4(2), the result is obtained. □

Observation 3.6. If G is a complete graph of order n, then $\gamma_{coe}(\overline{K_n}) = n$.
proof. It is obvious. □

Proposition 3.7. If G is a complete bipartite graph of order nm, then

$$\gamma_{coe}(\overline{K_{m,n}}) = \begin{Bmatrix} 2, & if\ m\ and\ n\ are\ odd \\ m+1, & if\ m\ is\ even\ and\ n\ is\ odd \\ m+n, & if\ m\ and\ n\ are\ even \end{Bmatrix}$$

Proof. It is clear that $\overline{K_{m,n}} \equiv K_m \cup K_n$, then, by using Observation 2.7(2), the result is obtained.

4. Conclusion

Through, this paper, a new domination parameter in the graphs it is called co-even domination number is introduced. Many of the results of this number are obtained especially to certain graphs as a path, cycle, star, wheel, complete, and complete bipartite.

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