International Journal of Advanced Science and Technology
Vol. 29, No.02, (2020), pp. 2487-2495

On Almost Contra θg*s−Continuous Functions In Topological Spaces

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Abstract

*In this paper, we introduce and investigate a new class of continuity called contra θg*s-continuous functions and almost contra θg*s-continuous functions. Some characterizations and several properties concerning the some is obtained.*

Keywords: *contra θg*s-continuous function, almost contra θg*s-continuous function.*

1. Introduction

Dontchev[8] in 1966, presented a new notion of continuous function called contra-continuity. This notion is a strong S-closedness in topological space. A new weaker form of this class of functions called contra semi continuous functions is introduced and investigated by Dontchev and Noiri[10]. Recently, Sathismohan[20] et al have introduced and studied the concepts of θg⋆-closed sets and obtained some interesting results. Further, he[21] studied the notion of θ-generalized star semi closed sets and invested its basic properties. This research work aims to introduce and study the concept of contra θg⋆s-continuous functions and almost contra θg⋆s-continuous functions in topological spaces and analyze some of their properties.

2. Preliminaries

Throughout this paper the space (X,τ) represents the topological spaces on which no separation axioms are assumed unless otherwise mentioned. Let A be a subset of a space (X, τ) then cl(A), int(A) and A^c denote the closure of A, interior of A and complement of A respectively.

Definition 2.1. A subset A of space (X, τ) is called

(1) semi-closed set [5], if int(cl(A))⊆A
(2) regular closed set [24], if A=cl(int(A))

The complements of the above mentioned closed sets are called their respective open sets. The semi-closure [5] (resp. r-closure [24]) of a subset A of X, denoted by scl(A) (resp. rcl(A)) is defined to be the intersection of all semi-closed (resp. r-closed) of (X, τ) containing A.

Definition 2.2. [25] A point x of a space (X, τ) is called θ-adherent point of a subset A of X if cl(U)∩A≠ϕ, for every open set U containing x. The set of all θ-adherents points of A is called the θ-closure of A and is denoted by $cl_\theta(A)$. A subset A of a space X is called θ-closed if and only if $A=cl_\theta(A)$. The complement of a θ-closed set is called θ-open.

Definition 2.3. [7] A point x of a space (X, τ) is called semi θ-cluster point of A if A∩scl(U)≠ϕ, for every semi-open set U containing x. The set of all semi θ-cluster points of A is called semi θ-closure of A and is denoted by $scl_\theta(A)$. Hence, a subset A is called semi θ-closed if $scl_\theta(A)=A$. The complement of a semi θ-closed set is called semi θ-open set.

Definition 2.4. A subset A of a space (X, τ) is called

(1) a generalized closed (briefly. g-closed) set [13], if cl(A)⊆U whenever A⊆U and U is open in (X, τ).
(2) a regular generalized closed (briefly. rg-closed) set [19], if cl(A)⊆U whenever A⊆U and U is r-open in (X, τ).
(3) a α generalized regular closed (briefly. αgr-closed) set [24], if αcl(A)⊆U whenever A⊆U and U is r-open in (X, τ).

(4) a regular ω-generalized closed (briefly. rωg-closed) set [14], if cl(int(A))⊆A whenever A⊆U and U is r-open in (X, τ).
(5) a θ-generalized closed (briefly. θg-closed) set [9], if $cl_\theta(A)$⊆U whenever A⊆U and U is open in (X, τ).
(6) a g^*sr-closed set [22], if rcl(A)⊆U whenever A⊆U and U is gs-open in (X, τ).
(7) a gpr-closed set [12], if pcl(A)⊆U whenever A⊆U and U is r-open in (X, τ).

(8) a θg^*-closed set [20], if $cl_\theta(A)$⊆U whenever A⊆U and U is g-open in (X, τ).

Definition 2.5. [21] The intersection of all θg*s-closed sets containing A is called θg*s-closure of A and is denoted by θg*scl(A). A set A is θg*s-closed set if and only if θg*scl(A)=A.

Definition 2.6. [26] A clopen set in a topological space is a set which is both open and closed.

Definition 2.7. [21] A function f : X→Y is called θg*s-continuous if $f^{-1}(V)$ is θg*s-closed set in X for every closed set V in Y.

Definition 2.8. A function f : (X, τ) →(Y, σ) is called
(1) contra continuous [8] if $f^{-1}(V)$ is θ-closed set of (X, τ) for every open set V of (Y, σ).
(2) contra θ-continuous [18] if $f^{-1}(V)$ is θ-closed set of (X, τ) for every open set V of (Y, σ).
(3) contra semi θ-continuous [7] if $f^{-1}(V)$ semi θ-closed set of (X, τ) for every open set V of (Y, σ).
(4) contra rg-continuous [19] if $f^{-1}(V)$ is rg-closed set of (X, τ) for every open set V of (Y, σ).
(5) contra αgr-continuous [24] if $f^{-1}(V)$ is αgr-closed set of (X, τ) for every open set V of (Y, σ).
(6) contra rωg-continuous [14] if $f^{-1}(V)$ is rωg-closed set of (X, τ) for every open set V of (Y, σ).
(7) contra gpr-continuous[12] if $f^{-1}(V)$ is gpr-closed set of (X, τ) for every open set V of (Y, σ).
(8) contra g^*sr-continuous [22] if $f^{-1}(V)$ is g^*sr-closed set of (X, τ) for every open set V of (Y, σ).
(9) contra θg^*-continuous [20] if $f^{-1}(V)$ is θg^*-closed set of (X, τ) for every open set V of (Y, σ).

Definition 2.9. A space (X, τ) is called a
(1) T_b-space [6] if every gs-closed set in it is closed.
(2) $_{\theta g}T_{1/2}$*-space [21] if every θg*s-closed set of (X, τ) is a closed set.
(3) locally indiscrete [15] if every open subset of X is closed in X.
(4) regular [1], if for each closed set F of X and each point x∉F there exists disjoint open sets U and V such that, F⊆V and x∈U.

Definition 2.10. A function f : X→Y is said to be
(1) almost continuous [17] if $f^{-1}(V)$ is open in X for each regular open set V of Y.
(2) perfectly continuous [16] if $f^{-1}(V)$ is clopen in X for each open set V of Y.
(3) R-map if $f^{-1}(V)$ [3] is regular open in X for each regular open set V of Y.

Definition 2.11. Let A be a subset of a space (X, τ), the set ⋂{U∈τ : A⊂U} is called the kernal of A [2, 4] and is denoted by ker(A).

Lemma 2.12. [11] The following properties hold for a subsets A and B of a space X
(1) x∈ker(A) if and only if A∩F=ϕ for any closed set F of X containing x.
(2) A⊂ker(A) and A=ker(A) if A is open in X.
(3) If A⊂B, then ker(A)⊂ker(B).

3. Contra θg*s-continuous functions

In this section, we define the concepts of contra θg*s-continuous functions and also we discuss some of its properties.

Definition 3.1. A function f : (X, τ)→(Y, σ) is called contra θg*s- continuous if f^{-1}(V) is θg*s-closed set in (X, τ) for every open set V in (Y, σ).

Example 3.2. Let X = Y = { a, b, c, d }, τ = {ϕ, {b}, {d}, {b, d}, X} and σ = {ϕ, {d}, {c, d}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping. Then f is contra θg*s-continuous.

Theorem 3.3. Let f be a function from (X, τ) to (Y, σ).

(1) If f is contra r-continuous then it is contra θg*s-continuous.
(2) If f is contra θ-continuous then it is contra θg*s-continuous.
(3) If f is contra semi θ-continuous then it is contra θg*s-continuous
(4) If f is contra g*sr-continuous then it is contra θg*s-continuous.
(5) If f is contra θg*-continuous then it is contraθg*s- continuous.

Proof: Let f : (X, τ)→(Y, σ) is contra regular continuous. Let V be any open set in Y. The inverse image f^{-1}(V) is regular closed in X. Since every regular closed set is θg*s-closed. Hence f^{-1}(V) is θg*s- closed in (X, τ). Hence f is contraθg*s-continuous.
The proof of (2) to (5) is obvious.
The converse of the above are not true in general as it can be seen from the following examples.

Example 3.4. Let X = Y = {a, b, c}, τ = {ϕ, {a}, {a, c}, X} and

σ = {ϕ, {c}, {b, c}, Y}. Define f : (X, τ)→(Y, σ) by f(a) =a, f(b) =c and f(c)=b. Then f is contra θg*s-continuous. Since f^{-1}(c)=b is not r-closed in X. So f is not contra r-continuous.

Example 3.5. Let X = Y = {a, b, c}, τ = {ϕ, {c}, {a, c}, {b, c}, X} and σ = {ϕ, {b, c}, Y}. Define f : (X, τ)→(Y, σ) by f(a)=c, f(b)=b and f(c)=a. Then f is contra θg*s-continuous. Since f^{-1}({b,c})={a, b} is not θ-closed. So f is not contra θ-continuous.

Example 3.6. Let X = Y = {a, b, c, d}, τ = {ϕ, {a, b, c}, X} and σ ={ϕ,{a, c, d},Y}. Define f : (X, τ)→(Y, σ) by identity mapping. Then f is contra θg*s-continuous. Since f^{-1}({a, c, d})={a, c, d} is not semi θ-closed. So f is not contra semi θ-continuous.

Example 3.7. Let X = Y = {a, b, c, d}, τ = { ϕ, {a}, {a, b}, {a, b, c}, X} and σ = {ϕ, {a, b, d}, Y}. Define f : (X, τ)→(Y, σ) by identity mapping. Then f is contra θg*s-continuous. Since f^{-1}({a,b,d})={ a , b, d } is not g*sr-closed. So f is not contra g*sr-continuous.

Example 3.8. Let X = Y = {a, b, c}, τ = {ϕ, {a}, {b}, {a, b}, X} and σ = {ϕ, {b}, {c}, {b, c}, Y}. Define f : (X, τ)→(Y, σ) by identity mapping. then f is contra θg*s-continuous. Since f^{-1}(b)=b is not θg*-closed. So f is not contra θg*-continuous.

Theorem 3.9. Let f be a function from (X, τ) to (Y, σ).

(1) If f is contra θg*s-continuous then it is contra rg-continuous.
(2) If f is contra θg*s-continuous then it is contra αgr-continuous.
(3) If f is contra θg*s-continuous then it is contra rωg-continuous.
(4) If f is contra θg*s-continuous then it is contra gpr-continuous.

Proof: Let f : (X, τ)→(Y, σ) is contra θg*s-continuous. Let V be any open set in Y. The inverse image f^{-1}(V) is θg*s-closed in X. Since every θg*s-closed set is rg-closed. Hence f^{-1}(V) is rg-closed in (X, τ). Hence f is contra rg-continuous.
The proof of (2) to (4) is obvious.

Example 3.10. Let X = Y = { a, b, c }, τ = { ϕ , {b}, {a, c}, X} and σ = {ϕ, {a}, {a, c}, Y}. Define f : (X, τ)→(Y, σ) by identity mapping. Then f is contra rg-continuous. Since $f^{-1}(a)=a$ is not θg*s- closed. So f is not contra θg*s-continuous.

Example 3.11. Let X = Y = {a, b, c, d}, τ = {ϕ, {d}, {a, b, d}, X} and σ = {ϕ, {d}, {b, c, d}, Y}. Define f : (X, τ)→(Y, σ) by f(a)=d, f(b)=b, f(c)=c and f(d)=a. Then f is contra αgr-continuous. Since $f^{-1}(d)=a$ is not θg*s-closed. So f is not contra θg*s-continuous.

Example 3.12. Let X = Y = {a, b, c, d}, τ = {ϕ, {a, c, d}, X} and σ = {ϕ, {d}, {c, d}, Y}. Define f : (X, τ)→(Y, σ) by identity mapping. Then f is contra rωg-continuous. Since $f^{-1}(d)=d$ is not θg*s- closed. So f is not contra θg*s-continuous.

Example 3.13. Let X = Y = { a, b, c, d }, τ = {ϕ, { a, b}, X} and σ = {ϕ, {a, b}, {c, d}, Y}. Define f : (X, τ)→(Y, σ) by identity mapping. Then f is contra gpr-continuous. Since $f^{-1}(\{c, d\})=\{c, d\}$ is not θg*s- closed. So f is not contra θg*s-continuous.

Remark 3.14. The concept of θg*s-continuity and contra θg*s-continuity is independent as show in the following examples.

Example 3.15. Let X = Y = {a, b, c}, τ = {ϕ, {b}, {a, c}, X}and σ = {ϕ,{b,c}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping. Then f is contra θg*s-continuous but not θg*s-continuous. Since $f^{-1}(\{a\})=\{a\}$ is not θg*s-closed in X where {a} is closed in Y .

Example 3.16. Let X = Y = {a, b, c, d}, τ = {ϕ, {a}, {a, b}, X} and σ = {ϕ, {c, d}, Y}. Define f : (X, τ)→(Y, σ) by f(a)=c, f(b)=d, f(c)=a and f(d)=b. Then f is θg*s-continuous but not contra θg*s- continuous. Since $f^{-1}(\{c,d\})=\{a,b\}$is not θg*s-closed in X where {c, d} is open in Y .

Remark 3.17. The composition of two contra θg*s-continuous functions need not be contra θg*s-continuous as seen from the following example.

Example 3.18. Let X = Y = Z = {a, b, c, d}, τ = {ϕ, {d}, {c, d}, X}, σ = { ϕ , { a, b, c}, Y} and η = { ϕ, {d}, {a, d}, Z}. Define f : (X, τ)→(Y, σ) and g : (Y, σ)→(Z, η) by identity mapping. Then f and and g are contra θg*s-continuous. But$(g\circ f)^{-1}(d)=f^{-1}(g^{-1}(d))=f^{-1}(d)=\{d\}$ which is not θg*s-closed in X. Hence g∘f is not contra θg*s-continuous.

Theorem 3.19. If f : (X, τ)→(Y, σ) is a function, then the following are equivalent.

(i) f is contra θg*s-continuous.
(ii) For every closed set F of Y, $f^{-1}(F)$ is θg*s-open set of X.
(iii) For each x∈X and each closed set F of Y containing f(x), there exists θg*s-open set U containing x such that f(U)⊂F.
(iv) For each x∈X and each open set V of Y containing f (x), there exists θg*s-closed set K not containing x such that $f^{-1}(V)\subset K$.
(v) f (θg*scl(A))⊂ker(A) for every subset A of X.
(vi) $\theta g^*scl(f^{-1}(B))\subset f^{-1}(ker(B))$ for every subset B of Y .

Proof: (i)⇒(ii) Let F be a closed set in Y, then Y−F is an open s
et in Y. By (i), $f^{-1}(Y-F) = X-f^{-1}(F)$ is θg*s-closed set in X. This implies $f^{-1}(F)$ is θg*s-open set in X. Therefore, (ii) holds.

(ii) ⇒ (i) Let G be an open set of Y. Then Y−G is a closed set in Y. By (ii), $f^{-1}(Y-G)$ is θg*s-open set in X. This implies $X-f^{-1}(G)$ is θg*s-open set in X, which implies $f^{-1}(G)$ is θg*s-closed set in X. Therefore, (i) holds.

(ii)⇒(iii) Let F be a closed set in Y containing f(x), then $x\in f^{-1}(F)$. By (ii), $f^{-1}(F)$ is θg*s-open containing x. Let $U=f^{-1}(F)$,implies $f(U)=f(f^{-1}(F))\subset F$. Therefore (iii) holds.

(iii)⇒(ii) Let F be a closed set in Y containing f(x), then $x\in f^{-1}(F)$. From (iii), there exists θg*s-open U_x containing x such that $f(U_x)\subset F$. That is $U_x\subset f^{-1}(F)$. Thus $f^{-1}(F)=\bigcap\{U_x$:

$x\in f^{-1}(F)\}$ is a $f^{-1}(F)$ is θg*s-open set of X.

(iii)⇒(iv) Let V be an open set in Y not containing f(x). Then Y−V is closed set in Y containing f(x). From (iii), there exists a θg*s-open set U in X containing x such that f(U)⊂Y−V . This implies $U\subset f^{-1}(Y-V)=X-f^{-1}(V)$. Hence, $f^{-1}(V)\subset X-U$, Set K=X−U, is θg*s-closed set not containing x in X.

(iv)⇒(iii) Let F be a closed set in Y containing f(x). Then Y−F is an open set in Y not containing f(x). From (iv), there exists θg*s- closed set K in X not containing x such that $f^{-1}(Y-F)\subset K$. This implies $X-f^{-1}(F)\subset K$. Hence, $X-K\subset f^{-1}(F)$, that is f(X−K)⊂F and X−K is θg*s-open set containing x in X.

(ii)⇒(v) Let A be any subset of X. Suppose y∉ker(f(A)). Then by lemma 2.12, there exists a closed set F in Y containing y such that f(A)∩F=ϕ. Thus we have, $A\cap f^{-1}(F)=\phi$. Therefore $A\subset X-f^{-1}(F)$. By (ii), $f^{-1}(F)$ is θg*s-open set in X and hence $X-f^{-1}(F)$ is θg*s-closed set in X. Therefore, $\theta g^*scl(X-f^{-1}(F))=X-f^{-1}(F)$. Now $A\subset X-f^{-1}(F)$, implies $\theta g^*scl(A)-\theta g^*scl(X-f^{-1}(F))=X-f^{-1}(F)$. Therefore $\theta g^*scl(A)\cap f^{-1}(F)=\phi$, implies f(θg*scl(A))∩F=ϕ and y∉θg*scl(A). Hence f(θg*scl(A))⊂ker(A) for every subset A of X.

(v)⇒(vi) Let B⊂Y, then $f^{-1}(B)\subset X$. By (iv) and lemma 2.12, we have, $f(\theta g^*scl(f^{-1}(B)))\subset ker(f(f^{-1}(B)))\subset ker(B)$. Thus, $\theta g^*scl(f^{-1}(B))\subset f^{-1}(ker(B))$ for every subset B of Y.

(vi)⇒(i) Let V be any open subset of Y . Then by (vi) and lemma 2.12, $\theta g^*scl(f^{-1}(V)\subset f^{-1}(ker(V))=f^{-1}(V)$ and $\theta g^*scl(f^{-1}(V))=f^{-1}(V)$. Therefore, $f^{-1}(V)$ is θg*s-closed set in X. This shows that f is contra θg*s-continuous.

Remark 3.20. If a function f : (X, τ)→(Y, σ) is contra θg*s- continuous and X is ${}_{\theta g}T_{1/2}$*-space, then f is contra continuous.

Definition 3.21. A space X is called locally θg*s-indiscrete if every θg*s-open set is closed in X.

Theorem 3.22. If a function f : (X, τ)→(Y, σ) is contra θg*s- continuous and X is locally θg*s-indiscrete space, then f is continuous. Proof: Let U be an open set in Y. Since f is contra θg*s-continuous and X is locally θg*s-indiscrete space, implies $f^{-1}(U)$ is an open set in X. Therefore f is continuous.

Theorem 3.23. (i) If a function f : X→Y is θg*s-coninuous and X is locally θg*s-indiscrete space, then f is contra continuous.

(ii) If a function f : X→Y is contra θg*s-continuous and X is T_b-space then f is contra continuous.

Proof: (i) Let V be an open set in Y. Since f is θg*s-continuous, $f^{-1}(V)$ is θg*s-open set in X and X is locally θg*s-indiscrete space, implies $f^{-1}(V)$ is a closed set in X. Therefore f is contra continuous.

(ii) Let V be an open set in Y. Since f is contra θg*s-continuous, $f^{-1}(V)$ is θg*s-closed set in X and every θg*s-closed set is gs-closed set and X is T_b-space, implies $f^{-1}(V)$ is closed set in X. Therefore f is contra continuous.

Remark 3.24. If f : X→Y is contra θg*s-continuous and g : Y→Z is continuous, then g∘f : X→Z is contra θg*s-continuous.

Theorem 3.25. Let f : X→Y is contra θg*s-continuous and g : Y→Z is θg*s-continuous. If Y is ${}_{\theta g}T_{1/2}$*-space, then g∘f : X→Z is contra θg*s-continuous.

Proof: Let V be any open set in Z. Since g is θg*s-continuous $g^{-1}(V)$ is θg*s-open in Y and Y is ${}_{\theta g}T_{1/2}$*-space implies $g^{-1}(V)$ is open in Y. Since f is contra θg*s-continuous $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is θg*s-closed sets in X. Therefore, g∘f is contra θg*s-continuous.

Definition 3.26. If f : X→Y is said to be strongly θg*s-open (or strongly θg*s-closed) if image of every θg*s-open (resp. θg*s-closed) set of X is θg*s-open (resp. θg*s-closed) set in Y .

Theorem 3.27. If f : X→Y is surjective θg*s-open (or strongly θg*s-closed) and g : Y→Z is a function such that g∘f : X→Z is contra θg*s-continuous, then g is contra θg*s-continuous.

Proof: Let V be any closed (resp. open) set in Z. Since g∘f is contra θg*s-continuous, implies $(g\circ f)^{-1}(V)=f^{-1}(g^{-1}(V))$ is θg*s-open (resp. θg*s-closed). Since f is surjective and strongly θg*s-open (or strongly θg*s-closed), implies $f(f^{-1}(g^{-1}(V)))=g^{-1}(V)$ is θg*s-open (or θg*s- closed). Therefore g is contra θg*s-continuous.

Lemma 3.28. If A is g-open and θg*s-closed in a space (X, τ), then
A is clopen.

Theorem 3.29. If a function f : X→Y is contra θg*s-continuous and g-continuous, then f is perfectly continuous.
Proof: Let U be an open set in Y. Since f is contra θg*s-continuous and g-continuous, $f^{-1}(U)$ is θg*s-closed and g-open, by Lemma 3.28, $f^{-1}(U)$ is clopen. Then f is perfectly continuous.

Theorem 3.30. If a function f : X→Y is contra θg*s-continuous and Y is regular, then f is θg*s-continuous.
Proof: Let x be an arbitrary point of X and U be an open set of Y containing f(x). Since Y is regular, there exists an open set W in Y containing f(x) such that cl(W)⊂U. Since f is contra θg*s- continuous, there exists V∈θg*sO(X, x) such that f(V)⊂cl(W). Then f(V)⊂ cl(W)⊂U. Hence f is θg*s-continuous.

4. Almost contra θg*s-continuous functions

In this section, we study the concepts of almost contra θg*s-continuous functions and also we discuss some of their properties.

Definition 4.1. A function f : X→Y is said to be almost contra θg*s-continuous if $f^{-1}(V)$ is θg*s-closed in X for each regular open set V in Y.

Example 4.2. Let X = Y = {a, b, c}, τ = {ϕ, {a}, {c}, {a, b}, {a, c}, X} and σ = {ϕ, {b}, {c}, {a, b}, {b, c}, Y}. Define f : (X, τ)→(Y, σ) by identity mapping. Then f : (X, τ)→(Y, σ) is almost contra θg*s- continuous.

Theorem 4.3. Let f be a function from (X, τ) to (Y, σ).
(1) If f is almost contra r-continuous then it is almost contra θg*s- continuous.
(2) If f is almost contra θ-continuous then it is almost contra θg*s- continuous.
(3) If f is almost contra semi-θ continuous then it is almost contra θg*s-continuous.
(4) If f is almost contra g*sr-continuous then it is almost contra θg*s-continuous.
(5) If f is almost contra θg*-continuous then it is almost contra θg*s-continuous.

Proof: Let f : (X, τ)→(Y, σ) is almost contra regular continuous. Let V be any regular open set in Y. The inverse image $f^{-1}(V)$ is regular closed in X. Since every regular closed set is θg*s-closed. Hence $f^{-1}(V)$ is θg*s-closed in (X, τ). Hence f is almost contra θg*s- continuous.
The proof of (2) to (5) is obvious.
The converse of the above are not true in general as it can be seen from the following examples.

Example 4.4. Let X = Y = {a, b, c}, τ = {ϕ, {a}, {b}, {a, b}, X} and σ = {ϕ, {c}, {a, b}, Y}. Define f : (X, τ)→(Y, σ) by identity mapping. Then f : (X, τ)→(Y, σ) is almost contra θg*s-continuous. Since $f^{-1}(\{a, b\})=\{a, b\}$ is not regular closed (resp. θ-closed, semi-θ closed, g*sr-closed and θg*-closed) is θg*s-closed).

Theorem 4.5. Let f be a function from (X, τ) to (Y, σ).
(1) If f is almost contra θg*s-continuous then it is almost contra rg-continuous.
(2) If f is almost contra θg*s-continuous then it is almost contra αgr-continuous.
(3) If f is almost contra θg*s-continuous then it is almost contra rωg-continuous.

(4) If f is almost contra θg*s-continuous then it is almost contra gpr-continuous.

Proof: Let f : (X, τ)→(Y, σ) is almost contra θg*s-continuous. Let V be any regular open set in Y. The inverse image $f^{-1}(V)$ is θg*s-closed in X. Since every θg*s-closed set is rg-closed. Hence $f^{-1}(V)$ is rg- closed in (X, τ). Hence f is almost contrarg-continuous.
The proof of (2) to (4) is obvious.
The converse of the above are not true in general as it can be seen from the following examples.

Example 4.6. Let X = Y = {a, b, c}, τ = {ϕ, {a}, {b}, {a, b}, {a, c}, X} and σ = {ϕ,{a},{c},{a,b},{a,c},Y}. Define f : (X, τ)→(Y, σ) by identity mapping. Then f : (X, τ) → (Y, σ) is almost contra rg- continuous (resp. almost contra αgr-continuous, almost contra rωg- continuous and almost contra gpr-continuous). Since $f^{-1}(\{a,b\}) = \{a,b\}$ is not θg*s-closed.

Theorem 4.7. If X is $_{\theta g}T_{1/2}$*-space and f : X →Y is almost contra θg*s-continuous, then it is almost contra continuous.
Proof: Let U be a regular open set in Y. Since f is almost contra θg*s-continuous $f^{-1}(U)$ is θg*s-closed set in X and X is $T_{\theta g*s}$-space, which implies $f^{-1}(U)$ is closed set in X. Therefore f is almost contra continuous.

Theorem 4.8. If a function f : X→Y is almost contra θg*s- continuous and X is locally θg*s-indiscrete space, then f is almost continuous.
Proof: Let U be a regular open set in Y . Since f is almost contra θg*s-continuous $f^{-1}(U)$ is θg*s-closed set in X and X is locally θg*s- indiscrete space, which implies $f^{-1}(U)$ is an open set in X. Therefore f is almost continuous.

Theorem 4.9. If f : X→Y is contra θg*s-continuous then it is almost contra θg*s-continuous.
Proof: Obvious, because every regular open set is open set.

Theorem 4.10. For a function f : X→Y the following are equivalent:
(i) f almost contra θg*s-continuous.
(ii) for every regular closed set F of Y, $f^{-1}(F)$ is θg*s-open set of X.
(iii) for each x∈X and each regular closed set F of Y containing f (x), there exists θg*s-open U containing x such that f(U)⊂F. (4) for each x∈X and each regular open set V of Y not containing f(x), there exists θg*s-closed set K not containing x such that $f^{-1}(V)$⊂K.
Proof: (i)⟹(ii) Let F be a regular closed set in Y, then Y−F is a regular open set in Y. By (i), $f^{-1}(Y-F)=X-f^{-1}(F)$ is θg*s-closed set in X. this implies $f^{-1}(F)$ is θg*s-open set in X. Therefore, (ii) holds.
(ii)⟹(i) Let G be a regular open set of Y. Then Y−G is a regular closed set in Y. By (ii), $f^{-1}(Y-G)$ is θg*s-open set in X. This implies $X-f^{-1}(G)$ is θg*s-open set in X, which implies $f^{-1}(G)$ is θg*s-closed set in X. Therefore, (i) holds.
(ii)⟹(iii) Let F be a regular closed set in Y containing f(x), which implies $x\in f^{-1}(F)$ By (ii), $f^{-1}(F)$ is θg*s-open in X containing x. Set $U=f^{-1}(F)$, which implies U is θg*s-open in X containing x and $f(U)=f(f^{-1}(F))\subset F$. Therefore (iii) holds.
(iii)⟹(ii) Let F be a regular closed set in Y containing f(x), which implies $x\in f^{-1}(F)$. From (iii), there exists θg*s-open U_x in X containing x such that $f(U_x)\subset F$. That is $U_x\subset f^{-1}(F)$. Thus $f^{-1}(F)=\cup\{U_x : x\in f^{-1}(F)\}$, which is union of θg*s-open sets. Therefore, $f^{-1}(F)$ is θg*s-open set of X.
(iii)⇒(iv) Let V be a regular open set in Y not containing f(x). Then Y−V is a regular closed set in Y containing f(x). From (iii), there exists a θg*s-open set U in X containing x such that f(U)⊂Y−V. This implies $U\subset f^{-1}(Y-V)=X-f^{-1}(V)$. Hence, $f^{-1}(V)\subset X-U$. Set K=X−U, then K is θg*s-closed set not containing x in X such that $f^{-1}(V)$⊂K.
(iii) ⇒ (iii) Let F be a regular closed set in Y containing f(x). Then Y−F is a regular open set in Y not containing f(x). From (iv), there exists θg*s-closed set K in X not containing x such that $f^{-1}(Y-F)\subset K$. This implies $X-f^{-1}(F)\subset K$. Hence, $X-K\subset f^{-1}(F)$, that is f(X−K)⊂F.

Set U=X−K, then U is θg*s-open set containing x in X such that f(U)⊂F.

Theorem 4.11. For a function f : X→Y the followings are equivalent:

(i) f is almost contra θg*s-continuous.

(ii) f^{-1}(int(cl(G))) is θg*s-closed set in X for every open subset G of Y.

(iii) f^{-1}(cl(int(F))) is θg*s-open set in X for every closed subset F of Y.

Proof: (i)⟹(ii) Let G be an open set in Y . Then int(cl(G)) is regular open set in Y . By (i), f^{-1}(int(cl(G)))∈θg*sC(X).

(ii)⟹(i) Proof is obvious.

(i)⟹(iii) Let F be a closed set in Y . Then cl(int(G)) is regular closed set in Y . By (i), f^{-1}(cl(int(G)))∈θg*sO(X).

(iii)⟹(i) Proof is obvious.

Theorem 4.12. For two function f : X→Y and g : Y→ Z, let g∘f : X→Z is a composition function. Then the following properties hold

(i) If f is almost contra θg*s-continuous and g is an R-map, then g∘f is almost contra θg*s-continuous.

(ii) If f is almost contra θg*s-continuous and g is perfectly continuous, then g∘f is θg*s-continuous and contra θg*s-continuous.

(iii) If f is contra θg*s-continuous and g is almost continuous, then g∘f is almost contra θg*s-continuous.

Proof: (i) Let V be any regular open set in Z. Since g is an R-map, g^{-1}(V) is regular open in Y . Since f is an almost contra θg*s- continuous $f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$ is θg*s-closed set in X. Therefore, g∘f is almost contra θg*s-continuous.

(ii) Let V be any open set in Z. Since g is perfectly continuous, g^{-1}(V) is clopen in Y. Since f is almost contra θg*s-continuous $f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$ is θg*s-open and θg*s-closed set in X. Therefore, g∘f is θg*s-continuous and contra θg*s-continuous.

(iii) Let V be any regular open set in Z. Since g is almost continuous, g^{-1}(V) is open in Y . Since f is contra θg*s-continuous

$f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$ is θg*s-closed set in X. Therefore, g∘f is almost contra θg*s-continuous.

5. Acknowledgments

The authors are grateful to the Journal of Advanced Science and Technology for accepting and publishing the paper.

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