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PUZZLING AND APUZZLING IN GRAPH THEORY

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Abstract
The notation G represents a graph with partition P among chromatic number with vertex coloring of graph using the numbers 1,2,…x, such that the addition of the numbers assigns a value to the partition segment are all same is puzzling and the addition of the numbers assigns a value to the partition segment are all different is apuzzling. In this article, the puzzling and apuzzling defined for various graphs and analyzed some properties in complete graph, bipartite graph, butterfly graph, friendship graph and Peterson graph.
Keywords: *Vertex coloring, chromatic number, puzzles, partition.*

1.Introduction

Graph theory is used in modeling and solving a lot of real world problems, games and puzzles. Graphs [1] are therefore mathematical structures used to model pairwise relations between objects. The set of nodes of a graph G is denoted as V(G). The set of edges of a graph G is denoted as E(G). Labeling a graph means placing a unique number at every node. A set of two vertices as every vertex is joined to the other. Considers if a movement between two vertices is possible, whatever its direction. Knowing joining makes it possible to find if it is possible to reach a vertices from another vertex within a graph.

2.Preliminaries

2.1 Definition

By vertex coloring is an assignment of labels or colors to each vertex of a graph such that no edges join two identically colored vertices.

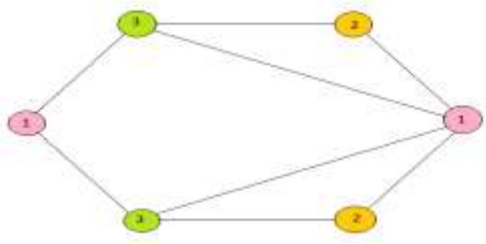

Figure 1. Vertex Colouring

2.2 Definition

In mathematics, [5] a graph partition is the reduction of a graph to a smaller graph by partitioning its set of nodes into mutually exclusive group. Edges of the original graph that cross between the graphs will produce edges in the partitioned graph.

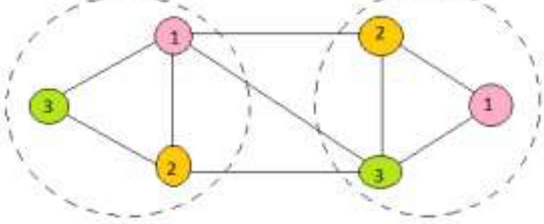

Figure 2.Graph Partition

2.3. Definition

[6] By chromatic number of a graph is the small scale number of colors needed to color the vertices of G so that no two adjacent vertices share the same color, i.e., the small scale value of K possible to obtain a K-coloring.

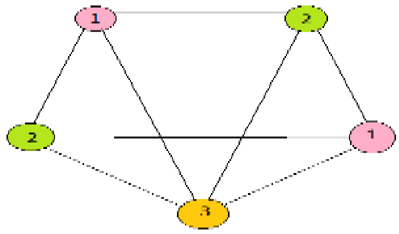

Figure 3.Chromatic Number

2.4 Definition

In [3], G be a connected graph with chromatic number χ(G). A puzzle on G is a partition such that there is exactly one vertex coloring with the property that the sum of the vertex labels of the partition pieces are all the same.

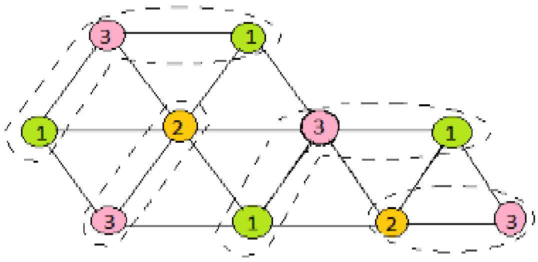

Figure 4. Puzzling Graph

2.5 Definition

In [3], An apuzzling on a graph G is a partition of G such that there is exactly one vertex coloring such that the sum of the vertex labels of the partition pieces are all different.

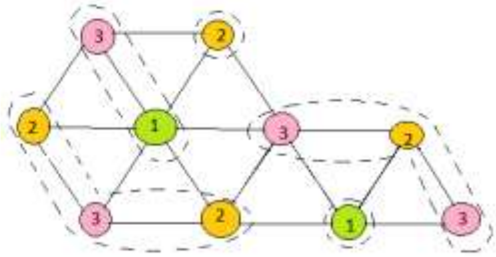

Figure 5.Apuzzling Graph

3. Puzzling and apuzzling in graphs

3.1 Theorem

The complete graph of n vertices are puzzling, if n>2.

Proof

Let G be a complete graph of n vertices that is Kn, chromatic number n. Complete graph vertices are n and the edges are n(n-1)/2. There is no equal coloring in complete graph. Now all the partition segment are all same length, is puzzling. In that segment are either odd or even the sums of the vertex labeling the partitions are same.

3.2 Example

In complete graph the vertices n=7 and the edges are 21, then the chromatic number also 7 (figure 6).

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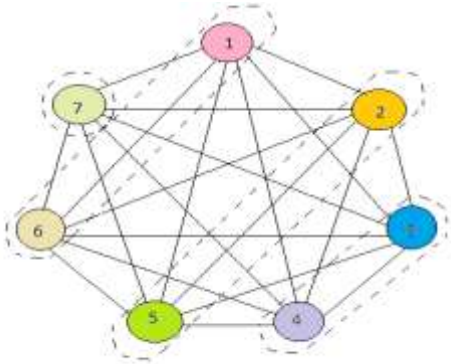

Figure 6. Complete Graph

Let p be the puzzle on kn and let us say that in the solution, the vertex is labeled one and all the other adjacent segment labeled the other colors which the property that the adjacent vertices does not possess the same color. Since all segment have the same sum, hence it is puzzling.

3.3 Theorem

The bipartite graph is apuzzling.

Proof

The vertices are break down into two dislocate sets such that no two vertices within the same set are dislocate. It has a chromatic number 2 (figure 7), there is no equal coloring in bipartite graph. In this all partition segment are are all different. In that segment are either odd or even the sums of the vertex labeling the partitions are different. Hence the bipartite graph is apuzzling.

3.4 Example

In bipartite graph, there is no equal color labeling. In this graph, it is apuzzling because the vertex labeling sum is different and not a puzzling.[7] This graph represents that there is no equal sum of edges in vertex.

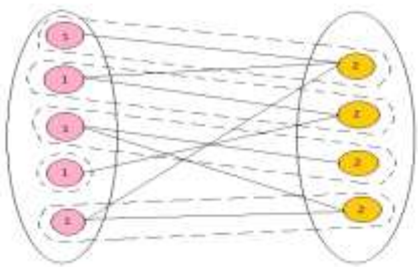

Figure 7.Bipartite Graph

3.5 Theorem

Butterfly graph is puzzling with constant vertices and edges.

Proof

Butterfly graph possess vertex 5, edges 6 with chromatic number 3 (figure 8). Butterfly graph is a consistent undirected graph. It has no equal coloring because,[4] it is also called a complete graph. In this graph the partition segment of vertex labeling sums all are same. So, it is a puzzling graph.

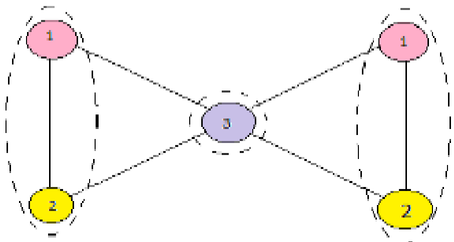

Figure 8.Butterfly Graph

3.6 Theorem

Friendship graph of 2n+1 vetices are puzzling, if n≥1.

Proof

Let the friendship graph Fn of 2n+1 vertices have chromatic number 3. [8] This graph represents the vertices 2n+1 and the edges 3n. There is no equal coloring in friendship graph. Now all the partition segment are all same length is puzzling. The partition segment of vertex labeling sums all are same. So, it is puzzling.

3.7 Example

In freindship graph, if n=5 and the vertices are 11, edges are 15 and the chromatic number is 3 (figure 9).

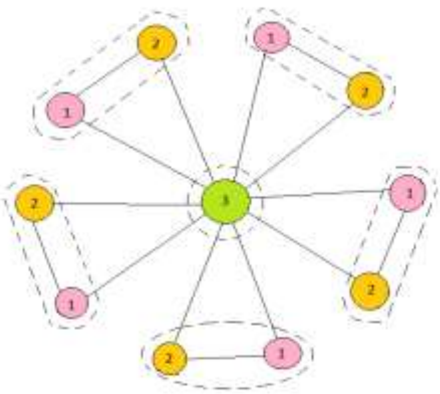

Figure 9. Friendship Graph

There is equal partition labeling and the sum of the vertex are all same. Hence, it is a puzzling.

3.8 Theorem

Peterson graph is puzzling since the peterson graph is having constant labeling.

Proof

This graph has[9] vertex 10, edges 15 and the chromatic number 3 (figure 10). It is a undirected graph,then it has no equal coloring. Hence it has a unique solution. If the length of the all segment of the partition is odd, then adjacent segment must have different sums.

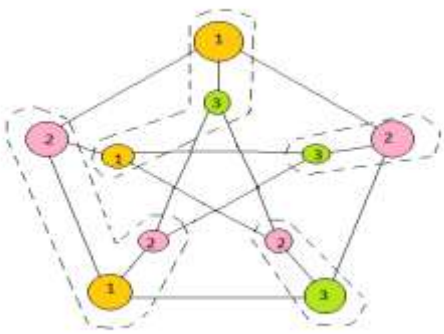

Figure 10. Peterson Graph

4. Conclusion

In this paper we studied the puzzling and apuzzling in some graphs.[2] The vertices are labelling some partitioning sum of the same or different. In graph theory the puzzles is known as the mean of tricky use with labelling of the partition to create the new graphs with vertices and edges.

5. Reference

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[9]www.mathworld.wolfram.com / Petersen graph.

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