

## Comparative Analysis on the solution of Travelling Salesman Problem for a Fuzzy Octagonal numbers by different methods

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### Abstract

Travelling salesman problem is used to find the shortest route for the salesman with minimum cost (or) time schedule. In this paper, the same is defined under fuzzy environment with eight parameters and is called octagonal fuzzy number. Hungarian method, matrix one's method and branch and bound methods are used to solve the octagonal fuzzy problem by  $\alpha$ -cut Ranking method. Further the solution by the three methods are compared.

**Keywords:** robust ranking, Hungarian method, one matrix method, branch and bound method.

### 1. Introduction

The travelling salesman problem was first introduced by Irish Mathematician W.R. Hamilton. Travelling salesman problem is a well-known popular and extensively studied problem in the field of combinatorial optimization. In the general form of travelling salesman problem, a salesman has to visits the entire cities only once and return to the home town with minimum cost. The design of the problem is simple. Fuzzy sets were proposed by Prof. L.A. Zadeh in 1965, to use the data and information feature of non-statistical ambiguity. Provide us a new mathematical tool to deal with uncertainty of information. One of the main applications of fuzzy arithmetic is accommodated the parameters and is represented by a fuzzy number. Numerical example also included to clear the optimization. Since then, fuzzy set theory has been rapidly developed. A ranking using  $\alpha$ -cut is introduced on octagonal fuzzy numbers. Using this ranking the fuzzy assignment problem or fuzzy travelling salesman problem is converted to a crisp valued problem, which can be solved using Hungarian method. The optimal solution can be got either as a crisp number.

### 2. Preliminaries

**2.1. Fuzzy Set:** A fuzzy set is characterized by a membership function mapping element of a domain, space, or the universe of discourse  $X$  to the unit interval  $[0, 1]$ , i.e.  $A = \{(x, \mu_A(x)) ; x \in X\}$ . Here  $\mu_A : X \rightarrow [0, 1]$  is a mapping called the degree of membership function of the fuzzy set  $A$  and  $\mu_A(x)$  is called the membership value of  $x \in X$  in the fuzzy set  $A$ . These membership grades are often represented by real numbers ranging from  $[0, 1]$ .

**2.2. Normal fuzzy set:** A fuzzy set  $A$  of the universe of discourse  $X$  is called a normal fuzzy set implying that there exist at least one  $x \in X$  such that  $\mu_A(x) = 1$ .

**2.3. Fuzzy number:** A fuzzy number  $\tilde{A}$  is a normal octagonal fuzzy number denoted by  $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ , where  $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$  are real numbers and its membership function  $\mu_{\tilde{A}}(x)$  is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ k \left( \frac{x - a_1}{a_2 - a_1} \right), & a_1 \leq x \leq a_2 \\ k, & a_2 \leq x \leq a_3 \\ k + (1 - k) \left( \frac{x - a_3}{a_4 - a_3} \right), & a_3 \leq x \leq a_4 \\ 1, & a_4 \leq x \leq a_5 \\ k + (1 - k) \left( \frac{a_6 - x}{a_6 - a_5} \right), & a_5 \leq x \leq a_6 \\ k, & a_6 \leq x \leq a_7 \\ k \left( \frac{a_8 - x}{a_8 - a_7} \right), & a_7 \leq x \leq a_8 \\ 0, & x \geq a_8 \end{cases} \quad \text{where } 0 < k < 1.$$

**2.4. Robust ranking technique:** The technique which satisfy compensation, linearity and additive properties and provides results which are consistent with human intuition. If  $\tilde{a}$  is a convex fuzzy number, the Robust ranking index is defined by  $R(\tilde{a}) = \int_0^1 (0.5) (\alpha_\alpha^L, \alpha_\alpha^U) d\alpha$ ,

Where,  $(\alpha_\alpha^L, \alpha_\alpha^U) = \{[(b - a)\alpha + a, d - (d - c)\alpha], [(f - e)\alpha + e, h - (h - g)\alpha]\}$  is a  $\alpha$ -level cut of a fuzzy number  $\tilde{a}$ .

Given a fuzzy set ‘A’ defined on ‘X’ and any number  $\alpha \in [0,1]$ , the  $\alpha$ -cut is denoted by  $A(\alpha)$  and is defined by the crisp set  $A(\alpha) = \{x: A(x) \geq \alpha\}$ . i.e.  $A(\alpha) = \{x: \mu(x) \geq \alpha, \alpha \in [0,1]\}$

**2.5. Algorithm for solving the fuzzy travelling salesman problem:**

Step 1: Find the least cost tour starting at A, travelling through the other cities exactly once and returning to A

Step 2: Compute the given octagonal fuzzy number to crisp value in the travelling salesman problem using  $\alpha$  cut ranking.

Then by using different methods the Critical path and the total distance travelled is calculated.

**2.5.1. Hungarian Method:**

Step 1: Row Reduction – Find the minimum value of each row and subtract it from each element in that row.

Step 2: Column Reduction – Find the minimum value of each column and subtract it from each element in that column.

Step 3: Mark all zeros in the resulting matrix using a minimum number of horizontal and vertical lines. If there is a zero in each row and column then an optimal assignment exists among the zeros.

Step 4: If any row or column is not marked with zero then find the minimum value that is not covered by a line in Step 3. Subtract that minimum value from all uncovered elements, and add that minimum value to all elements that are covered twice.

**2.5.2. One Matrix Method:**

It is similar to that the Hungarian method that the minimum value subtracted is changed to dividing the minimum value to the each row and column then finally the ones are marked in the resulting matrix.

**2.5.3. Branch and Bound Method:**

Step 1: A row (column) is to be reduced iff it contains at least one zero and all the remaining entries are non-zero.

Step 2: Consider upper bound as  $\infty$ .

Step 3: Initial node is to split the other branches and compute the cost of each node.

Step 4: The reduced cost for every node is  $C$  (Initial node, branch node) +  $r$  +  $\hat{r}$  where  $r$  is the reduced cost of initial node and  $\hat{r}$  is the reduced cost of current node.

Step 5: Compute the least cost branch and terminate the other.

Step 6: Stop when only one branch survives.

Step 7: Finally find the optimum result.

### 3. Illustrative Example

Consider the following Fuzzy travelling salesman problem

	A	B	C	D	E
A	$\infty$	{4,5,8,9,10,11,12,13}	{2,3,9,10,12,13,14,15}	{2,3,4,5,8,9,12,13}	{4,5,8,9,13,14,16,17}
B	{4,5,7,8,10,11,12,13}	$\infty$	{4,5,9,10,14,15,16,17}	{5,6,7,8,9,10,13,14}	{8,9,10,11,13,14,17,18}
C	{5,6,10,11,12,13,15,18}	{6,7,10,11,13,14,16,17}	$\infty$	{8,10,11,12,13,14,15,17}	{7,8,10,11,12,13,14,15}
D	{9,10,12,13,14,15,16,17}	{3,4,9,10,12,13,15,16}	{4,5,8,9,10,11,14,15}	$\infty$	{6,7,8,9,11,12,13,14}
E	{6,7,10,11,12,13,15,16}	{5,6,7,8,9,10,12,13}	{8,9,10,11,12,13,17,18}	{6,7,9,10,12,13,15,16}	$\infty$

#### Solution:

The fuzzy Travelling Salesman problem can be formulated in the following ranking technique

$$R(\bar{a}) = \int_0^1 (0.5) (\alpha_{\bar{a}}^L, \alpha_{\bar{a}}^U) d\alpha$$

$$\text{Where, } (\alpha_{\bar{a}}^L, \alpha_{\bar{a}}^U) = \{[(b-a)\alpha + a, d - (d-c)\alpha], [(f-e)\alpha + e, h - (h-g)\alpha]\}$$

Then,

$$\begin{aligned} R_{12} (4,5,8,9,10,11,12,13) &= \int_0^1 (0.5) \{[(b-a)\alpha + a, d - (d-c)\alpha], [(f-e)\alpha + e, h - (h-g)\alpha]\} d\alpha \\ &= \int_0^1 (0.5) \{[\alpha + 4 + 9 - \alpha], [\alpha + 10 + 13 - \alpha]\} d\alpha \\ &= \int_0^1 (0.5) [36] d\alpha = \int_0^1 (18) d\alpha \end{aligned}$$

$$R_{12} = 18$$

$$\begin{aligned} R_{13} (2,3,9,10,12,13,14,15) &= \int_0^1 (0.5) \{[\alpha + 2 + 10 - \alpha], [\alpha + 12 + 15 - \alpha]\} d\alpha \\ &= \int_0^1 (0.5) [39] d\alpha = \int_0^1 (19.5) d\alpha \end{aligned}$$

$$R_{13} = 19.5$$

$$\begin{aligned} R_{14} (2,3,4,5,8,9,12,13) &= \int_0^1 (0.5) \{[\alpha + 2 + 5 - \alpha], [\alpha + 8 + 13 - \alpha]\} d\alpha \\ &= \int_0^1 (0.5) [28] d\alpha = \int_0^1 (14) d\alpha \end{aligned}$$

$$R_{14} = 14$$

$$\begin{aligned}R_{15} (4,5,8,9,13,14,16,17) &= \int_0^1 (0.5)[\{\alpha + 4 + 9 - \alpha\}, \{\alpha + 13 + 17 - \alpha\}] d\alpha \\ &= \int_0^1 (0.5)[43] d\alpha = \int_0^1 (21.5) d\alpha\end{aligned}$$

$$R_{15} = 21.5$$

$$\begin{aligned}R_{21} (4,5,7,8,10,11,12,13) &= \int_0^1 (0.5)[\{\alpha + 4 + 8 - \alpha\}, \{\alpha + 10 + 13 - \alpha\}] d\alpha \\ &= \int_0^1 (0.5)[35] d\alpha = \int_0^1 (17.5) d\alpha\end{aligned}$$

$$R_{21} = 17.5$$

$$\begin{aligned}R_{23} (4,5,9,10,14,15,16,17) &= \int_0^1 (0.5)[\{\alpha + 4 + 10 - \alpha\}, \{\alpha + 14 + 17 - \alpha\}] d\alpha \\ &= \int_0^1 (0.5)[45] d\alpha = \int_0^1 (22.5) d\alpha\end{aligned}$$

$$R_{23} = 22.5$$

$$\begin{aligned}R_{24} (5,6,7,8,9,10,13,14) &= \int_0^1 (0.5)[\{\alpha + 5 + 8 - \alpha\}, \{\alpha + 9 + 14 - \alpha\}] d\alpha \\ &= \int_0^1 (0.5)[36] d\alpha = \int_0^1 (18) d\alpha\end{aligned}$$

$$R_{24} = 18$$

$$\begin{aligned}R_{25} (8,9,10,11,13,14,17,18) &= \int_0^1 (0.5)[\{\alpha + 8 + 11 - \alpha\}, \{\alpha + 13 + 18 - \alpha\}] d\alpha \\ &= \int_0^1 (0.5)[50] d\alpha = \int_0^1 (25) d\alpha\end{aligned}$$

$$R_{25} = 25$$

$$\begin{aligned}R_{31} (5,6,10,11,12,13,15,18) &= \int_0^1 (0.5)[\{\alpha + 5 + 11 - \alpha\}, \{\alpha + 12 + 16 - \alpha\}] d\alpha \\ &= \int_0^1 (0.5)[44] d\alpha = \int_0^1 (22) d\alpha\end{aligned}$$

$$R_{31} = 22$$

$$\begin{aligned}R_{32} (6,7,10,11,13,14,16,17) &= \int_0^1 (0.5)[\{\alpha + 6 + 11 - \alpha\}, \{\alpha + 13 + 17 - \alpha\}] d\alpha \\ &= \int_0^1 (0.5)[47] d\alpha = \int_0^1 (23.5) d\alpha\end{aligned}$$

$$R_{32} = 23.5$$

$$\begin{aligned}R_{34} (8,10,11,12,13,14,15,17) &= \int_0^1 (0.5)[\{2\alpha + 8 + 12 - \alpha\}, \{\alpha + 13 + 17 - 2\alpha\}] d\alpha \\ &= \int_0^1 (0.5)[50] d\alpha = \int_0^1 (25) d\alpha\end{aligned}$$

$$R_{34} = 25$$

$$\begin{aligned}R_{35} (7,8,10,11,12,13,14,15) &= \int_0^1 (0.5)[\{\alpha + 7 + 11 - \alpha\}, \{\alpha + 12 + 15 - \alpha\}] d\alpha \\ &= \int_0^1 (0.5)[45] d\alpha = \int_0^1 (22.5) d\alpha\end{aligned}$$

$$R_{35} = 22.5$$

$$\begin{aligned}R_{41} (9,10,12,13,14,15,16,17) &= \int_0^1 (0.5)[\{\alpha + 9 + 13 - \alpha\}, \{\alpha + 14 + 17 - \alpha\}] d\alpha \\ &= \int_0^1 (0.5)[53] d\alpha = \int_0^1 (26.5) d\alpha\end{aligned}$$

$$R_{41} = 26.5$$

$$R_{42} (3,4,9,10,12,13,15,16) = \int_0^1 (0.5)[\{\alpha + 3 + 10 - \alpha\}, \{\alpha + 12 + 16 - \alpha\}] d\alpha$$

$$= \int_0^1 (0.5)[41] d\alpha = \int_0^1 (20.5) d\alpha$$

$$R_{42} = 20.5$$

$$R_{43} (4,5,8,9,10,11,14,15) = \int_0^1 (0.5)[\{\alpha + 4 + 9 - \alpha\}, \{\alpha + 10 + 15 - \alpha\}] d\alpha$$

$$= \int_0^1 (0.5)[38] d\alpha = \int_0^1 (19) d\alpha$$

$$R_{43} = 19$$

$$R_{45} (6,7,8,9,11,12,13,14) = \int_0^1 (0.5)[\{\alpha + 6 + 9 - \alpha\}, \{\alpha + 11 + 14 - \alpha\}] d\alpha$$

$$= \int_0^1 (0.5)[40] d\alpha = \int_0^1 (20) d\alpha$$

$$R_{45} = 20$$

$$R_{51} (6,7,10,11,12,13,15,16) = \int_0^1 (0.5)[\{\alpha + 6 + 11 - \alpha\}, \{\alpha + 12 + 16 - \alpha\}] d\alpha$$

$$= \int_0^1 (0.5)[45] d\alpha = \int_0^1 (22.5) d\alpha$$

$$R_{51} = 22.5$$

$$R_{52} (5,6,7,8,9,10,12,13) = \int_0^1 (0.5)[\{\alpha + 5 + 8 - \alpha\}, \{\alpha + 9 + 13 - \alpha\}] d\alpha$$

$$= \int_0^1 (0.5)[37] d\alpha = \int_0^1 (18.5) d\alpha$$

$$R_{52} = 18.5$$

$$R_{53} (8,9,10,11,12,13,17,18) = \int_0^1 (0.5)[\{\alpha + 8 + 11 - \alpha\}, \{\alpha + 12 + 18 - \alpha\}] d\alpha$$

$$= \int_0^1 (0.5)[49] d\alpha = \int_0^1 (24.5) d\alpha$$

$$R_{53} = 24.5$$

$$R_{54} (6,7,9,10,12,13,15,16) = \int_0^1 (0.5)[\{\alpha + 6 + 10 - \alpha\}, \{\alpha + 12 + 16 - \alpha\}] d\alpha$$

$$= \int_0^1 (0.5)[44] d\alpha = \int_0^1 (22) d\alpha$$

$$R_{54} = 22$$

The corresponding table after ranking,

	A	B	C	D	E
A	$\infty$	18	19.5	14	21.5
B	17.5	$\infty$	22.5	18	25
C	22	23.5	$\infty$	25	22.5
D	26.5	20.5	19	$\infty$	20
E	22.5	18.5	24.5	22	$\infty$

Using Hungarian method, initially taken row reduction

	A	B	C	D	E	Min
A	$\infty$	18	19.5	14	21.5	<b>14</b>
B	17.5	$\infty$	22.5	18	25	<b>17.5</b>
C	22	23.5	$\infty$	25	22.5	<b>22</b>
D	26.5	20.5	19	$\infty$	20	<b>19</b>
E	22.5	18.5	24.5	22	$\infty$	<b>18.5</b>

Subtract the minimum value from the respective rows,

	A	B	C	D	E
A	$\infty$	4	5.5	0	7.5
B	0	$\infty$	5	0.5	7.5
C	0	1.5	$\infty$	3	0.5
D	7.5	1.5	0	$\infty$	1
E	4	0	6	3.5	$\infty$

Then column reduction

	A	B	C	D	E
A	$\infty$	4	5.5	0	7.5
B	0	$\infty$	5	0.5	7.5
C	0	1.5	$\infty$	3	0.5
D	7.5	1.5	0	$\infty$	1
E	4	0	6	3.5	$\infty$
Min	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0.5</b>

Subtract the minimum value from the respective columns,

	A	B	C	D	E
A	$\infty$	4	5.5	0	7
B	0	$\infty$	5	0.5	7
C	0	1.5	$\infty$	3	0
D	7.5	1.5	0	$\infty$	0.5
E	4	0	6	3.5	$\infty$

Then solution is,

	A	B	C	D	E
A	$\infty$	4	5.5	<b>0</b>	7
B	<b>0</b>	$\infty$	5	0.5	7
C	<del><math>\infty</math></del>	1.5	$\infty$	3	<b>0</b>
D	7.5	1.5	<b>0</b>	$\infty$	0.5
E	4	<b>0</b>	6	3.5	$\infty$

Using the allotment the solution of the problem can be obtained in the form of octagonal fuzzy number,

	A	B	C	D	E
A	$\infty$	{4,5,8,9,10,11,12,13}	{2,3,9,10,12,13,14,15}	{2,3,4,5,8,9,12,13}	{4,5,8,9,13,14,16,17}
B	{4,5,7,8,10,11,12,13}	$\infty$	{4,5,9,10,14,15,16,17}	{5,6,7,8,9,10,13,14}	{8,9,10,11,13,14,17,18}
C	{5,6,10,11,12,13,15,18}	{6,7,10,11,13,14,16,17}	$\infty$	{8,10,11,12,13,14,15,17}	{7,8,10,11,12,13,14,15}
D	{9,10,12,13,14,15,16,17}	{3,4,9,10,12,13,15,16}	{4,5,8,9,10,11,14,15}	$\infty$	{6,7,8,9,11,12,13,14}
E	{6,7,10,11,12,13,15,16}	{5,6,7,8,9,10,12,13}	{8,9,10,11,12,13,17,18}	{6,7,9,10,12,13,15,16}	$\infty$

The optimum travelling schedule is  $A \rightarrow D \rightarrow C \rightarrow E \rightarrow B \rightarrow A$

$$\begin{aligned} \text{Total distance travelled} &= \{2,3,4,5,8,9,12,13\} + \{4,5,7,8,10,11,12,13\} + \{7,8,10,11,12,13,14,15\} + \\ & \{4,5,8,9,10,11,14,15\} + \{5,6,7,8,9,10,12,13\} \\ &= (23,28,36,41,49,54,64,69) \end{aligned}$$

Total distance travelled = 91

By using one matrix method, initially by row reduction

	A	B	C	D	E	Min
A	$\infty$	18	19.5	14	21.5	<b>14</b>
B	17.5	$\infty$	22.5	18	25	<b>17.5</b>
C	22	23.5	$\infty$	25	22.5	<b>22</b>
D	26.5	20.5	19	$\infty$	20	<b>19</b>
E	22.5	18.5	24.5	22	$\infty$	<b>18.5</b>

Divide the minimum value from the respective rows,

	A	B	C	D	E
A	$\infty$	1.3	1.4	1	1.5
B	1	$\infty$	1.3	1.03	1.4
C	1.02	1.07	$\infty$	25	1
D	1.39	1.08	1	$\infty$	1.05
E	1.2	1	1.3	1.19	$\infty$

Then by column reduction,

	A	B	C	D	E
A	$\infty$	1.3	1.4	1	1.5

B	1	$\infty$	1.3	1.03	1.4
C	1.02	1.07	$\infty$	25	1
D	1.39	1.08	1	$\infty$	1.05
E	1.2	1	1.3	1.19	$\infty$
<b>Min</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>

Divide the minimum value from the respective columns,

The solution is,

	A	B	C	D	E
A	$\infty$	1.3	1.4	1	1.5
B	1	$\infty$	1.3	1.03	1.4
C	1.02	1.07	$\infty$	25	1
D	1.39	1.08	1	$\infty$	1.05
E	1.2	1	1.3	1.19	$\infty$

The optimum travelling schedule is  $A \rightarrow D \rightarrow C \rightarrow E \rightarrow B \rightarrow A$

Total distance travelled by the fuzzy octagonal number is,  $\{2,3,4,5,8,9,12,13\} + \{4,5,7,8,10,11,12,13\} + \{7,8,10,11,12,13,14,15\} + \{4,5,8,9,10,11,14,15\} + \{5,6,7,8,9,10,12,13\}$   
 $= (23,28,36,41,49,54,64,69)$

Total distance travelled = 91

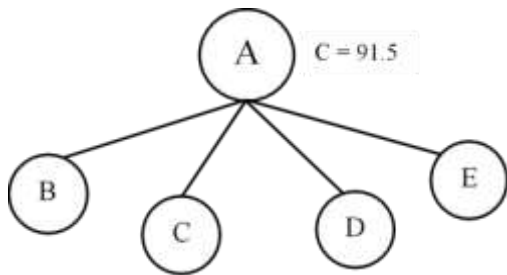
Using branch and bound technique,

	A	B	C	D	E
A	$\infty$	18	19.5	14	21.5
B	17.5	$\infty$	22.5	18	25
C	22	23.5	$\infty$	25	22.5
D	26.5	20.5	19	$\infty$	20
E	22.5	18.5	24.5	22	$\infty$

Applying row reduction and column reduction, the above matrix becomes

	A	B	C	D	E	Min
A	$\infty$	4	5.5	0	7	14
B	0	$\infty$	5	0.5	7	17.5
C	0	1.5	$\infty$	3	0	22
D	7.5	1.5	0	$\infty$	0.5	19
E	4	0	6	3.5	$\infty$	18.5
<b>Min</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0.5</b>	<b>r = 91.5</b>





B = Make row A and column B as  $\infty$ ,

	A	B	C	D	E	Reduction
A	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	
B	0	$\infty$	5	0.5	7	0
C	0	$\infty$	$\infty$	3	0	0
D	7.5	$\infty$	0	$\infty$	0.5	0
E	0.5	$\infty$	2.5	0	$\infty$	3.5
Reduction	0		0	0	0	<b><math>\hat{r}=3.5</math></b>

$$C(A,B) + r + \hat{r} = 4 + 91.5 + 3.5 = 99$$

C = Make row A and column C as  $\infty$ ,

	A	B	C	D	E	Reduction
A	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	
B	0	$\infty$	$\infty$	0.5	7.5	0
C	0	1.5	$\infty$	3	0	0
D	7.5	1.5	$\infty$	$\infty$	0.5	0.5
E	4	0	$\infty$	3.5	$\infty$	0
Reduction	0	0		0	0.5	<b><math>\hat{r}=1</math></b>

$$C(A,C) + r + \hat{r} = 5.5 + 91.5 + 1 = 98$$

D = Make row A and column D as  $\infty$ ,

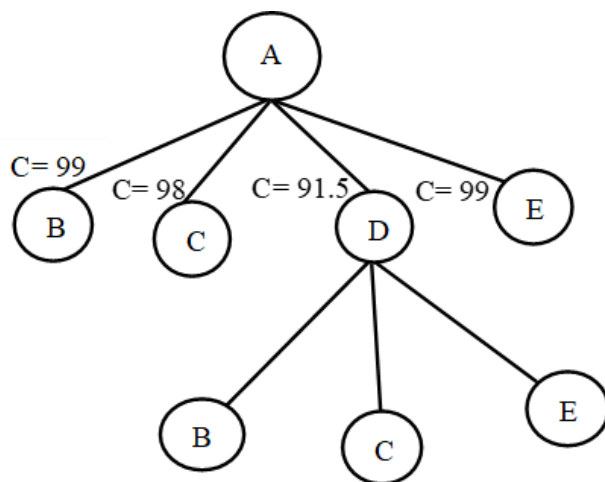
	A	B	C	D	E	Reduction
A	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	
B	0	$\infty$	5	$\infty$	7	0
C	0	1.5	$\infty$	$\infty$	0	0
D	7.5	1.5	0	$\infty$	0.5	0
E	4	0	6	$\infty$	$\infty$	0
Reduction	0	0	0		0	<b><math>\hat{r}=0</math></b>

$$C(A,D) + r + \hat{r} = 0 + 91.5 + 0 = 91.5$$

E = Make row A and column E as  $\infty$ ,

	A	B	C	D	E	Reduction
A	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	
B	0	$\infty$	5	0	$\infty$	0
C	0	1.5	$\infty$	2.5	$\infty$	0
D	7.5	1.5	0	$\infty$	$\infty$	0
E	4	0	6	3	$\infty$	0
Reduction	0	0	0	0.5		<b><math>\hat{r}=0.5</math></b>

$$C(A, E) + r + \hat{r} = 7 + 91.5 + 0.5 = 99$$



B = Make D row and B column as  $\infty$ ,

	A	B	C	D	E	Reduction
A	$\infty$	$\infty$	3	0	7	0
B	0	$\infty$	2.5	0.5	7	0
C	0	$\infty$	$\infty$	3	0	0
D	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	
E	0.5	$\infty$	0	0	$\infty$	3.5
Reduction	0		2.5	0	0	<b><math>\hat{r}=6</math></b>

$$C(D, B) + r + \hat{r} = 1.5 + 91.5 + 6 = 99$$

C = Make D row and C column as  $\infty$ ,

	A	B	C	D	E	Reduction
A	$\infty$	4	$\infty$	0	7	0
B	0	$\infty$	$\infty$	0.5	7	0
C	0	1.5	$\infty$	3	0	0
D	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	
E	4	0	$\infty$	3.5	$\infty$	0

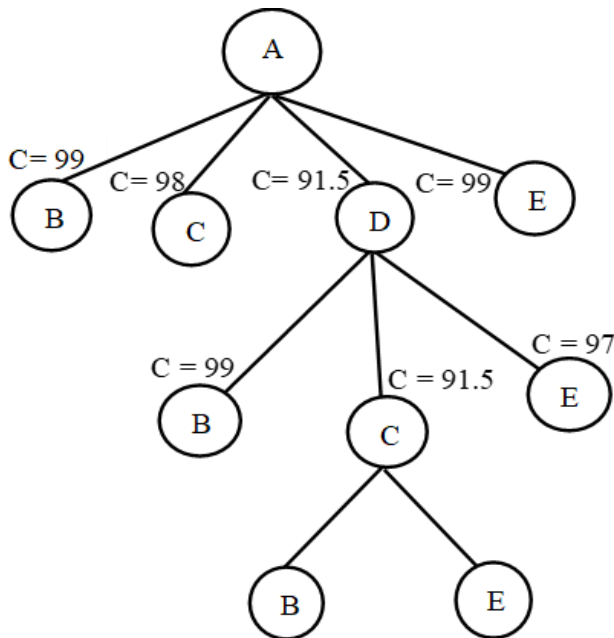
Reduction	0	0		0	0	$\hat{r} = 0$
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$C(D, C) + r + \hat{r} = 0 + 91.5 + 0 = 91.5$

E = Make D row and E column as  $\infty$ ,

	A	B	C	D	E	Reduction
A	$\infty$	4	0.5	0	$\infty$	0
B	0	$\infty$	0	0.5	$\infty$	0
C	0	1.5	$\infty$	3	$\infty$	0
D	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	
E	4	0	1	3.5	$\infty$	0
Reduction	0	0	5	0		$\hat{r} = 5$

$C(D, E) + r + \hat{r} = 0.5 + 91.5 + 5 = 97$



B = Make C row and B column as  $\infty$ ,

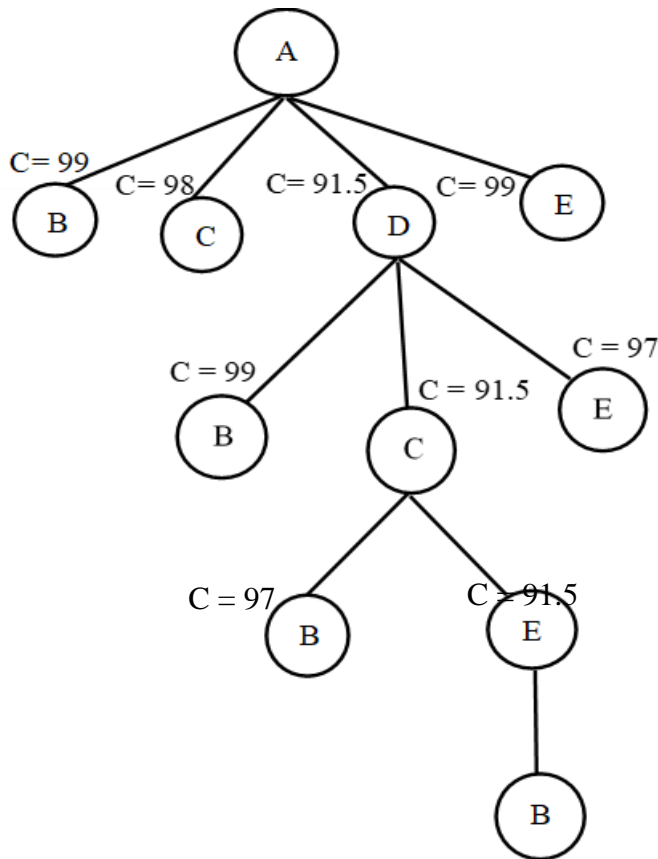
	A	B	C	D	E	Reduction
A	$\infty$	$\infty$	5.5	0	6.5	0
B	0	$\infty$	5	0.5	6.5	0
C	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	
D	7.5	$\infty$	0	$\infty$	0	0
E	0.5	$\infty$	2.5	0	$\infty$	3.5
Reduction	0		0	0	0.5	$\hat{r} = 4$

$C(C, B) + r + \hat{r} = 1.5 + 91.5 + 4 = 97$

E = Make C row and E column as  $\infty$ ,

	A	B	C	D	E	Reduction
A	$\infty$	4	5.5	0	$\infty$	0
B	0	$\infty$	5	0.5	$\infty$	0
C	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	
D	7.5	1.5	0	$\infty$	$\infty$	0
E	4	0	6	3.5	$\infty$	0
Reduction	0	0	0	0		$\hat{r} = 0$

$$C(C, E) + r + \hat{r} = 0 + 91.5 + 0 = 91.5$$



B = Make E row and B column as  $\infty$ ,

	A	B	C	D	E	Reduction
A	$\infty$	$\infty$	5.5	0	7	0
B	0	$\infty$	5	0.5	7	0
C	0	$\infty$	$\infty$	3	0	0
D	7.5	$\infty$	0	$\infty$	0.5	0
E	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	
Reduction	0		0	0		$\hat{r} = 0$

$$C(E, B) + r + \hat{r} = 0 + 91.5 + 0 = 91.5$$

The travelling schedule is

$$A \rightarrow D \rightarrow C \rightarrow E \rightarrow B \rightarrow A$$

Total distance travelled = 91.5

#### 4. Conclusion

In this paper, we derived the fuzzy travelling salesman problem using octagonal fuzzy numbers in three different methods with  $\alpha$ -cut ranking technique. Also a numerical example is discussed and also observed the answers.

	Hungarian Method	One Matrix Method	Branch and Bound method
Total distance travelled	91	91	91.5
Critical path	A → D → C → E → B → A	A → D → C → E → B → A	A → D → C → E → B → A

The three methods .i.e., Hungarian Method, One Matrix Method, Branch and Bound Method are useful for deriving the Critical path and Total distance travelled for the Fuzzy Travelling Salesman Problem (FTSP). In this Branch and Bound Method is very efficient for finding the critical path for FTSP.

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