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Analysis of Students' Algebraic Thinking Skills and Realistic Mathematics Education Approach to Help Students Learn Function

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Abstract

This research aims to determine algebraic thinking skills of students on learning function and to describe Realistic Mathematics Education Approach as the alternative way to promote students' algebraic thinking skills. The research is conducted at a junior high school involving twenty-seven students by analyzing the results of the students' answers on the problems provided as well as the results of interviews of students. The problems were given to see achievement of students' algebraic thinking's indicators. The indicators are students are able to: a) use relationship patterns to analyze situation; b) move between different representation; c) make and use symbolic, visual or spatial notations, and words or sentences in solving mathematical problems; d) explore problem solving related to other topics or fields of science; e) model and solve the problem. Based on the results of this study, it can be concluded that algebraic thinking skill of students still on low level and need to be improved. Students have to work with problem situations that arise in various contexts that are close to students' daily life or make sense to them. This is in accordance with the characteristics of the Realistic Mathematics Education.

Keywords: *algebraic thinking skill, realistic mathematics education, function*

1. Introduction

Algebra is a branch of mathematics related to the study of the quantity, relationships, and structure that are formed. To learn these things, use symbols to represent something uncertain, in this case variables, parameters, or something unknown. Algebra relates to expressions using symbols and with extended numbers to resolve equations, analyze functional relationships, and to determine the structure of a representative system consisting of expressions and relationship. Activities such as resolving equations, analyzing functional relationships, and determining the structure of a representational system are not the purpose of algebra but rather a tool to model real-world phenomena and solve problems that related to various situations [1].

Algebra can be used as a tool for generalizing and solving various problems, such as advanced mathematical problems, science, business, economics, commerce, computing, and other problems in daily life. The many problems in daily life can be solved by algebra, make it important to learn. Algebra is also important to learn as a provision for the next life, both in work and preparation for further studies.

In general, students still have difficulty in learning algebra [2]. Students with low algebraic thinking skills tend to fail in solving algebraic problems, weak in simplifying algebraic equations and expressions, and have difficulty in interpreting graphs of functions. These difficulties make students learn by memorization without understanding the concept. Thus, students can't solve high-level non-routine problems.

In everyday life, students often find situations related to functional relationships. Therefore, they can bring a lot of relevant knowledge into the classroom. This knowledge can help students reason through algebraic problems. The problems described in the context of everyday life can be solved by more students than the same problem presented only with mathematical equations. However, if the mathematical understanding that students bring to class is not associated with formal algebra learning, then it will not support new learning [3].

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The concept of function has become one of the basic ideas in modern mathematics that underlies almost all disciplines. But it proved to be one of the most difficult concept to master in learning mathematics [4].

In fact, the concept of function is often taught without connecting to everyday contexts. Formal mathematical symbolism often introduced prematurely to students which can lead to misconceptions about functions. When the concept of function is taught using a mathematical context that is not understood by students, students may have difficulty understanding the concept of the function.

When teaching functions, some teachers prefer to start with definitions, rather than providing situations that can lead students to form their own definitions of functions. Or students may also be asked to substitute an integer in the function *f(x)* to find the output of a single real number. Although students can evaluate this value correctly, they have difficulty understanding the essence of the notion of function. Such learning is problematic because it will encourage students to memorize procedures without understanding them, which will make it difficult for students to transfer or associate the concepts learned with new situations.

Learning functions can include the main components of algebraic thinking skills. In a broad sense, algebraic thinking includes a series of understandings that are needed to translate information, or events into mathematical language to explain and predict a phenomenon. Therefore, analysis of students' algebraic thinking skills on learning functions become important for choosing and determining the right approach in teaching functions.

2. Methodology

The research in this study is a qualitative research design. The data were collected from the results of essay test about algebraic thinking skills on function materials and interviews. The test was given to 27 eighth grade students in Jakarta after learning functions. Then, interviews were conducted on 6 students who were selected by purposive sampling to confirm the results of the test and obtain deeper information about students' algebraic thinking skills. Furthermore, the data were analyzed descriptively. The results of the analysis of algebraic thinking skills and literature studies are then used to design appropriate learning approach in teaching function.

3. Results and Discussion

3.1. Students' Algebraic Thinking Skills

In [5] explains that thinking algebra can be interpreted as an approach to quantitative situations emphasizing aspects of general relations with tools that do not necessarily require a letter symbol, but can eventually be used as a cognitive support to introduce and retain more traditional school algebra material. Further, in [5] categorizes the school algebra according to the following activity: a). Generational activities, involving the formation of expressions and equations of algebraic objects.; b) Transformational activities, such as factoring, parsing, substitution, summing and multiplying polynomial expressions, simplifying expressions, working with equivalence expressions and equations; c) Global meta-level activities, activities in which algebra is used as a tool but not limited to algebra, including problem solving, modeling, observing structures, studying changes, generalization, relationship analysis, justifying, evidence, and predicts.

This study adopts algebraic thinking activities proposed by [5] as the basis for describing students' algebraic thinking skills. This is because the opinion already includes the opinions of other experts about the algebraic thinking skills. Indicators for each activity in algebraic thinking skills are presented in Table 1.

Table 1. Indicators of Algebraic Thinking Skills

Activity	Indicators

Generational	1. Students are able to use relationship patterns to analyze situation.
Transformational	2. Students are able to move between different representation. 3. Students are able to make and use symbolic, visual or spatial notations, and words or sentences in solving mathematical problems.
Global Meta-Level	4. Students are able to explore problem solving related to other topics or fields of science. 5. Students are able to model and solve the problem.

To find out the achievement of each indicator by students, several problems about functions have been given. The analysis for each problem is as follows.

3.1.1. Analysis problem 1

1. A student observed the plant on his school yard. He noted its leaf color, root type, and height. The following table shows the results.

Plant	Leaf Color	Root Type	Height (cm)
Guava	green	tap root	217,5
Keladi	green and red	fibrous root	25,7
Puring	green, yellow, and red	tap root	90
Cactus	-	tap root	27,3
Paku	green	fibrous root	3,5

a. Present the relationship between plant with leaf color, plant with root type, and plant with the height, each in the form of other representation.
Hint: You can present the data in the form of a diagram, graph, or other representation that you think is most appropriate.
b. From that data, which relationship is a function and which is not a function? Explain.
c. Explain what you know about the function.

Figure 1. Problem 1

Almost all students represented data on problem 1 with the arrow diagram. Representations of ordered pairs and Cartesian coordinates did not appear on the students' answers. It showed that indicator 2: “students are able to move between different representation” had begun to be achieved but still needed to be improved. Then, there were still some students who error in determining which relationships are functions and which are not functions. Some students had been able to show it correctly. But most of them were still wrong in concluding the definition of function. It showed that indicator 1: “students are able to use relationship patterns to analyze situation” had not been well achieved.

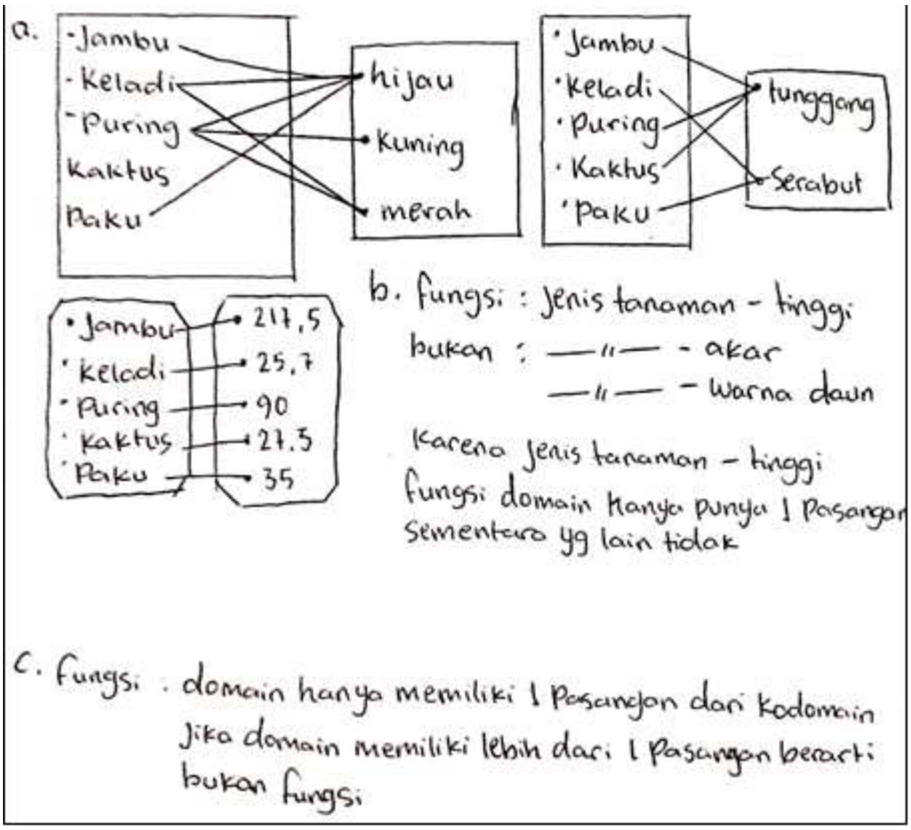

Figure 2. Example of Student's Answer to Problem 1

To follow up on the results of students' answers, interviews were conducted with several students. The interview transcript between researcher (R) and one of student (S) is presented as follows.

R : From the data in problem 1, which relationships are functions?
S1 : Plant and the height.
R : Why?
S1 : Because domain of function only has one pair.
R : What about the relationships that are not functions?
S1 : Plant and the root type as well as plant and the leaf color. Because the domain has more than one pair.
R : Which domain?
S1 : This. (pointed to the arrow diagram that connecting the plant and the leaf color, "puring" and "keladi" have more than one leaf color). And this (pointed to the arrow diagram that connecting the plant and the root type, "jambu", "puring" and "kaktus" have the same root type, that is tap root as well as "keladi" and "paku" have the same root type, that is fibrous root)
R : So, what do you think about function?
S1 : A function is domain area that can only have one pair.

3.1.2. Analysis problem 2

2. The following table shows the height of the plant from week to week.

Weeks	Height (cm)
1	7
2	12

3	17
4	22
5	27

a. Present the data in the form of other representation that you think is most appropriate.
b. Based on the given plant height patterns, predict the plant heights at sixth and seventh weeks. Explain your reasons.
c. Determine functions that map plant age (in weeks) to plant height (in cm).

Figure 3. Problem 2

Most of students can predict the plant heights at sixth and seventh weeks. It showed that indicator 1: "students are able to use relationship patterns to analyze situation" for this problem had been well achieved. But for the problem 2.c. most of students could not determine the functions. It showed that indicator 1: "students are able to make and use symbolic, visual or spatial notations, and words or sentences in solving mathematical problems." still on low level. It might be because students do not understand the concept of function well.

Based on the results of students' answer and interview with some students, there are three categories of students' answers in interpreting the concept of function.

1. A function is defined as a relationship between domain and codomain, where each element of domain is paired with only one element of codomain.
2. A function is an unbranched data collection.
3. A function is domain area that can only have one pair.

3.1.3 Analysis problem 3

3. Yusuf takes two hours to drive a car from city A to city B. While Amir takes three hours to travel the same. Yusuf drove a car with a speed of 12 km/hour faster than Amir. Determine the distance of city A and city B.

Figure 4. Problem 3

This problem was given to see the students' achievement of indicator 4: "students are able to explore problem solving related to other topics or fields of science" and indicator 5: "students are able to model and solve the problem". Most of students could not complete this problem well. Here is the example of student's answer.

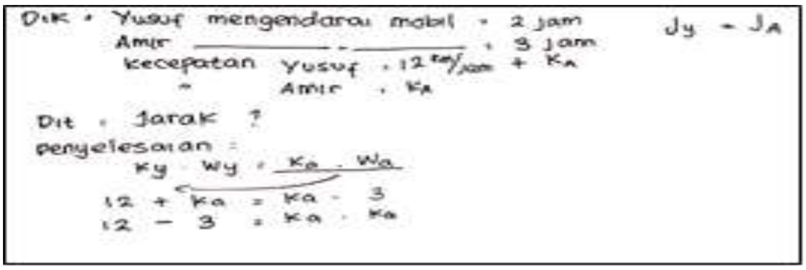

Figure 5. Example of Student's Answer to Problem 3

The answer showed that student made errors on model the problem deals with other field of science. Thus, he could not solve the problem.

3.2. Realistic Mathematics Education Approach on Teaching Function

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Realistic Mathematics Education (RME) is a learning theory specifically for mathematics developed in the Netherlands. As a theory, RME has a philosophy and characteristics. The emergence of RME is based on a philosophy about mathematics as a human activity [6]. Human activity is first and foremost in mathematics, where doing mathematics is more important than seeing mathematics as a ready-made product. Mathematics as a human activity can mean two things. First, mathematics is constructed from human activities. Second, mathematics can be implemented in human activities. According to [7], mathematics learning is the process of doing mathematics which leads to mathematics as a product. This condition is contrary to traditional mathematics learning that uses the results of other people's activities as a starting point for learning. In mathematics education, teaching mathematics does not have to be the same as how mathematics was discovered by mathematicians.

According to [8], there are three principles in RME: a) guided reinvention and progressive mathematizing; b) didactical phenomenology; c) self-developed models. Besides these three principles, there are five characteristics of RME. The five characteristics are as follows: a) use of context in phenomenological exploration; b) use of models with vertical instruments; c) use of students' contribution; d) interactivity in learning process; e) intertwining.

In [9] visualizes the process of mathematization as the process of forming an iceberg (iceberg). The Iceberg Model or the model of the iceberg is a visual metaphor that describes the process of progressive formalization through informal, pre-formal, and formal representation. The iceberg model consists of the tip of the iceberg and a much larger area below it is defined as "floating capacity". The tip of the iceberg represents formal procedures or symbolic representations. Before reaching this formal level, students must have the opportunity to deal with informal contexts and pre-formal representations. These informal and pre-formal representations and strategies play an important role in understanding formal mathematics. Students initially meet a realistic context which then motivates the use of mathematical language. Next they use a more structured pre-formal model that supports their understanding of formal notation. An example of a visualization of the mathematical process in material functions is illustrated as follows.

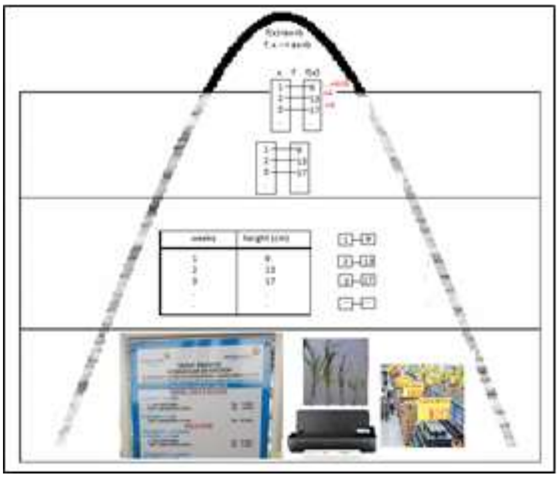

Figure 6. Visualization of the iceberg model of the mathematical process on the material function

In [7] formulated three learning principles about functions that are in line with principles in Realistic Mathematics Education. The first principle, the importance of building new knowledge based on students' prior understanding and knowledge. This is in line with the principles in PMR: Guided reinvention and progressive mathematizing. Mathematics learning is a process undertaken by students to rediscover a mathematical concept with the guidance of the teacher. This discovery process can be carried out by utilizing prior knowledge. The second principle, students need a strong conceptual understanding of the function and also procedural skills to solve the given problem. The third principle, the importance of student involvement in the metacognitive process. Learning must facilitate students to be able to operate at a higher level of abstraction. Students must be involved in monitoring their problem solving and reflecting on the strategies adopted and the solutions obtained. This principle is connected in the development of formal mathematical knowledge with a long process that includes more informal forms of modeling in generalizing types of mathematical knowledge. Contextual problems will demand a model that directs students' informal knowledge to formal knowledge so that students will develop mathematical tools and understanding in the given problem.

4. Conclusion

Based on the results of this study, it can be concluded that students' algebraic thinking skills on function materials is still low, so it need to be improved. One of the learning approach that can facilitate the development of students' algebraic thinking skills on learning function is the Realistic Mathematics Education Approach. Furthermore, this research can be developed next by designing learning about function with Realistic Mathematics approach to improve students' algebraic thinking skills.

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