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Non-uniform heat sink/source and heat flux effects on Casson-fluid with an inclined magnetic field

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Abstract

The study deals with the slip effect on Casson-fluid flow over a stretching-sheet with inclined magnetic field, the radiations from the thermal sources and the heat generated through sink/source. The base boundary-layer equations that governs are modified into normal differential equations by making use of the similarity transformation and are analytically solved using hypergeometric function. The results obtained are projected using various graphs and appropriate tables for different flow characters. It is identified that the rate in which the heat is transferred increases with the increase in inclined magnetic field. Also found that the fluid temperature in the boundary-layer region rises for increasing the Casson-parameter.

Keywords: *Casson-Fluid, Heat Flux, magnetic Field, Sink/Source and Slip-flow.*

1. Introduction

In recent years, researches on casson-fluid issue that has different physical efforts are keenly focused owing to the wide range of its applications in the nuclear reactors, polymer manufacturers , medical domain, etc[1-7]. The classical examples of these kind of fluid are honey, jelly and soups from tomato., etc. Human blood also falls in this kind. It comes under the non-Newtonian models of fluids. It was initially introduced by casson in 1995. Recent researches include various effects on the magnetic field of the boundary layer flows. with different geometries surface in electrically conducting fluids [8-15] due to its numerous applications in geophysics, astrophysics, magnetic drug targeting, sensors and engineering. Keeping in this mind the objective of the present study is to investigate the effects of non-uniform heat sink/source, thermal-radiation and inclined-magnetic field over a stretching sheet on Casson-fluid with slip boundary condition. The governing equation are solved analytically using hyper geometric function and results are discussed with the help of the graphs for various physical parameters..

2. Formulations of the problem

Taking into consideration of a steady and incompressible 2D linear boundary-layer slip's flow in case of casson-fluid in a stretching sheet that has an applied acute angle γ aligned magnetic-field B_0 along *y*- direction. If $\gamma = 90^o$ then $\sin 90^o = 1$ It becomes a transverse magnetic field.

2.1 Flow Analysis

The equations that governs the given scenario is given as $v_y + u_x = 0$ **(1)**

$$u_x u + u_y v = \left(1 + \frac{1}{\beta}\right) \vartheta u_{yy} - \frac{\sigma B_0^2}{\rho} u \sin^2 \gamma \tag{2}$$

Where ϑ - kinematic viscosity, *u*and *v* are velocity component of *x*–direction and *y*-direction respectively, β - Casson-fluid parameter ($\beta = \mu_B \sqrt{2\pi_c} / p_y$) electrical conductivity and ρ - fluid density.The velocity field boundary conditions are

If $y \to \infty$ as $u \to 0$, $u = ax + lu_y, v = v_w$ at $y = 0$ (3)

By appllying the given similarity-transformation,

$$u = axf_\eta,\ v = -f(a\vartheta)^{0.5} \text{ and } \eta = \left(\frac{a}{\vartheta}\right)^{0.5} y \tag{4}$$

When the eq(4) is applied in (1) , the trivial satisfication is obtained and hence the (2) and (3) will take the form as

$$\left(\frac{1}{\beta} + 1\right) f_{\eta\eta\eta} + f_{\eta\eta} f - f_\eta^2 - Mn\ f_\eta \sin^2 \gamma = 0 \tag{5}$$

With the respective boundary-conditions.

$f_\eta = Lf_{\eta\eta} + 1$ at $\eta = 0$,

If $\eta \to \infty$ then $f_\eta \to 0$ (6)

The differentiation with respect to η is denoted by subscript η. Here $\left(Mn = \frac{\sigma B_0^2}{\rho a}\right)$ magnetic based parameter and $\left(L = l\sqrt{a/\vartheta}\right)$slip-paramter.

The analytical solution of (5) with boundary-conditions (6) as in the form of,

$$f(\eta) = \left(\frac{1 - e^{-\alpha\eta}}{\alpha}\right) X \tag{7}$$

Where $X = (L\alpha + 1)^{-1}$,

$$\alpha = -\frac{1+\beta}{3(L+L\beta)} - \frac{\alpha_1}{3(2^{2/3})(L+L\beta)(\alpha_2 + \sqrt{(\alpha_2^2 + 4\alpha_1^3)^{1/3}})} + \frac{\alpha_2 + \sqrt{(\alpha_2^2 + 4\alpha_1^3)^{1/3}}}{6(2^{1/3})(L+L\beta)}$$

$$\alpha_1 = -4(1+\beta)^2 + 6(L+L\beta)(-L\,Mn\,\beta + LMn\beta\cos 2\gamma)$$

$$\alpha_2 = -16 - (48 - 216L^2 - 72L^2Mn + 72L^2Mn\cos 2\gamma)\beta$$
$$-(48 - 432L^2 - 144L^2Mn + 144L^2Mn\cos 2\gamma)\beta^2$$
$$-(16 - 216L^2 - 72L^2Mn + 72L^2Mn\cos 2\gamma)\beta^3$$

Using (7) in (4), the velocity components are

$$u = \frac{ax}{L\alpha+1}e^{-\alpha\eta}$$ and $$v = -\sqrt{a\vartheta}\left(\frac{1-e^{-\alpha\eta}}{\alpha(L\alpha+1)}\right)$$

The wall shearing stress for stretching-sheet is given by

$$\tau_w = \left(\vartheta u_y\right)_{y=0} \tag{8}$$

The skin-friction that is obtained locally (the frictioanl-drag) is re-written as

$$C_f = \frac{\tau_w}{\rho u_w^2} = \mathrm{Re}_x^{-0.5}\left(1+\frac{1}{\beta}\right)f_{\eta\eta}(0) \tag{9}$$

Where $\mathrm{Re}_x = \frac{xu_w}{\vartheta}$ Reynolds number.

2.2. Analysis of heat transfer

The thermal Boundary-layers' equation of the casson-fluids which is incpmpressable is given as

$$uT_x + vT_y = \frac{k}{\rho c_p}T_{yy} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{q'''}{\rho c_p} \tag{10}$$

where T is the temperature, $k$denoters the thermal conductivity and

ρ represents the density and $c_p$denotes the specific-heat of the pressure which is kept constant.. The for radiation heat flux approximation Rosseland diffusion is given by

$$q_r = -\frac{4\sigma^*}{3k^*}T_y^4 \tag{11}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. Further, T^4 shall be explained through the taylor series. Thus when expanding T^4 through T_∞ and by the neglection of the terms of higher orders , we tend to get,

$$T^4 \cong 4TT_\infty^3 - 3T_\infty^4$$

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q''' is the temperature and space dependent internal heat absorptionlgenerationwhich can define as

$$q''' = \left(\frac{ku_w(x)}{x\vartheta}\right)\left[A^*\left(T_w - T_\infty\right)f_\eta + B^*\left(T - T_\infty\right)\right] \tag{12}$$

Where the A^* and the B^* are the parameters on the given space and the temperatute which depend on the heat generated/observed internally. It is also to be noted that the A^* and B^* are greater than 0 and A^*, B^* is less than 0 which corresponda to the internal heat which is genrated and the apsorption that takes place internally. The boundary conditions are given by

$$-kT_y = q_w = A\left(\frac{x}{l}\right)^2 \quad \text{at } y = 0,$$

$$T \to T_\infty \quad \text{as} \quad y \to \infty \tag{13}$$

Where l is the characteristic length, T_w and T_∞ denotes the tempreatures on the sheet and the tempreature of the given fluid are kept in a distance from the corresponding sheet. The non-dimensional value of the tempreature is thus defined as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{14}$$

Put (4), (11) and (12) in (10), non-dimensional form of temperature equation as follows

$$\theta_{\eta\eta}\omega + f\,\mathrm{Pr}\,\theta_\eta - 2\,\mathrm{Pr}\,\theta\, f_\eta + A^* f_\eta + B^*\theta = 0 \tag{15}$$

where Prandtl number $\mathrm{Pr} = \mu c_p / k$.

The boundary-conditions in (13) take the form,

$$\theta_\eta(\eta) = -1 \quad \text{at } \eta = 0,$$

$$\theta(\eta) \to 0 \text{ as } \eta \to \infty \tag{16}$$

The solution of (15) represented in-terms of the confluent and hypergeometric functions along

with the boundary-conditions in (16) is

$$\theta(\eta) = c_1 e^{-\alpha\left(\frac{a0+bo}{2}\right)\eta} M\left(\frac{b0 + a0 - 4}{2}, b0 + 1, \frac{-\mathrm{Pr}\,X}{\omega\alpha^2} e^{-\alpha\eta}\right) - c_2 e^{-\alpha\eta} - c_3 e^{-2\alpha\eta} \quad (17) \tag{17}$$

Where $a0 = \frac{\Pr X}{\omega \alpha^2}$, $b0 = \sqrt{a0^2 - \frac{4B^*}{\omega \alpha^2}}$, $c_3 = \frac{A^* X}{\omega \ \alpha^2 \left(1 - a0 + \frac{B^*}{\omega \alpha^2}\right)}$

$$c_1 = \frac{-1 - \alpha c_2 - 2\alpha c_3}{\left[-\alpha \left(\frac{bo + ao}{2}\right) M\left(\frac{b0 + a0 - 4}{2}, b0 + 1, \frac{-\Pr X}{\omega \alpha^2}\right) + \left(\frac{bo + ao - 4}{2(1 - b0)}\right) \frac{\Pr X}{\omega \alpha} M\left(\frac{b0 + a0 - 2}{2}, b0 + 2, \frac{-\Pr X}{\omega \alpha^2}\right) \right]}$$

The non-dimensional wall temperature obtained from (17) as

$$\theta(0) = c_1 M\left(\frac{b0 + a0 - 4}{2}, b0 + 1, \frac{-\Pr X}{\alpha^2}\right) - c_2 - c_3 \qquad (18)$$

3. Results and Discussion

The sink/source effects which are non-uniform in a streched sheet with considerable slip ad with inclined magnetic fields and also thermal effects are discussed. The similarity transformation are then applid for the conversion into non-linear Partial Diffrential Equations to a ODE which is also non-linear. In-order to research on the solutions which are bench-marked, a comparison is made between the present study and the Turkyilmazoglu [15] for some limitations of the fluids that are Newtonian with no aligned angles , slip-velocity and heat which is random and not uniform from the Sink/Source with certain parameters for radiation. The results were found to be exemplary as dpicted in the table 1. Figure 1 depicts the effect on the Casson-parameter and the slip-parameters with velocity having a profile with L = 2, β = 0.5, Mn=1 and $\gamma=45^0$. It is to be noted that both of the parameters considered are decelerated towards the boundary layer. The effect in which the magnetic parameters and the profile of the velocity with L = 2, β = 0.5, Mn=1 and $\gamma=45^0$is depicted in the figure 2. It is here to be seen that the increase in magnetic parameter is diminising the velocity profile owing to the increase of Lorentz force's resistance along the flow. The magnetic fields that are inclined is found to reduce the thickness of the boundary-layer as far as the momentum is considered.

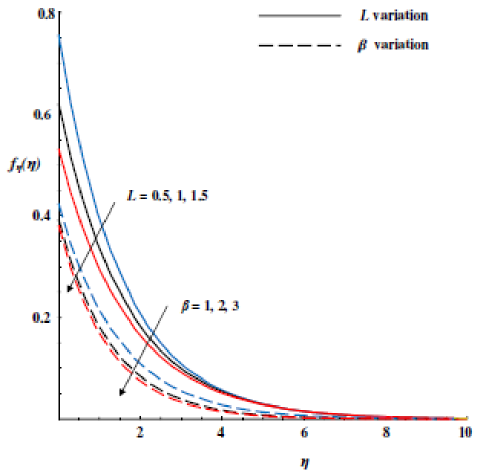

Fig: 1 Effect of Slip and Casson parameters on velocity

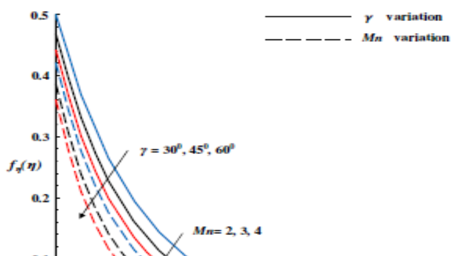

Fig: 2 Effect of angle and magnetic parameters on velocity

Figure 3 projects the effects on the slip and the casson-parameters in the tempreature's profile when the value of with L = 0.2, β = 0.3, N=5, A*=0.1, B*=0.1, Pr= 0.71, Mn=0.4 and $\gamma=45^0$. The presence of the casson-parameters and the slip-parameters leads to the arguemtn of the tempreature profile. The effects of the magnetic aswell as the angle parameters in the profile of the tempreature is presented in Fig.4 where L = 0.2, β = 0.3, N=5, A*=0.1, B*=0.1, Pr= 0.71, Mn=0.4 and $\gamma=45^0$. Owing to the enhanced magentic fields' strength and the Lorentz force which is resistive and found associated with the magnetic field which makes the boundary layer more thinner. The combination of effects from magnetic as well as the aligned paramters thus increases the value of the thermal boundary-layers' thickness.

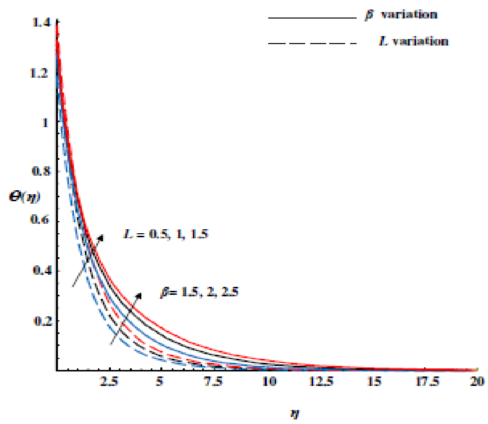

Fig: 3 Effect of Slip and Casson parameters on temperature

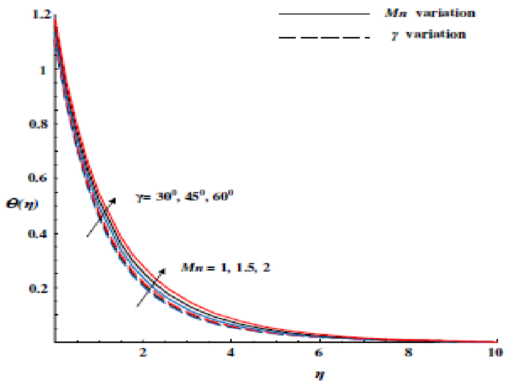

Fig: 4 Effect of magnetic and angle parameters on temperature

The results on the effect that the non-uniform paramter of the sink/source is presented in the Figure 5 which has a profile with L = 0.2, β = 0.3, N=5, A*=0.1, B*=0.1, Pr= 0.71, Mn=0.4 and $\gamma=45^0$.It is understood from the results that if the A^* greater than 0, and also B^* is greater thn 0, the value of the tempreature is increased for a given heat source and then reduced when the values of $A^{*\text{ and}}$ B^* are less than 0 in the heat sink. The combination of effect in case of a sip-paramneter that has a non-uniform heat from sink/source is found to lead towards the condensed thermal boundary layer.

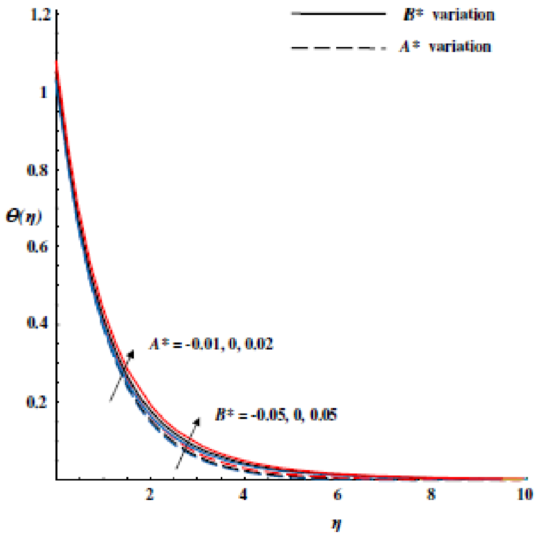

Fig: 5 Effect of A* and B* parameters on temperature

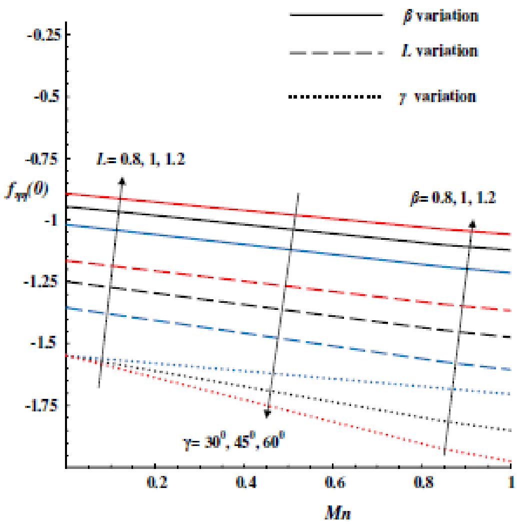

Fig: 6 Effect of magnetic, Slip, angle and Casson parameters on local skin-friction

The combination of the effects of M_n with an aligned angles , the Casson-parameter along with the slip-parameters on the skin-frictions' coefficient obtained locally is shown in the figure 6 that has $f_{\eta\eta}(0)$ are then considered to be the Y-axis and the M_n is taken as the X axis. It is seen that the local skin-friction is decreased with the increasing values of M_n and γ and also the local skin's friction co-efficient's arguments with the slip aswell as Casson-parameters.

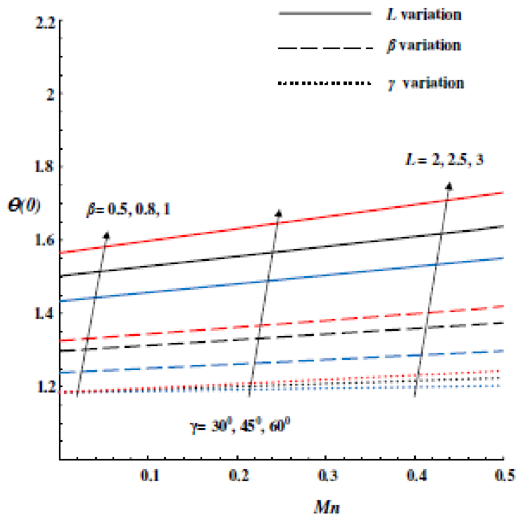

Fig: 7 Effect non-dimensional wall temperature Vs Mn

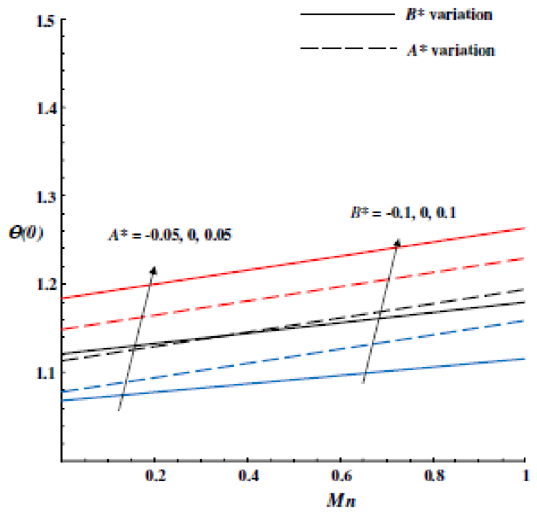

Fig: 8 Effect non-dimensional wall temperature with Mn

The combined results of the magnetic and the Casson – Parameter along with the slip and non-uniform heat from sink/source on a non-dimensional tempereature are depicted in the Figures 7 and 8. In the X-Axis, the magnetic – parameter is then taken and in the Y axis, non-dimensional tempreature is considered. The non-dimensional walls' tempreature is increased with the values of M_n as in figures 7-8. The figure 7 represents the effect on the Casson , Slip and the angle parameters on the local Nu. The increased value of the angle , casson and the slip-based parameters forms the argument of the non-dimensionalwalls' tempreature. In the figure 8, the effect of the non-uniform heat from the sink/source with the parameters A^{*} and B^{*} is shown. The combined effcts of the Mn and the un-even hear from sink/source per parameter are found to increase the non-dimensional tempreatures of the wall.

Table 1. Bench-mark solution for $\theta(0)$.

Mn	Pr	Turkyilmazoglu [15]	current results
0	1	0.75000	0.75000
	5	---------	0.30152
1	1	0.82252	0.82252
	5	---------	0.31179

4. Conclusion

The major results that are obtained in the present research are as described. First, the velocity pertaining to non-Newtonian fluids seems to reduce with the increase in Casson-Parameter with magnetic-field inclined and with velocity slip-parameters. It is also found that the increased values of the magnetic field that is aligned, the Casson parameter and the velocity- slip parameters along with the magnetic field parameters increases the temperature profile. An notable increase in the aligned angle of the magnetic field tend to decelerate the skin-friction but the same increases the dimensional wall temperature. At last, it is also found that he angle belonging to the magnetic field actually controls the strength of the magnetic field

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