International Journal of Advanced Science and Technology
Vol. 29, No.02, (2020), pp. 2181-2189

A Study and Construction of Tightened- Normal- Tightened Sampling Scheme by Attributes under the Conditions of Zero-Inflated Poisson Distribution

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Abstract

No defect has found in the sampling inspection frequently and the manufacturing process is well fitted and maintained proper way. The suitable probability distribution of the number of defects is a zero-inflated Poisson (ZIP) distribution. This article gives a construction of Tightened- Normal- Tightened (TNT) sampling scheme by attributes using unity values are considered, under the conditions of zero-inflated Poisson distribution, when the number of defects follows a ZIP distribution. The operating characteristic (OC) function of the TNT sampling Scheme was derived. Scheme parameters were obtained for some sets of values of (P_N, P_T, p_1, α, p_2, β). Numerical illustrations are given to describe the determination of TNT by attributes under ZIP distribution and to study its performance in comparison with TNT by attributes under Poisson distribution.

Keywords: *Acceptance sampling; Operating Characteristic function; Sampling inspection by attributes; TNT; Zero-inflated Poisson distribution.*

1. Introduction

Acceptance sampling is a quality monitoring technique of the manufacturing product at all stages like, buying raw material qualities, checking product quality at every stage and the manufactured product quality at what the management has fixed. In an industry, during inspection of all manufacturing products errors may arise. So, it is practically impossible to do full inspection in many industries due to lack of time, high cost, and risks regarding product liability, etc. In modern industries, sampling plan design should be used in situations like checking incoming raw materials, where the entire production depends on the quality of the raw materials. In case of launching new products, the processed products should be checked for the quality assurance before being delivered to the market. In all these cases, acceptance sampling is used in order to make decisions either to accept or reject the lots based on the inspection of the samples. Acceptance sampling can be broadly classified into acceptance sampling by attributes and acceptance sampling by variables. Acceptance sampling by attributes consists of special sampling plan that is Tightened Normal Tightened sampling scheme. The Tightened Normal Tightened (TNT) sampling scheme devised by Calvin (1977). This scheme utilizes c=0 for different sample sizes of sampling plans and together with switching rules of Tightened to Normal and Normal to tightened, to build up the OC curve. This TNT scheme was designated as TNT-(n_1, n_2; 0). Another type of TNT scheme was developed by Soundararajan and Vijayaraghavan (1992) it was designated as TNT - (n; c_1, c_2). This is also a scheme involving the switching between two sampling plans. When the product is forthcoming in a stream of lots and given acceptance numbers are maintained, the TNT - (n; c_1, c_2) scheme is particularly appropriate. This scheme utilizes the single sampling normal plan with a sample size n and acceptance number c_2, as well as the single sampling tightened plan with a sample size n and acceptance number c_1. The sampling plans (n, c_1) and (n, c_2), together with the switching rules, build up the shoulder of the OC curve in a manner similar to that of the switching rules of M1L-STD105D (1963).

Today, due to the automation development the production processes are well designed. Once the fixed specifications of manufacturing units in the automation technology produce items that are nil defective. Sometimes it is a long duration process where some defects occur.

So many defects do not occur and few times defects occur. These types of situations in existing plans cannot be used. This situation more suitable to apply zero-inflated Poisson (ZIP) distribution. The ZIP distribution can be viewed as a mixture of a distribution which degenerates at zero and a Poisson distribution. ZIP distribution has been used in a wide range of disciplines such as agriculture, epidemiology, econometrics, public health, process control, medicine, manufacturing, etc. Some of the applications of ZIP distribution can be found in Bohning et al. (1999), Lambert (1992), Naya et al. (2008), Ridout et al. (1998), and Yang et al. (2011). Construction of control charts using ZIP distribution are discussed in Sim and Lim (2008) and Xie et al. (2001). Some theoretical aspects of ZIP distributions are mentioned in McLachlan and Peel (2000). Many researchers use and find valuable results to society improvement. This distribution was used in acceptance sampling plan, Loganathan and Shalini (2014) have attempts to determine SSPs by attributes under the conditions of ZIP distribution.

In this article attempts to determine TNT by attributes under the conditions of ZIP distribution. A brief introduction to this distribution and the operating characteristic (OC) function of TNT under the conditions of ZIP distribution are given. Designing TNT is presented and Illustrations are given.

2. Conditions for Application

- The production is steady so that results of past, present and future lots are broadly indicative of a continuing process.
- Lots are submitted sequentially in the order of production.
- Inspection is attributes with stable quality between lots for calculation of the OC curves.
- The sample units are selected from a big lot and production is continuous.
- Human involvement should be less in the manufacturing process.

3. OC Function of TNT under the Conditions of ZIP Distribution

TNT by attributes is specified by three parameters, say, N, n, c_N and c_T. The operating procedure of a TNT can be described as follows:

The operating procedure of TNT (n; C1, C2) plan is as follows:

1. Inspect using Tightened inspection with the sample size n and acceptance number c_T.

2. Switch to Normal inspection when 't' lots are accepted under Tightened inspection.

3. Inspect using Normal inspection with sample size n and acceptance number c_N.

4. Switch to Tightened inspection after a rejection if an additional lot is rejected in the next ‘s‘ lots.

the OC function of the TNT scheme is given by

$$P_a(p) = \frac{P_T(1-P_N^s)(1-P_T^t)(1-P_N) + P_N P_T^t(1-P_T)(2-P_N^s)}{(1-P_N^s)(1-P_T^t)(1-P_N) + P_T^t(1-P_T)(2-P_N^s)} \qquad (1)$$

The OC function of TNT scheme corresponds to the scheme OC function of MIL-STD-105D for s =4 and t =5 (Hald and Thyregod [10]; Dodge [11]; Calvin [1]).

In general, the values of $P_a(p)$ can be computed for different values of p using hyper-geometric probabilities. According to Schilling and Neubauer (2009), when $n/N \le 0.10$, n is large, p is small such that $np < 5$, the number of defects in the sample is distributed according to the Poisson distribution with mean $\lambda = np$. In this case, the OC function can be evaluated using the Poisson (λ) probabilities. When the manufacturing process is properly aligned, occurrence of defects would be a rare event. The number of defects for many sampled products would be zero. Under such circumstances, the appropriate probability distribution of the number of defects in the sampled products is a ZIP distribution.

The probability mass function of the ZIP (ϕ, λ) distribution is given by Lambert (1992) and McLachlan and Peel (2000)

$$P(X = x|\varphi,\lambda) = \varphi f(x) + (1-\varphi) P(X = x|\lambda) \quad (2)$$

where $$f(x) = \begin{cases} 1 \; if \; x = 0 \\ 0 \; if \; x \neq 0 \end{cases}$$

and $$P(X = x/\lambda) = \frac{e^{-\lambda}\lambda^x}{x!} \; when x = 0,1,2... \quad (3)$$

The above probability mass function can also be expressed as

$$(P(X = x/\phi,\lambda) =) \begin{cases} \varphi + (1-\varphi)e^{-\lambda}, when\; x = 0 \\ (1-\varphi)\frac{e^{-\lambda}\lambda^x}{x!}, \text{when } x = 1,2 \ldots .0 < \varphi < 1, \lambda > 0 \end{cases} \quad (4)$$

In this distribution, ϕ may be termed as the mixing proportion. ϕ and λ are the parameters of the ZIP distribution. According to McLachlan and Peel (2000), a ZIP distribution is a special kind of mixture distribution. The OC function of the TNT under the conditions of ZIP (ϕ, λ) distribution can be defined as

$$P_a(p) = \frac{P_T(1-P_N^s)(1-P_T^t)(1-P_N) + P_N P_T^t (1-P_T)(2-P_N^s)}{(1-P_N^s)(1-P_T^t)(1-P_N) + P_T^t(1-P_T)(2-P_N^s)} \quad (5)$$

where P_T and P_N are the proportion of lots expected to be accepted using tightened (n_σ, c_T) and normal (n_σ, c_N) attributes single sampling plans respectively. Under the assumption of Poisson distribution, the expression for P_T and P_N are given by

$$P_T = \varphi + (1-\varphi)e^{-\lambda} + \sum_{x=1}^{c_T}(1-\varphi)\frac{e^{-\lambda}\lambda^x}{x!} \quad (6)$$

$$P_N = \varphi + (1-\varphi)e^{-\lambda} + \sum_{x=1}^{c_N}(1-\varphi)\frac{e^{-\lambda}\lambda^x}{x!} \quad (7)$$

respectively. Equations (6) and (7) are substituted in (1) to find $P_a(p)$ values for given p, s, t, n, c_T, and c_N. As the individual values of x follows Poisson distribution with mean np. where $\lambda = np$. When c = 0, the lot acceptance probability becomes as

$$P_T = P_N = \varphi + (1-\varphi)e^{-\lambda} \quad (8)$$

Hence, for specified (p_1, α, p_2, β) and ϕ a zero-acceptance number sampling plan can be determined from

$$n = \frac{1}{p}\log e \frac{1-\phi}{P_a(p)-\phi} \quad (9)$$

satisfying $P_a(p_1) = 1-\alpha$ and $P_a(p_2) = \beta$. Here, p_1, α, p_2, and β denote, respectively, acceptable quality level, producer's risk, limiting quality level, and consumer's risk.

4. Designing TNT

For given ϕ and (p_1, α, p_2, β) a TNT is determined satisfying the conditions

$$P_a(p_1) = 1-\alpha \quad (10)$$

$$P_a(p_2) = \beta \quad (11)$$

so as to meet the requirements of both producer and consumer. The plan parameters n, c_T and c_N can be obtained for each set of values of ϕ, p_1, α, p_2, and β applying unity values approach. The values np_1 and np_2 satisfying respectively Equations (6) and (7) are termed as unity values (Schilling and Neubauer (2009)). The plan parameters can be tabulated for various combinations of (p_1, α, p_2, β). The use of operating ratio $R = np_2/np_1$ reduces the number of tables. Moreover, the operating ratio may be considered as a measure of discrimination of the sampling plans.

The plan parameters are determined for some sets of values of ϕ, p_1, α, p_2 and β under the conditions of the ZIP (ϕ, λ) distribution. The unity values are computed for various combinations of (ϕ, c, $P_a(p)$) and are given in Table 1. The values taken for ϕ are 0.0001, 0.01, 0.05, and 0.09 and for $P_a(p)$ are 0.99, 0.95, 0.90, 0.50, 0.20, and 0.10. The values considered for c_T and c_N in this computation are c_T = 1(1)15 and c_N = 1(1)15. The operating ratio values calculated corresponding to (α = 0.05, β = 0.10), (α = 0.10, β = 0.20), ϕ = 0.0001, 0.01, 0.05, 0.09 are listed in Table 2.

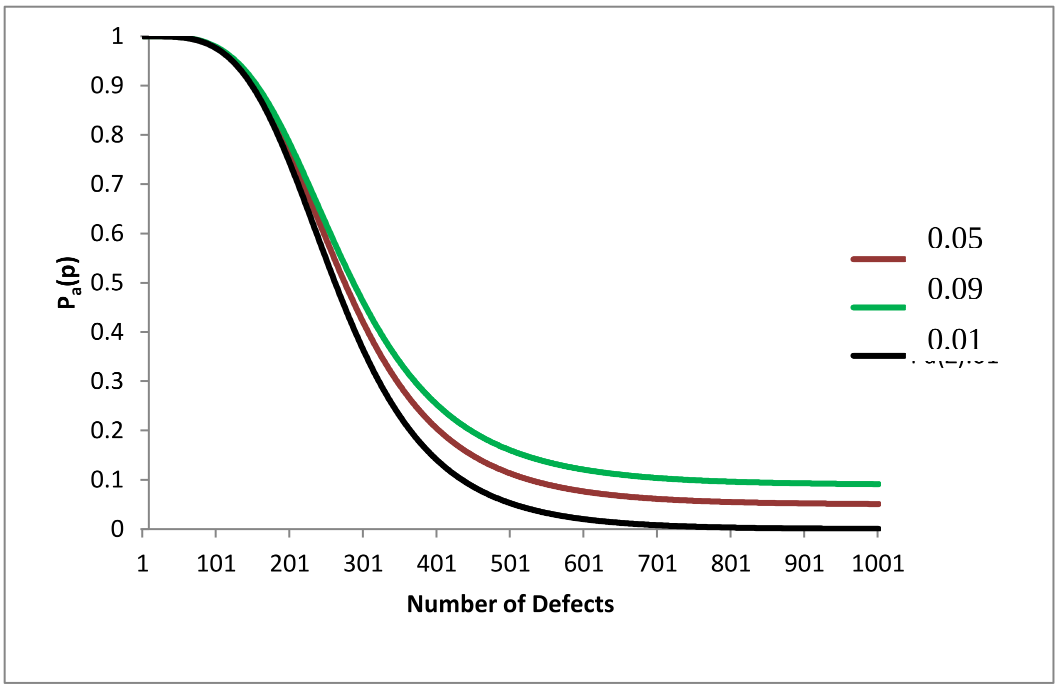

Figure 1: OC Curves of TNT (150; 5,1) with φ= 0.01, 0.05 and 0.09

For a given strength (p_1, α, p_2, β) and the value of ϕ, the plan parameters can be determined from these tables applying the following procedure:

Step 1 Compute the operating ratio $R = p_2/p_1$.

Step 2 Determine the acceptance number c from Table 2 corresponding to the values of ϕ, α, β, and to the value of R or it nearest.

Step 3 Determine the unity values np_1 and np_2 from Table 1 and calculate the sample size n as

Illustrations

The procedure of selecting sampling plans for a specified strength is described in this section with numerical illustrations. The significance of the attributes TNT under the conditions of ZIP distribution is

ISSN: 2005-4238 IJAST

also highlighted.

Illustration 1

Suppose that ϕ = 0.05, p_1 = 0.05, α = 0.05, p_2 = 0.5, and β = 0.10. For these values, the operating ratio can be calculated as R = 10. The acceptance number can be found from Table 2 as the acceptance number in the normal plan is c_N = 4 and tightened plan c_T = 2. The unity values corresponding to the values of ϕ, p_1, α, p_2, and β are obtained from Table 1 as np_1 =0.79944 and np_2 =1.3793.
Since

$$\frac{np_1}{p_1} = \frac{0.79944}{0.05} \approx 16 \; and \; \frac{np_2}{p_2} = \frac{7.9944}{0.5} \approx 16,$$

The number of products to be inspected is 16. Therefore, the sampling plan for the given specification scheme is (16; 4, 2).

According to Calvin (1977), the TNT under the conditions of Poisson distribution is also (16; 4, 2). It indicates that the sampling scheme under the conditions of Poisson distribution can be obtained under the conditions of ZIP distribution.

It may also be noted that observing non defects would not be an event occurring frequently when ϕ is small. In such case, the TNT under the conditions of ZIP distribution would be same as the TNT under the conditions of Poisson distribution. Hence, the TNT designed under the conditions of Poisson distribution will be a special case of the TNT under ZIP distribution.

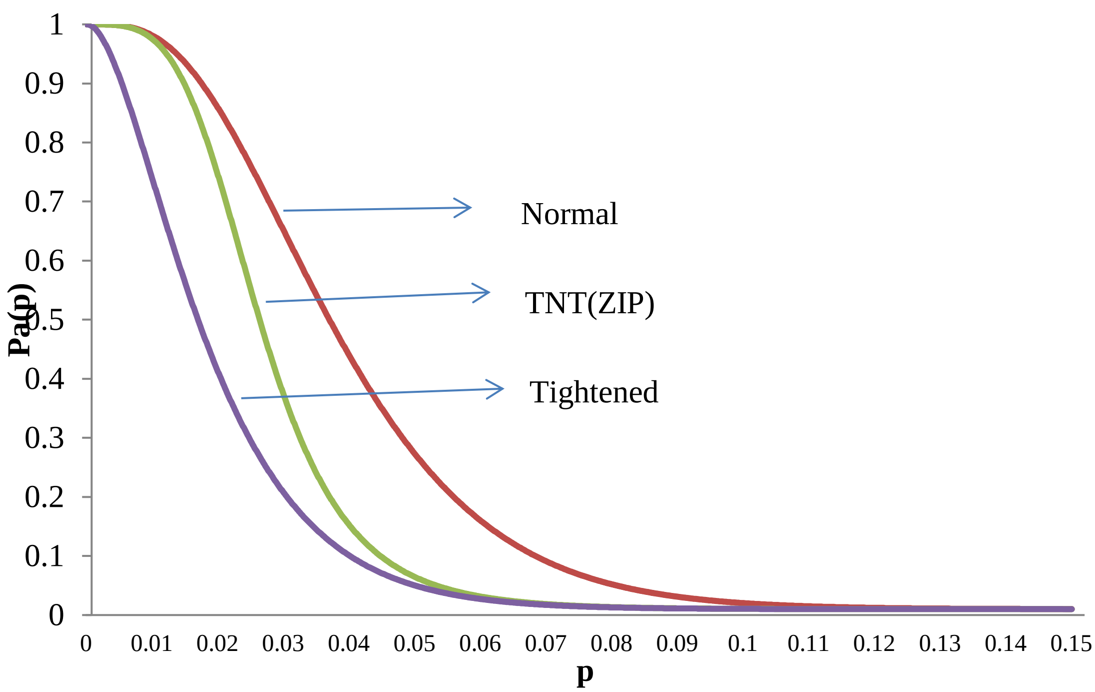

Figure 2: OC Curve of TNT (ZIP) (150; 5, 1) with φ = 0.01

Table 1: Unity values of TNT under the conditions of ZIP distribution

φ	CN	CT	Np					
			0.99	**0.95**	**0.9**	**0.5**	**0.2**	**0.1**
0.0001	2	1	0.149446	0.361927	0.573767	2.625667	4.271438	5.321352
	3	2	0.439143	0.832572	1.159545	3.62439	5.51205	6.67467
	3	1	0.149357	0.363015	0.581791	3.597978	5.51205	6.67467
	4	3	0.804664	1.379268	1.805023	4.621139	6.720957	7.994443
	4	2	0.441903	0.837061	1.175736	4.592801	6.720957	7.994443
	4	1	0.14937	0.363046	0.583141	4.577723	6.720957	7.994443
	5	4	1.267157	1.987879	2.506554	5.61993	7.900864	9.268288
	5	3	0.826244	1.386051	1.837857	5.588987	7.900864	9.268288
	5	2	0.443333	0.836239	1.179885	5.571213	7.900864	9.268288
	5	1	0.149378	0.36305	0.58333	5.562856	7.900864	9.268288
0.01	2	1	0.150375	0.361974	0.578496	2.646196	4.343924	5.462298
	3	2	0.434603	0.832266	1.163812	3.64805	5.586728	6.830365
	3	1	0.157196	0.365104	0.585331	3.621648	5.586728	6.830365
	4	3	0.814569	1.383844	1.81197	4.648123	6.800302	8.157776
	4	2	0.44398	0.840473	1.181087	4.619578	6.800302	8.157776
	4	1	0.150148	0.365117	0.586706	4.604496	6.800302	8.157776
	5	4	1.255052	1.988539	2.515015	5.649883	7.991293	9.447448
0.01	5	3	0.821544	1.383274	1.843027	5.618553	7.991293	9.447448
	5	2	0.434287	0.834989	1.18526	5.600786	7.991293	9.447448
	5	1	0.149378	0.36305	0.58333	5.592402	7.991293	9.447448
0.05	2	1	0.150148	0.373516	0.592573	2.738048	4.644924	6.222343
	3	2	0.440423	0.84665	1.185741	3.753515	5.924664	7.672815
	3	1	0.160365	0.37381	0.600294	3.723584	5.924664	7.672815
	4	3	0.826048	1.40304	1.84109	4.764009	7.170768	9.047306
	4	2	0.45074	0.854797	1.19673	4.734737	7.170768	9.047306
	4	1	0.153409	0.373838	0.601739	4.719655	7.170768	9.047306
	5	4	1.287115	2.017414	2.548595	5.777099	8.385004	10.41471
	5	3	0.838895	1.410264	1.87365	5.745559	8.385004	10.4147

								1
	5	2	0.45158	0.85401	1.208046	5.727766	8.385004	10.41471
	5	1	0.153414	0.373843	0.601973	5.719504	8.385004	10.41471

Table 1 (Continue…)

φ	**CN**	**CT**	**np**					
			0.99	**0.95**	**0.9**	**0.5**	**0.2**	**0.1**
0.09	2	1	0.164176	0.3823	0.609304	2.832325	5.05045	8.381171
	3	2	0.450071	0.861998	1.209253	3.863981	6.373367	9.938991
	3	1	0.160456	0.379827	0.616937	3.837998	6.373367	9.938991
	4	3	0.843581	1.41755	1.875432	4.892376	7.664177	11.49767
	4	2	0.450581	0.86455	1.227776	4.8627	7.664177	11.49767
	4	1	0.162693	0.384316	0.617969	4.847691	7.664177	11.49767
	5	4	1.282414	2.03858	2.593005	5.918277	8.910891	12.91873
	5	3	0.84313	1.422561	1.902966	5.886463	8.910891	12.91873
	5	2	0.447976	0.864976	1.232372	5.861066	8.910891	12.91873
	5	1	0.153414	0.383808	0.617817	5.861066	8.910891	12.91873

Table 2: Operating ratios of TNT under the conditions of ZIP distribution

φ	CN	CT	R	
			$\alpha = 0.05$ $\beta = 0.10$	$\alpha = 0.10$ $\beta = 0.20$
0.0001	2	1	14.70284	28.58181
	3	2	8.016927	12.55185
	3	1	18.38675	36.90514
	4	3	5.796148	8.352504
	4	2	9.550608	15.20914
	4	1	22.02044	44.99537
	5	4	4.6624	6.23511
	5	3	6.68683	9.562384
	5	2	11.0833	17.82152
	5	1	25.52893	52.89164
0.01	2	1	15.0903	28.88722
	3	2	8.206954	12.85478
	3	1	18.70799	35.53986
	4	3	5.89501	8.348341

	4	2	9.706178	15.31668
	4	1	22.34292	45.29054
	5	4	4.750949	6.367299
	5	3	6.829775	9.727164
	5	2	11.31446	18.40095
	5	1	26.02242	53.49701

Table 2 (Continue…)

φ	CN	CT	R	
			$\alpha = 0.05$ $\beta = 0.10$	$\alpha = 0.10$ $\beta = 0.20$
0.05	2	1	16.65883	30.93555
	3	2	9.062554	13.45222
	3	1	20.526	36.9449
	4	3	6.44836	8.680812
	4	2	10.58416	15.90887
	4	1	24.20112	46.74272
	5	4	5.162405	6.514571
	5	3	7.384935	9.995294
	5	2	12.19506	18.56813
	5	1	27.85849	54.65611
0.09	2	1	21.92304	30.76242
	3	2	11.53018	14.16081
	3	1	26.16715	39.72036
	4	3	8.110942	9.08529
	4	2	13.29902	17.00952
	4	1	29.91723	47.10831
	5	4	6.33712	6.948527
	5	3	9.081316	10.56882

5. Conclusion

Zero Inflated Poisson distribution is studied under the single sampling plan and implemented in the system which latter be in the schemes as well. In ZIPD a parameter called mixing proportion is introduced based on the situation and the need one can fix the value between 0 and 1. In a production process being well-monitored, occurrence of non defects would be more frequent and a ZIP distribution is the appropriate probability distribution to the number of defects per unit. The OC function of the TNT under the conditions of ZIP distribution has been derived. The procedure for designing TNT using unity values under the conditions of ZIP distribution is presented. Tables providing the TNT are presented for some specified strength. The TNT under Poisson distribution become special cases of the TNT under ZIP distribution. Also, the TNT under ZIP distribution reduces both producer's risk as well as consumer's risk. When inspection errors are taken into account, similar procedure can be followed to determine the plan parameters corresponding to the apparent lot fraction nonconforming.

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