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Forced Flow of a Non-Newtonian Second-Order Fluid between Two Infinite Discs with Exponential Porosity Ratio

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Abstract

The steady forced flow of an incompressible second- order fluid between two infinite discs with exponential porosity ratio has been considered. The obtained non-dimensional governing differential equations are solved by regular perturbation technique assuming flow Reynolds number as perturbation parameter. The effects of second order parameters ($\tau = \tau_1 + \tau_2$), forced and porosity ratio parameters M and e^N respectively on the velocity components have been investigated.

Keywords: *Forced flow, second-order fluid, two infinite discs, exponential porosity ratio. Subject Classification: Fluid Mechanics*

1. Introduction

Steady forced flow of a viscous incompressible fluid against a rotating disc was first studied by Schlichting and Truckenbrodt [1], Jain [2] extended this problem for Reiner-Rivilin fluid. Thereafter Srivastava and Sharma [3] worked on this problem in case of second-order fluid. Sharma and Prakash [4] have discussed the problem of forced flow of a second-order fluid when the disc is subjected to uniform high suction.

Sharma and Gupta [5] have considered the problem of forced flow of a non-Newtonian second-order fluid between two infinite discs. Sharma and Singh[6] have extended the problem of Sharma and Gupta[5] by taking the infinite discs of uniform porosity. Singh and Agarwal [7] have discussed the heat transfer in the forced flow of visco-inelastic fluid between two infinite discs thereafter this problem of heat transfer in the forced flow of Reiner-Rivlin fluid between two infinite discs of different permeability is discussed by Singh, Singhal and Kumar [8]. Fully developed flow of non-Newtonian fluids is used by M. Devakar et al. [9] in a straight uniform square duct through porous medium.

The present paper deals with the forced steady flow of incompressible second - order fluid between two infinite discs of exponential porosity ratio. The lower disc is subjected to uniform injection wherever the upper discs is subjected to different suction with exponential ratio of injection to suction.

2. Formulation of the Problem

The constitutive equation of an incompressible second-order fluid as suggested by Coleman and Noll[9] can be written as:

$$\tau_{ij} = -p\delta_{ij} + 2\mu_1 d_{ij} + 2\mu_2 e_{ij} + 4\mu_3 c_{ij} \qquad \ldots(1)$$

where

$$d_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

$$e_{ij} = \frac{1}{2}(a_{i,j} + a_{j,i}) + u^{m}_{,i}\, u_{m,j}$$

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and

$$c_{ij} = d_i^m d_{mj} \qquad \ldots(2)$$

τ_{ij} is the stress tensor, p is the hydrostatic pressure, μ_1, μ_2 & μ_3 are the coefficient of Newtonian viscosity, elastico-viscosity & cross-viscosity respectively. δ_{ij} the kronecker delta tensor and u_i, a_i are the velocity and the acceleration vectors.

Equation (1) together with the momentum equation for no extraneous forces

$$\rho\left(\frac{\partial u_i}{\partial t} + u^m u_i, m\right) = \tau^m{}_i, m \qquad \ldots(3)$$

and the equation of continuity for steady flow of an incompressible fluid

$$u^i, i = 0 \qquad \ldots(4)$$

where ρ is the density of fluid and (,) represents covariant differentiation, forms the set of governing equations.

In a three dimensional cylindrical set of co-ordinates (r, θ, z), the system consists of two infinite porous discs of infinite radius and different permeability coinciding with the plane $z = 0$ and $z = d$. The lower disc $(z = 0)$ is rotating with constant angular velocity Ω about the z-axis and the upper disc is creating a symmetrical radial velocity 'ar' which may be possible by using a sink at the edge of the upper infinite circular disc. Uniform injection w_0 is applied on the lower disc and the upper disc is subjected to uniform suction e^N w_o The value of e^N as N $\to -\infty$ becomes zero which indicates that the upper disc will become impermeable as N $\to -\infty$

Assuming u, v and w as the radial, transverse and axial components of the velocity respectively, the relevant boundary conditions of the problem are:

$$z = 0: \quad u = 0 \quad v = r\Omega \quad w = w_0$$

$$z = d: \quad u = ar \quad v = 0 \quad w = e^N w_0, \ N \text{ is an integer.} \qquad \ldots(5)$$

The non-dimensional form of velocity components satisfying the continuity equation (4) and that of pressure are

$$u = r\Omega F'(\zeta),$$
$$v = r\Omega G(\zeta),$$
$$w = -2d\Omega F(\zeta) \qquad \ldots(6)$$

and

$$p = \Omega\mu_1 - p_1(\zeta) + R\frac{r^2}{d^2}(2\tau_1 + \tau_2)(F''^2 + G'^2) + \lambda\frac{r^2}{d^2} \qquad \ldots(7)$$

where $F(\zeta)$ and $G(\zeta)$ are non-dimensional functions of the dimensionless variable $\zeta\left(= z/d\right)$, $R\left(= \Omega\rho d^2/\mu_1\right)$ is Reynolds number, $\tau_1\left(= \mu_2/\rho d^2\right)$ and $\tau_2\left(= \mu_3/\rho d^2\right)$ are dimensionless parameters representing elastico-viscous and cross-viscous effects respectively, λ is a parameter which depend upon the Reynolds number R and primes denote differentiation with respect to ζ.

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Substituting equations (6) and (7) in the equation of motion (3) and using the constitutive equation (1) we obtain

$$R(F'^2 - 2FF''' - G^2) = F''' - R[(\tau + \tau_1)F''^2 + (3\tau + \tau_1)G'^2$$
$$+2\tau_1 FF^{iv} + 2\tau_2 F'F'''] - 2\lambda \qquad \ldots(8)$$

$$2R(F'G - FG') = G'' + 2R[\tau F''G' - \tau_1 FG''' - \tau_2 F'G''] \qquad \ldots(9)$$

$$4RFF' = P_1' - 2F'' + 4R\{(7\tau + 4\tau_1)F'F'' + \tau_1 FF'''\} \qquad \ldots(10)$$

where $\tau = \tau_1 + \tau_2$ represents total second-order effects. Using the velocity field (3.6), the boundary conditions (3.5) are transformed to

$$\zeta = 0: \quad F = -A \quad F' = 0 \quad G = 1$$
$$\zeta = 1: \quad F = -e^N A \quad F' = M \quad G = 0 \qquad \ldots(1)$$

where $A = \left(w_0/2\Omega d\right)$ is the suction parameter and $M\left(= a/\Omega\right)$ is a dimensionless forced parameter assumed to be small $(M \le 1)$.

3. Solution of the Problem

For slow rotational flow, the Reynold's number R will be small. Hence a regular perturbation technique can be developed by expanding F, G, λ in the ascending powers of R as follows:

$$F = -A + \sum_{n=0}^{\infty} F_n R^n,$$

$$G = \sum_{n=0}^{\infty} G_n R^n,$$

and

$$\lambda = \sum_{n=0}^{\infty} \lambda_n R^n \qquad \ldots(12)$$

For finding out the values of the functions F, G and λ, we substitute the series (12) into equations (8) and (9). Equating the terms independent of R, coefficient of R and R^2, we obtain the following two sets of six nonlinear partial differential equations:

$$F_0''' = 2\lambda_0,$$

$$F_1''' = F_0'^2 - G_0^2 - 2(F_0 - A)F_0'' + (\tau + \tau_1)F_0''^2 + (3\tau + \tau_1)G_0'^2$$
$$+2\tau_1(F_0 - A)F_0^{iv} + 2\tau_2 F_0'F_0''' + 2\lambda_1,$$

$$F_2''' = 2[F_0'F_1' - G_0G_1 - (F_0 - A)F_1'' - F_1F_0'' + (\tau + \tau_1)F_0''F_1''$$
$$+ (3\tau + \tau_1)G_0'G_1' + \tau_1\{(F_0 - A)F_1^{iv} + F_1F_0^{iv}\}$$

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$$+\tau_2(F_0{}'F_1{}''' + F_1{}'F_0{}''') + \lambda_2] \qquad \ldots(13)$$

$$G_0{}'' = 0,$$

$$G_1{}'' = 2[F_0'G_0 - (F_0 - A)G_0{}' - \tau F_0{}''G_0{}' + \tau_1(F_0 - A)G_0{}''' + \tau_2 F_0{}'G_0{}''],$$

$$G_2{}'' = 2[F_0{}'G_1 + F_1{}'G_0 - (F_0 - A)G_1{}' - F_1G_0{}' - \tau(F_0{}''G_1{}' + F_1{}''G_0{}')$$

$$+\tau_1\{(F_0 - A)G_1{}''' + F_1G_0{}'''\} + \tau_2(F_0{}'G_1{}'' + F_1{}'G_0{}'')] \qquad \ldots(14)$$

Solving (13) and (14) subject to the boundary conditions.

$$F_n(0) = 0, \qquad F_n{}'(0) = 0 \qquad \forall\, n = 0, 1, 2, 3, \ldots.,$$

$$F_0(1) = (1 - e^N)A \qquad Fn(1) = 0 \qquad \forall\, n \geq 1,$$

$$F_0{}'(1) = M, \qquad F_n{}'(1) = 0 \qquad \forall\, n \geq 1,$$

$$G_0(0) = 1, \qquad G_n(0) = 0 \qquad \forall\, n \geq 1,$$

$$G_n(1) = 0 \qquad \forall\, n \geq 0 \qquad \ldots(15)$$

Making use of the series (12) and velocity field (3.6), the dimensionless velocity components U, V, W and λ correct to $O(R^2)$ are obtained as:

$$U = \frac{u}{r\Omega} = F'(\zeta) = F_0{}'(\zeta) + RF_1{}'(\zeta) + R^2F_2{}'(\zeta)$$

$$= 3\beta_1\zeta^2 - 2\beta_2\zeta + R\left(\frac{1}{30}\beta_3\zeta^6 + \frac{1}{20}\beta_4\zeta^5 + \frac{1}{12}\beta_5\zeta^4 + \frac{1}{6}\beta_6\zeta^3\right.$$

$$\left. + \frac{1}{2}\beta_7\zeta^2 + 6\beta_9\zeta^2 + \beta_{10}\zeta\right) + R^2\left(\frac{1}{45}\beta_{86}\zeta^{10} + \frac{1}{36}\beta_{87}\zeta^9\right.$$

$$+\frac{1}{28}\beta_{88}\zeta^8 + \frac{1}{21}\beta_{89}\zeta^7 + \frac{1}{15}\beta_{90}\zeta^6 + \frac{1}{10}\beta_{91}\zeta^5 + \frac{1}{6}\beta_{92}\zeta^4$$

$$\left. + \frac{1}{3}\beta_{93}\zeta^3 + (\beta_{94} + \beta_{97})\zeta^2 + \beta_{98}\zeta\right) \qquad \ldots(16)$$

$$V = \frac{v}{r\Omega} = G(\zeta) = G_0(\zeta) + RG_1(\zeta) + R^2G_2(\zeta)$$

$$= 1 - \zeta + R\left(-\frac{1}{5}\beta_1\zeta^5 + \frac{1}{4}\beta_{13}\zeta^4 - \frac{1}{6}\beta_{11}\zeta^3 - \frac{1}{2}\beta_{12}\zeta^2 - \beta_{14}\zeta\right)$$

$$+R^2\left(\frac{1}{36}\beta_{29}\zeta^9 + \frac{1}{28}\beta_{30}\zeta^8 + \frac{1}{21}\beta_{31}\zeta^7 + \frac{1}{15}\beta_{32}\zeta^6\right.$$

$$\left. + \frac{1}{10}\beta_{33}\zeta^5 + \frac{1}{6}\beta_{34}\zeta^4 + \frac{1}{3}\beta_{35}\zeta^3 + \beta_{36}\zeta^2 - \beta_{37}\zeta\right) \qquad \ldots(17)$$

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$$W = \frac{w}{2d\Omega} = -F(\zeta) = -(-A + F_0(\zeta) + RF_1(\zeta) + R^2F_2(\zeta))$$
$$= A - \beta_1\zeta^3 + \beta_2\zeta^2 - R\left(\frac{1}{210}\beta_3\zeta^7 + \frac{1}{120}\beta_4\zeta^6 + \frac{1}{60}\beta_5\zeta^5 + \frac{1}{24}\beta_6\zeta^4 + \frac{1}{6}\beta_7\zeta^3 + 2\beta_9\zeta^3 + \frac{1}{2}\beta_{10}\zeta^2\right)$$
$$-R^2\left(\frac{1}{495}\beta_{86}\zeta^{11} + \frac{1}{360}\beta_{87}\zeta^{10} + \frac{1}{252}\beta_{88}\zeta^9 + \frac{1}{168}\beta_{89}\zeta^8 + \frac{1}{105}\beta_{90}\zeta^7 + \frac{1}{60}\beta_{91}\zeta^6 + \frac{1}{30}\beta_{92}\zeta^5 + \frac{1}{12}\beta_{93}\zeta^4 + \frac{1}{3}(\beta_{94} + \beta_{97})\zeta^3 + \frac{1}{2}\beta_{98}\zeta^2\right) \quad \ldots(18)$$

and

$$\lambda = \lambda_0 + R\lambda_1 + R^2\lambda_2$$
$$\lambda = 3\beta_1 + 6R\beta_9 + R^2\beta_{97} \quad \ldots(19)$$

where

$$\tau = \tau_1 + \tau_2, \qquad T_1 = \tau + \tau_1, \qquad T_2 = 3\tau + \tau_1, \qquad \tau_1 = -0.2\tau_2,$$
$$\beta_1 = M + 2A(e^N - 1), \qquad \beta_2 = M + 3A(e^N - 1),$$
$$\beta_3 = -3\beta_1^2, \qquad \beta_4 = 4\beta_1\beta_2,$$
$$\beta_5 = -1 + 36T_1\beta_1^2 + 36\tau_2\beta_1^2,$$
$$\beta_6 = 2 + 12\beta_1 A - 24\beta_1\beta_2(T_1 + \tau_2),$$
$$\beta_7 = -1 - 4A\beta_2 + 4T_1\beta_2^2 + T_2,$$
$$\beta_8 = \frac{1}{30}\beta_3 + \frac{1}{20}\beta_4 + \frac{1}{12}\beta_5 + \frac{1}{6}\beta_6 + \frac{1}{2}\beta_7,$$
$$\beta_9 = \frac{1}{210}\beta_3 + \frac{1}{120}\beta_4 + \frac{1}{60}\beta_5 + \frac{1}{24}\beta_6 + \frac{1}{6}\beta_7 - \frac{1}{2}\beta_8,$$
$$\beta_{10} = -(6\beta_9 + \beta_8), \qquad \beta_{11} = 4(\beta_2 - 3\tau\beta_1),$$
$$\beta_{12} = 2(A + 2\tau\beta_2), \qquad \beta_{13} = \left(2\beta_1 + \frac{2}{3}\beta_2\right),$$
$$\beta_{14} = -\frac{1}{5}\beta_1 + \frac{1}{4}\beta_{13} - \frac{1}{6}\beta_{11} - \frac{1}{2}\beta_{12},$$
$$\beta_{15} = -6\beta_1^2 - \frac{1}{5}\beta_3, \qquad \beta_{16} = 6\beta_1\beta_{13} - \frac{1}{4}\beta_4 + 2\beta_1\beta_2,$$

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$\beta_{17} = 3\beta_1\beta_{11} + \frac{1}{3}\beta_5 + 2\beta_2\beta_{13}$, $\beta_{18} = 6\beta_1\beta_{12} + \frac{1}{2}\beta_6 - \beta_2\beta_{11}$,

$\alpha_{19} = 6\beta_1\beta_{14} + \beta_7 + 12\beta_9 - 2\beta_2\beta_{12}$,

$\beta_{20} = -12\beta_1^2$, $\beta_{21} = 12\beta_1^2 + 16\beta_1\beta_2$,

$\beta_{22} = \beta_1\beta_{11} + 12\beta_1\beta_2 + 4\beta_2^2$, $\beta_{23} = \beta_2\beta_{11} + 12A\beta_1$,

$\beta_{24} = 12A\beta_1 + 4A\beta_2$, $\beta_{25} = 18\beta_1^2 + 14\beta_1\beta_2$,

$\beta_{26} = 3\beta_1\beta_{11} + 12\beta_1\beta_2 + 4\beta_2^2$, $\beta_{27} = 3\beta_1\beta_{12} - 2\beta_2\beta_{11}$,

$\beta_{28} = 2\beta_2\beta_{12}$, $\beta_{29} = \frac{2}{5}\beta_1^2 - \frac{1}{35}\beta_3$,

$\beta_{30} = \frac{1}{30}\beta_3 - \frac{1}{24}\beta_4 - \frac{1}{4}\beta_1\beta_{13} - \frac{3}{5}\beta_1\beta_2$,

$\beta_{31} = \frac{1}{2}\beta_2\beta_{13} + \frac{1}{20}\beta_4 - \frac{1}{15}\beta_5 + \tau(\beta_{20} - \beta_{15})$,

$$\beta_{32} = -\frac{1}{2}\beta_1\beta_{12} - \frac{1}{6}\beta_2\beta_{11} + \frac{1}{12}\beta_5 - \frac{1}{8}\beta_6 - A\beta_1 - \tau\beta_{16} + \tau_1\beta_{21} + \tau_2\beta_{25},$$

$$\beta_{33} = -2\beta_1\beta_{14} + \frac{1}{6}\beta_6 - \frac{1}{3}\beta_7 - 4\beta_9 + A\beta_{13} + \tau\beta_{17} - \tau_1\beta_{22} - \tau_2\beta_{26},$$

$$\beta_{34} = \beta_2\beta_{14} + 6\beta_9 - \frac{1}{2}\beta_{10} - \frac{1}{2}A\beta_{11} + \tau\beta_{18} + \tau_1\beta_{23} - \tau_2\beta_{27} + \frac{1}{2}\beta_7$$

$\beta_{35} = \beta_{10} - A\beta_{12} + \tau\beta_{19} - \tau_1\beta_{24} + \tau_2\beta_{28}$,

$\beta_{36} = -A\beta_{14} + \tau\beta_{10} + \tau_1 A\beta_{11} - 2\tau\beta_2\beta_{14}$,

$$\beta_{37} = \frac{1}{36}\beta_{29} + \frac{1}{28}\beta_{30} + \frac{1}{21}\beta_{31} + \frac{1}{15}\beta_{32} + \frac{1}{10}\beta_{33} + \frac{1}{6}\beta_{34} + \frac{1}{3}\beta_{35} + \beta_{36},$$

$\beta_{38} = \frac{1}{10}\beta_1\beta_3$, $\beta_{39} = \frac{3}{20}\beta_1\beta_4 - \frac{1}{15}\beta_2\beta_3$,

$\beta_{40} = \frac{1}{4}\beta_1\beta_5 - \frac{1}{10}\beta_2\beta_4$, $\beta_{41} = \frac{1}{2}\beta_1\beta_6 - \frac{1}{6}\beta_2\beta_5$,

$\beta_{42} = \frac{3}{2}\beta_1\beta_7 + 18\beta_1\beta_9 - \frac{1}{3}\beta_2\beta_6$,

$\beta_{43} = 3\beta_1\beta_{10} - \beta_2\beta_7 - 12\beta_2\beta_9$, $\beta_{44} = 2\beta_2\beta_{10}$,

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$$\beta_{45} = \frac{1}{5}\beta_1, \qquad \beta_{46} = \frac{1}{5}\beta_1 + \frac{1}{4}\beta_{13},$$

$$\beta_{47} = \frac{1}{4}\beta_{13} + \frac{1}{6}\beta_{11}, \qquad \beta_{48} = \frac{1}{6}\beta_{11} - \frac{1}{2}\beta_{12},$$

$$\beta_{49} = \frac{1}{2}\beta_{12} - \beta_{14}, \qquad \beta_{50} = \frac{1}{5}\beta_1\beta_3,$$

$$\beta_{51} = \frac{1}{4}\beta_1\beta_4 - \frac{1}{5}\beta_2\beta_3, \qquad \beta_{52} = \frac{1}{3}\beta_1\beta_5 - \frac{1}{4}\beta_2\beta_4,$$

$$\beta_{53} = \frac{1}{2}\beta_1\beta_6 - \frac{1}{3}\beta_2\beta_5 - \frac{1}{5}A\beta_3,$$

$$\beta_{54} = \beta_1\beta_7 + 12\beta_1\beta_9 - \frac{1}{2}\beta_2\beta_6 - \frac{1}{4}A\beta_4,$$

$$\beta_{55} = \beta_1\beta_{10} - \beta_2\beta_7 - 12\beta_2\beta_9 - \frac{1}{3}A\beta_5,$$

$$\beta_{56} = \beta_2\beta_{10} + \frac{1}{2}A\beta_6, \qquad \beta_{57} = A\beta_7 + 12A\beta_9,$$

$$\beta_{58} = A\beta_{10}, \qquad \beta_{59} = \frac{3}{105}\beta_1\beta_3,$$

$$\beta_{60} = \frac{1}{20}\beta_1\beta_4 - \frac{1}{105}\beta_2\beta_3, \qquad \beta_{61} = \frac{1}{10}\beta_1\beta_5 - \frac{1}{60}\beta_2\beta_4,$$

$$\beta_{62} = \frac{1}{4}\beta_1\beta_6 - \frac{1}{30}\beta_2\beta_5, \qquad \beta_{63} = \beta_1\beta_7 + 12\beta_1\beta_9 - \frac{1}{12}\beta_2\beta_6,$$

$$\beta_{64} = 3\beta_1\beta_{10} - \frac{1}{3}\beta_2\beta_7 - 4\beta_2\beta_9, \qquad \beta_{65} = \beta_2\beta_{10},$$

$$\beta_{66} = \frac{6}{5}\beta_1\beta_3, \qquad \beta_{67} = \frac{6}{4}\beta_1\beta_4 - \frac{2}{5}\beta_2\beta_3,$$

$$\beta_{68} = 2\beta_1\beta_5 - \frac{1}{2}\beta_2\beta_4, \qquad \beta_{69} = 3\beta_1\beta_6 - \frac{2}{3}\beta_2\beta_5,$$

$$\beta_{70} = 6\beta_1\beta_7 + 72\beta_1\beta_9 - \beta_2\beta_6,$$

$$\beta_{71} = 6\beta_1\beta_{10} - 2\beta_2\beta_7 - 24\beta_2\beta_9,$$

$$\beta_{72} = 2\beta_2\beta_{10}, \qquad \beta_{73} = 4\beta_1\beta_3,$$

$$\beta_{74} = 3\beta_1\beta_4 - 4\beta_2\beta_3, \qquad \beta_{75} = 2\beta_1\beta_5 - 3\beta_2\beta_4,$$

$$\beta_{76} = \beta_1\beta_6 - 2\beta_2\beta_5 - 4A\beta_3, \qquad \beta_{77} = \beta_2\beta_6 + 3A\beta_4,$$

$$\beta_{78} = 2A\beta_5, \qquad \beta_{79} = A\beta_6,$$

$$\beta_{80} = \frac{16}{5}\beta_1\beta_3, \qquad \beta_{81} = \frac{33}{10}\beta_1\beta_4 - 2\beta_2\beta_3,$$

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$$\beta_{82} = \frac{7}{2}\beta_1\beta_5 - 2\beta_2\beta_4, \qquad \beta_{83} = 4\beta_1\beta_6 - 2\beta_2\beta_5,$$

$$\beta_{84} = 6\beta_1\beta_7 - 2\beta_2\beta_6 + 72\beta_1\beta_9,$$

$$\beta_{85} = 2\beta_2\beta_7 + 24\beta_2\beta_9 - 6\beta_1\beta_{10},$$

$$\beta_{86} = \beta_{38} - \beta_{50} - \beta_{59}, \qquad \beta_{87} = \beta_{39} - \beta_{51} - \beta_{60},$$

$$\beta_{88} = \beta_{40} - \beta_{45} - \beta_{52} - \beta_{61} + \beta_{66}T_1 + \beta_{73}\tau_1 + \beta_{80}\tau_2,$$

$$\beta_{89} = \beta_{41} + \beta_{46} - \beta_{53} - \beta_{62} + \beta_{67}T_1 + \beta_{74}\tau_1 + \beta_{81}\tau_2,$$

$$\beta_{90} = \beta_{42} - \beta_{47} - \beta_{54} - \beta_{63} + \beta_1T_2 + \beta_{68}T_1 + \beta_{75}\tau_1 + \beta_{82}\tau_2,$$

$$\beta_{91} = \beta_{43} + \beta_{48} - \beta_{55} - \beta_{64} - \beta_{13}T_2 + \beta_{69}T_1 + \beta_{76}\tau_1 + \beta_{83}\tau_2,$$

$$\beta_{92} = -\beta_{44} + \beta_{49} + \beta_{56} + \beta_{65} + \frac{1}{2}\beta_{11}T_2 + \beta_{70}T_1 - \beta_{77}\tau_1 + \beta_{84}\tau_2,$$

$$\beta_{93} = \beta_{14} + \beta_{57} + \beta_{12}T_2 + \beta_{71}T_1 - \beta_{78}\tau_1 - \beta_{85}\tau_2,$$

$$\beta_{94} = \beta_{58} + \beta_{14}T_2 - \beta_{72}T_1 - \beta_{79}\tau_1,$$

$$\beta_{95} = \frac{1}{90}\beta_{86} + \frac{1}{72}\beta_{87} + \frac{1}{56}\beta_{88} + \frac{1}{42}\beta_{89} + \frac{1}{30}\beta_{90} + \frac{1}{20}\beta_{91} + \frac{1}{12}\beta_{92} + \frac{1}{6}\beta_{93} + \frac{1}{2}\beta_{94},$$

$$\beta_{96} = \frac{1}{990}\beta_{86} + \frac{1}{720}\beta_{87} + \frac{1}{504}\beta_{88} + \frac{1}{336}\beta_{89} + \frac{1}{210}\beta_{90} + \frac{1}{120}\beta_{91} + \frac{1}{60}\beta_{92} + \frac{1}{24}\beta_{93} + \frac{1}{6}\beta_{94},$$

$$\beta_{97} = 12\beta_{96} - 6\beta_{95}, \qquad \beta_{98} = -2\beta_{95} - \beta_{97}.$$

4. Results and Discussions

The values of the velocity components have been carried out for different value of the parameter A, M, e^N and τ by taking R=0.5. The velocity functions are calculated on neglecting terms containing R^3 and higher powers of R. The variation of dimensionless radial velocity U with ζ at e^N=e, τ =1, R=0.5 for different values of A=0, 1, 2 in cases of M=0, 0.5 and 1 is shown in fig. 1. It is observed from the numerical data and graphs that at A=1 and 2 the radial velocity U increases with an increase in the forced parameter M in the gaplength region $0 \le \zeta \le 0.45$ approximately wherever it's behaviour is reversed in the gaplength region $0.45 < \zeta < 1$. At A=0 the radial velocity decreases with an increase in the forced parameter M in the gaplength region $0 < \zeta \le 0.6$ with it's reverse behaviour in the region $0.6 < \zeta < 1$. The value of the radial velocity is zero on the lower disc and equal to M at the upper disc.

The behaviour of the radial velocity at R=0.5, A=1, e^N=e for different values of the forced parameter M=0, 0.5, 1 and second - order parameter τ =0, 1, 2 is represented through fig. 2 and at R=0.5, A=1 τ =1 for different values of M=0, 0.5, 1 and $e^N= e^{0.5}$, e, $e^{1.5}$ through fig. 3 respectively. It is evident from both the figures that for the second-order parameter τ =1, 2 and exponential porosity ratio e^N=e, $e^{1.5}$ the radial velocity increases with an increase in forced parameter M in the region $0 < \zeta$

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< 0.45 wherever decreases in the region 0.45 < ζ < 1 which shows that the behaviour of radial velocity in fig. 2 and fig. 3 is similar to it's behaviour in fig.1. It is also observed that at M-0, o.5, 1 the radial velocity increases with an increase in second-order parameter τ and exponential ratio e^N in the gaplength region 0 < ζ < 0.45 with it's reversed behaviour in the region 0.45 ≤ ζ < 1 approximately. The fluid flows radially towards the centre of the discs (i.e. U<0) , in case of τ =0, M =0, 0.5, 1 in whole of the gaplength approximately except at the upper disc wherever in case of $e^N=e^{0.5}$ the behaviour is similar to the case of τ =0 except in case of $e^N=e^{0.5}$, M=1.

Fig. 4 exhibits the behaviour of the transverse velocity V with ζ at τ =1, R=0.5, e^N=e for different values of the forced parameter M=0, 0.5, 1 and suction parameter A=0, 1, 2. It is observed from the fig. 4 that the transverse velocity decreases with an increase in suction parameter A for fixed value of M wherever increase with an increase in M for fixed value of the suction parameter A. The transverse velocity is minimum at ζ=0.4 for all values of M in case of A=2 wherever it is minimum at ζ=0.5 & for all values of M, at A=1. Transverse velocity is maximum (=1) at the lower disc. The variation of V with ζ is linear for all values of M at A=0 and decreases with an increase in M throughout the gaplength.

The variation of the dimensionless transverse velocity V with ζ at R=0.5, A=1, e^N=e for different values of forced parameter M=0, 0.5, 1 and second order parameter τ =0, 1, 2 is shown through fig. 5. At τ =2 the values of V decreases with an increases in M upto ζ=0.45 wherever increases with an increase in M in the region 0.45 ≤ ζ < 1. At τ =1 the transverse velocity decreases with an increase in M throughout the gaplength. At τ =0 all the branches of M are being overlapped. It is also evident from this figure that transverse velocity decreases with an increase in τ throughout the gaplength. It is also clear from fig. 6 that the transverse velocity decreases with an increase in e^N and M both throughout the gaplength.

The variation of the dimensionless velocity W with ζ at τ =1, R=0.5, e^N=e for different values of A=0, 1,2 and M=0, R=0.5, e^N=e for different values A=0, 1,2 and M=0, 0.5,1 is sown through fig. 7. It is evident from this figure that at A=1, 2 the axial velocity decreases with an increase in force parameter M in whole of the region where ever at A=0 it increases with an increase in M. It is clear from the figure 8 that the axial velocity decreases with an increase in M at τ =1, 2 wherever increases at τ =0 throughout the gaplength the dimensionless velocity is also decreasing with an increase in τ for all values of M. The axial velocity is 1 at the lower disc and equal to 2.718282 approximately at the upper disc. The variation of the axial velocity W with ζ at R=0.5, A=1, τ =1 for different values of M=0, 1,2 and $e^N=e^{0.5}$, $e^{1.5}$ is represented through figure 9. It is observed that the behaviour of W with e^N and M in the figure 9 is similar to the behaviour of W in figure 8 with τ and M.

5. Conclusions

For N=0, the exponential porosity ratio e^N in this paper becomes unity which converted the results of this paper into the results of Sharma and Singh [6]. As the value of e^N as N $\rightarrow -\infty$ is equal to zero the boundary conditions represents that the upper disc will become nonporous that is the upper disc will not subject to suction or injection. Since the values of as e^N as N tends to infinity becomes infinite, the upper disc will subject to infinite suction or injection which is not possible practically. Hence N is taken as integer (not of large magnitude).

It is concluded that the radial velocity increases with an increase in M, τ and e^N in the region 0 < ζ ≤ 0.45 and decreases in the rest of the region. The transverse and axial velocities are decreasing with an increase in M, A, e^N in whole of the region approximately.

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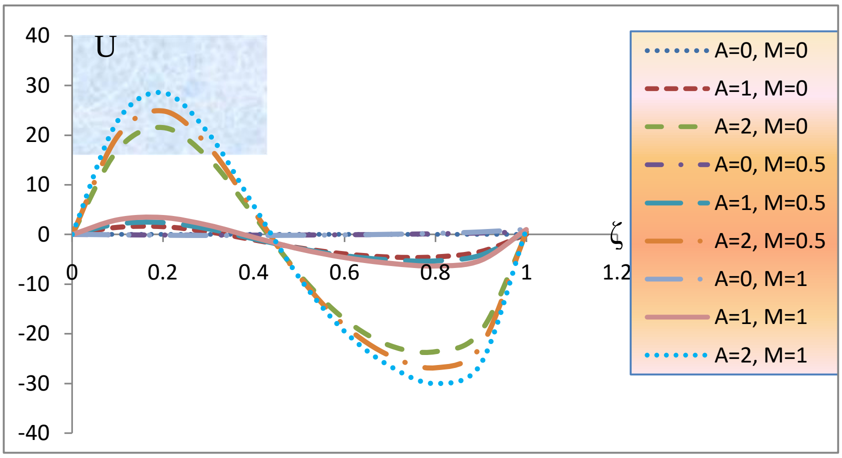

Figure-1: Variation of Radial velocity U with ζ at τ=1, R=0.5, e^N=e for different values of M and A.

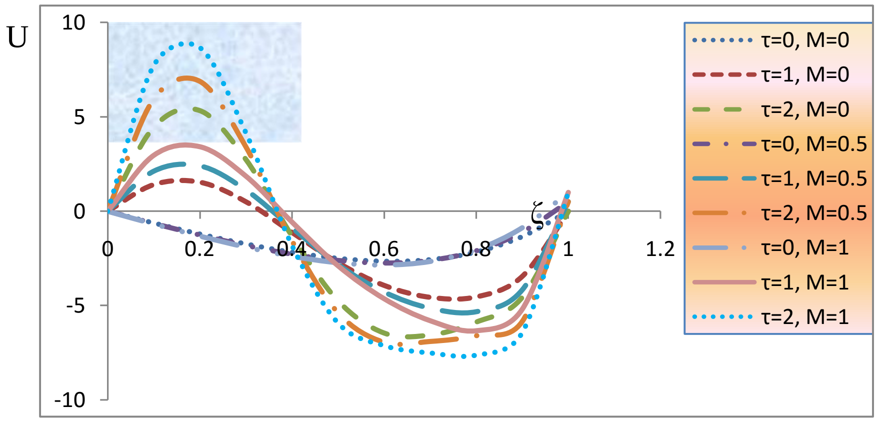

Figure-2: Variation of Radial velocity U with ζ at R=0.5, A=1, e^N=e for different values of M and τ.

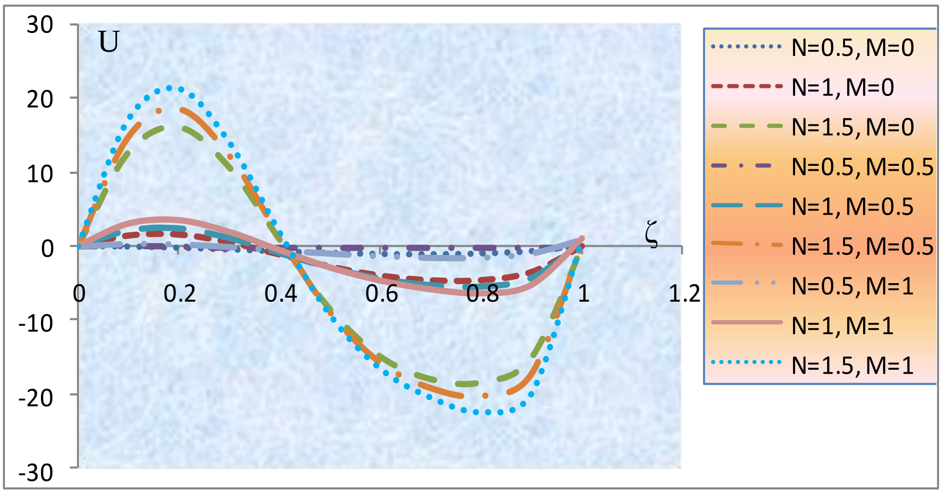

Figure-3: Variation of Radial velocity U with ζ at R=0.5, A=1, τ =1 for different values of M and N.

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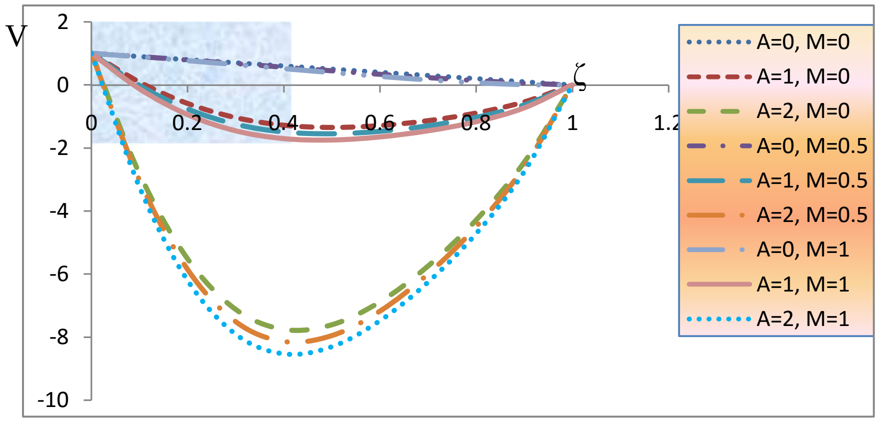

Figure-4: Variation of Transverse velocity V with ζ at τ=1, R=0.5, e^N=e for different values of M and A.

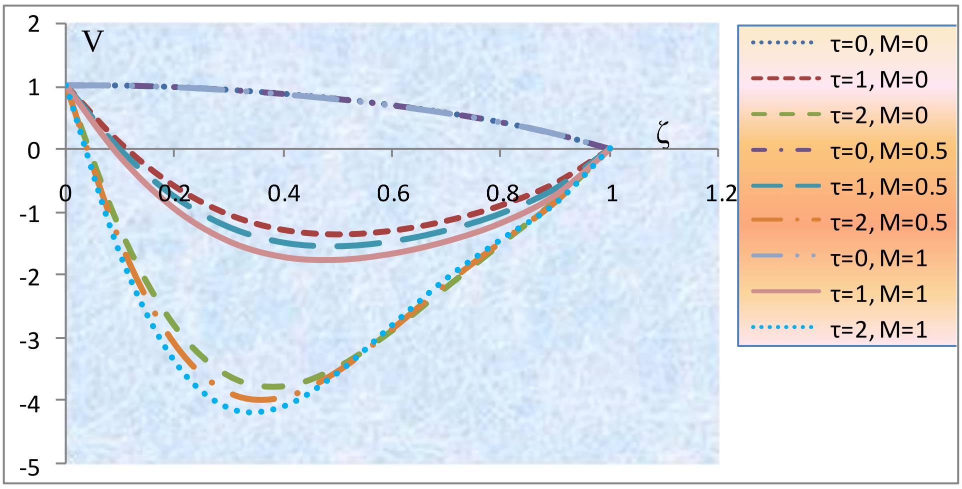

Figure-5: Variation of Transverse velocity V with ζ at R=0.5, A=1, e^N=e for different values of M and τ.

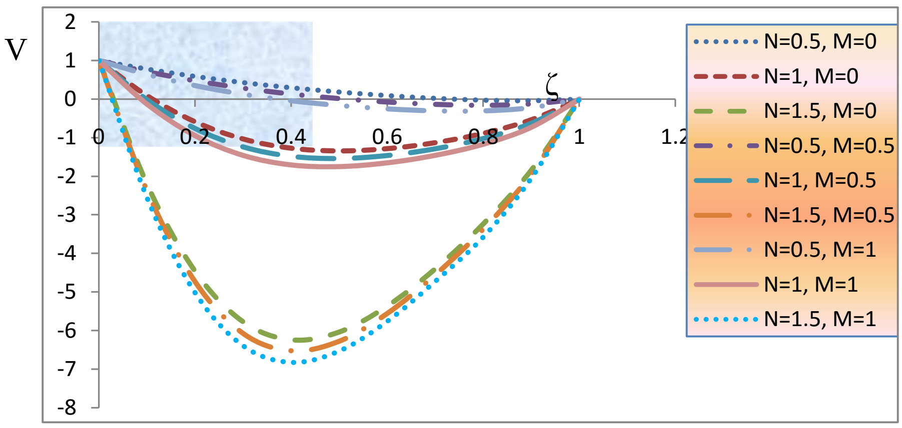

Figure-6: Variation of Transverse velocity V with ζ at R=0.5, A=1, τ =1 for different values of M and N.

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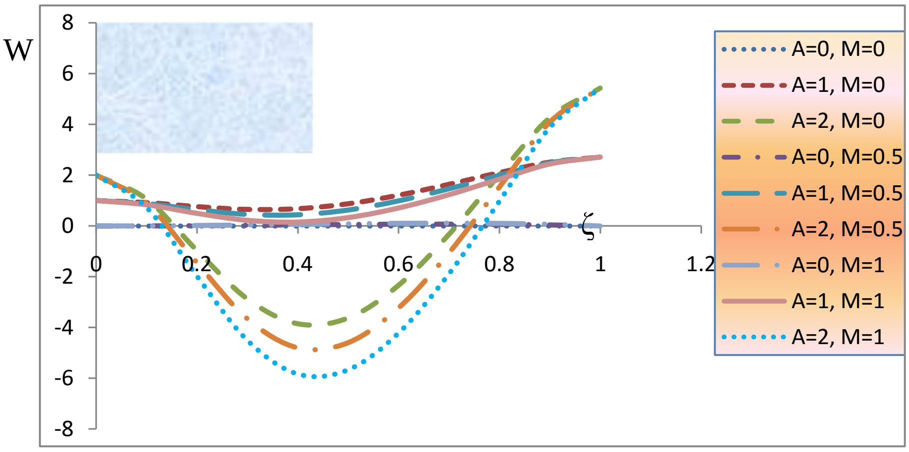

Figure-7: Variation of Axial velocity W with ζ at τ=1, R=0.5, e^N=e for different values of M and A.

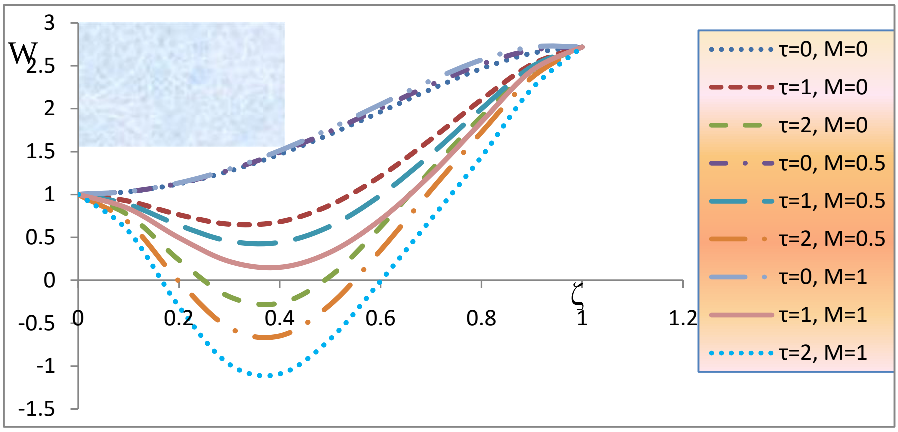

Figure-8: Variation of Axial velocity W with ζ at R=0.5, A=1, e^N=e for different values of M and τ.

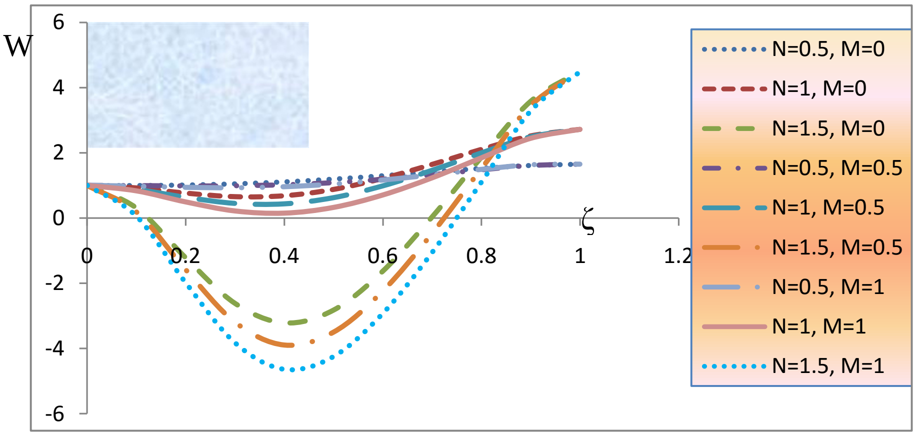

Figure-9: Variation of Axial velocity W with ζ at R=0.5, A=1, τ =1 for different values of M and N.

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