Helical surfaces and their application in engineering design

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Abstract. *Ruled surfaces are formed by the movement of a straight line and are known to be used in architecture as a way of simplifying for complex structures. It is necessary to notice that right helicoids are minimal surfaces.Through the centuries helicoids have been applied in architecture mostly for staircase construction as to its minimal space requirement. Nevertheless, helical ruled surfaces are being adopted not only as a single element, but as an overall arrangement of elements or as a single structure. Moreover, the outcome gives place to an innovative variation which will be mentioned and analyzed.*

Keywords: *surfaces, heliciod, developable, right, design, architecture, generatrix, parametric.*

Introduction

The aim of this article is to understand, compare and analyze the existing helicoidal surfaces and its classification; giving a variety of observations in the architectural field, and thus promoting innovative ideas for new appliances in architectural design (Fig. 1). Mathcad v.14 has been used for the projection of the surfaces given the parametric equation and the initial conditions as it will be seen below. In order to understand it's rationalization it is necessary to explain the definitions and geometry being involved. Furthermore, helical ruled surfaces have an extended application in different fields depending on the function given. For instance, some fields make use of helicoids as helical conveyers, support anchors, screws, etc [1], [2].

Figure 1 Entrance staircase in the Louvre by the Pyramid [3].

Geometry and classification of ruled helical surfaces

Ruled helical surfaces are known as surfaces "Gauches", before Jean Nicolas Pierre Hachette presented them their name. In order to explain the differences between helical surfaces it is necessary to clarify its definitions and geometric properties [4]. In the last years, great scientific publications were made about analytical methods of linear analysis of helical shells. In which some theoretical methods and analysis of engineering structures were described.

Descriptive geometry considers a surface as the axis of a movable curve, where the ruling motion of this curve limits the shape and position of the surface; we call this curve the generatrix of surface. The generatrix might lean on to one or more curve called directrix lines. A surface is defined when we can assign the generatrix line for each point.

The first attemp to classify the ruled surfaces applied to engineering design has been found in [1]. In the classification of the ruled helical surfaces [1], [5] it is possible to categorize surfaces in five types, depending on the values of the next equation:

$$r = r(u, v) = \boldsymbol{a}(v) + u\boldsymbol{b}(v) \quad (1)$$

***Where $a(v)$ is the radius vector of directrix curve, $b(v)$ is the directrix vector of rectilinear generatrix. The conditions $b''(v) \neq 0$ and $(a', b', b'') = 0$ hold true for Catalan's surfaces. A ruled surface cannot be a surface of positive Gaussian curvature. The Gaussian curvature of ruled surface is negative or equal to zero.

Right helicoid surface -The helix is a surface generated by curve, plane or twisted, which is rotates about a fixed line an axis and at the same time is translated in the direction of the axis with a velocity the bears a constant ratio to the rate at which the curve is being rotated. [6] The right helicoid (figure 2) is a specific type of helicoid in which the generator is a straight line perpendicular to the axis of rotation. The right helicoid is considered as a minimal surface, it has mean curvature zero. Parametrical equations: $\boldsymbol{x = a\,cosv}, \quad \boldsymbol{y = a\,sinv}, \quad \boldsymbol{z = kv}.$

The code used in mathcad for the surface construction is the following:

$M := 80$ $N := 80$ $c := 2\pi$, $i := 0 \ldots M$ $j := 0 \ldots M$, $\alpha 0 := 2$, $\alpha n := 10$, $\Delta\alpha := \frac{\alpha n - \alpha 0}{M}$,

$\beta 0 := 0$, $\beta n := 6\pi$, $\Delta\beta := \frac{\beta n - \beta 0}{N}$

$\alpha_i := \alpha 0 + i * \Delta\alpha$, $\beta_j := \beta 0 + j * \Delta\beta$

$\mathbf{x}_{i,j} := a * \cos(\beta_j)$, $\mathbf{y}_{i,j} := a * \sin(\beta_j)$, $\mathbf{z}_{i,j} := c * \beta_j$

a. Side View b. Top view c. Isometric view

Figure 2 Right helicoid surface

Oblique helicoid - An oblique helicoid (figure 3) is a helical ruled surface formed by a direct straight line that intersects the axis of the helicoid under constant angle α not equal 90° and rotates with constant angular speed around this axis and moves simultaneously with constant speed along the same

axis (figure 3). The speeds of these movements are proportional. In addition, we can conclude that if

the angular speed of the straight generatrix is equal to zero, this generatrix will form a plane; and if

speed of translation is equal to zero then the generation straight line Will form a conical Surface of

revolution.

Parametric equations:

$$\boldsymbol{x = x(u,v) = ucosv, \quad y = y(u,v) = usinv, \quad z = cv + ku}$$

The code used in mathcad for the surface construction is the following:

$M := 100.$ N:= 100 . b:= 1. $a := 3,$ $i := 0 \ldots M$ j $:= 0 \ldots M$

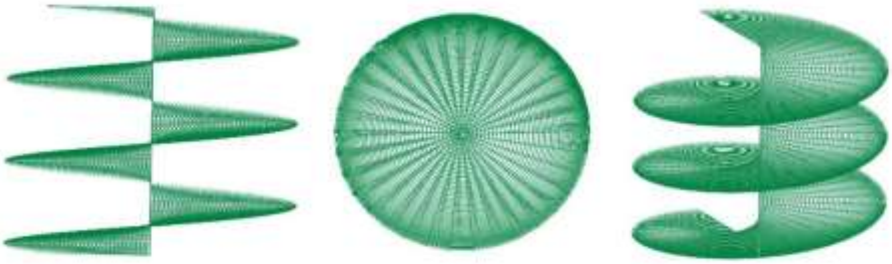
$\alpha 0 := 0, \ \alpha n := \pi, \ \ \Delta\alpha := \frac{\alpha n - \alpha 0}{M},$ $\beta 0 := 0, \ \beta n := 6\pi, \ \ \Delta\beta := \frac{\beta n - \beta 0}{N}$

$\alpha_i := \alpha 0 + i * \Delta\alpha, \quad \beta_j := \beta 0 + j * \Delta\beta$

$\boldsymbol{x_{i,j}} := a * \sin(\alpha_i) * \cos(\beta_j) \quad \boldsymbol{y_{i,j}} := a * \sin(\alpha_i) * \sin(\beta_j)$

$\boldsymbol{z_{i,j}} := b * \beta_j + a * \cos(\beta_j)$

,



a. Side View b. Top view c. Isometric view

Figure 3 Oblique helicoid

$$\boldsymbol{k_r} = 0, \quad \boldsymbol{k_r} = \frac{N}{B^2}, \quad \boldsymbol{K} < 0, \quad \boldsymbol{cosx} = \frac{ck}{\sqrt{r^2 + c^2}}. \qquad \boldsymbol{\rho} = cv\,tanx$$

$$\boldsymbol{A}^2 = 1 + k^2. \quad \boldsymbol{F} = ck. \quad \boldsymbol{B}^2 = r^2 + c^2. \quad \boldsymbol{L}^2 = 0, \quad \boldsymbol{M} = -\frac{c}{\sqrt{B^2 - F^2}}, \quad \boldsymbol{N} = \frac{kr^2}{\sqrt{B^2 - F^2}},$$

Developable helicoid – The curvilinear coordinate u (or v= const) is a rectilinear generatrix, tangel

to the helical cuspidal edge, but lines v(u=const) are helixes. The curvilinear coordinates u, v are parametric, non-orthogonal conjugate curves. If a lane circular area with inside radius a_n is taken and cut it along a tangent to the inside contour, then this circle area can be transformed into any lying on a cylinder with radius a, where $\boldsymbol{a} = a_n \, cos^2\varphi$, here $\boldsymbol{\varphi}$ is the helix angle [7].

Developable helicoid (figure 4) has the parametric equations in the form:

$$\boldsymbol{x}(\boldsymbol{u},\boldsymbol{v}) = \boldsymbol{a}\cos\boldsymbol{v} - \boldsymbol{au}\,\sin\left(\frac{v}{m}\right), \boldsymbol{y}(\boldsymbol{u},\boldsymbol{v}) = \boldsymbol{a}\sin\boldsymbol{v} + \boldsymbol{au}\cos\left(\frac{v}{m}\right), \boldsymbol{z}(\boldsymbol{u},\boldsymbol{v}) = \boldsymbol{bv} + \boldsymbol{b}\left(\frac{v}{m}\right)$$

The code used in mathcad for the surface construction is the following:

$M := 80,$ N $:= 80, b := 1.\ a := 1\ m := \sqrt{a^2 + b^2}$, $i := 0 \ldots M,$ j $:= 0 \ldots M$,

$\alpha 0 := 0,\ \alpha n := 8,\ \Delta\alpha := \frac{\alpha n - \alpha 0}{M},$ $\beta 0 := 0,\ \beta n := 10\pi,\ \Delta\beta := \frac{\beta n - \beta 0}{N}$

,

$\alpha_i := \alpha 0 + i * \Delta\alpha, \quad \beta_j := \beta 0 + j * \Delta\beta$

$\boldsymbol{x_{i,j}} := a * \cos(\beta_j) - a * \alpha_i * \sin\left(\frac{\beta_j}{m}\right) \quad \boldsymbol{y_{i,j}} := a * \sin(\beta_j) + a * \alpha_i * \sin\left(\frac{\beta_j}{m}\right)$

,

$\boldsymbol{z_{i,j}} := b * \beta_j + b * \frac{\beta_j}{m}$

a. Side View b. Top view c. Isometric view

Figure 4 Developable helicoid

Pseudo-developable helicoids for general types - Hence, a pseudo developable helicoid of general types (figure 5) is formed by projections of tangent lines of the helix of the constant pitch on the plane perpendicular to the helix. This Surface is a particular case of a convolute helicoid. The surface of pseudo developable helicoid is used in design of drills for wooden products. General parametric equations of an open oblique helicoid:

$$x = a\cos v - u\cos\varepsilon\sin v, \quad y = a\sin v + u\cos\varepsilon\cos v, \quad z = bv + u\sin\varepsilon$$

The code used in mathcad for the surface construction is the following:

$b := 10 \;\; a := 2, \varepsilon := 4\pi \quad i := 0 \ldots M, \quad := 0 \ldots M$

$M := 80.$ N:= 80. , j

$\alpha0 := 0, \;\; \alpha n := 20, \;\; \Delta\alpha := \frac{\alpha n - \alpha 0}{M} \quad \beta 0 := 0, \;\; \beta n := 8\pi, \;\; \Delta\beta := \frac{\beta n - \beta 0}{N}$

,

$\alpha_i := \alpha 0 + i * \Delta\alpha, \quad \beta_j := \beta 0 + j * \Delta\beta$

$x_{i,j} := a * \cos(\beta_j) - \alpha_i * \cos(\varepsilon) * \sin(\beta_j)$,

$y_{i,j} := a * \sin(\beta_j) + \alpha_i * \cos(\varepsilon) * \cos(\beta_j), \quad z_{i,j} := b * \beta_j + \beta_j * \sin(\varepsilon)$

a. Side view b. Top view c. Isometric view

Figure 5 Pseudo-developable helicoids for general types

***where u: distance form a helical directrix till the corresponding point on the surfaces. v: angle form the axis Ox in the direction of

the axis Oy; α eccentricity of the helicoid. Coordinate lines $u(v = const)$ coincide with rectilinear generatrixes of the surfaces, and $v(u = const)$ are equidistant helical in the relation of the directrix helix (u=0), fig.4 pseudo developable helicoid is shown when $0 < v < 4\pi$; and where ε is an angle between a straight generatrix of the surface and a horizontal plane.

Pseudo developable helicoid- It can be considered to be a special case of the surface pseudo developable helicoid of general type but when $\varepsilon = 0$. The surface Pseudo developable helicoid is a helical trace of uniform motion of horizontal rectilinear generatrix on the given helical guideline. Moreover, the straight generatrix on the plane will coincide with the tangents of the developable helicoid (figure 6). Parametric equations:

$$\boldsymbol{x(u,v) = a\cos v - u\sin v}, \quad \boldsymbol{y(u,v) = a\sin v - u\cos v}, \quad \boldsymbol{z(u,v) = bv}$$

The code used in mathcad for the surface construction is the following:

$M := 80$ $N := 80$ $b := 10$ $a := 2$, $i := 0 \ldots M$ $j := 0 \ldots M$, $\alpha 0 := 0$, $\alpha n := 10$, $\Delta\alpha := \frac{\alpha n - \alpha 0}{M}$, ,
$\beta 0 := 0$, $\beta n := 8\pi$, $\Delta\beta := \frac{\beta n - \beta 0}{N}$ $\alpha_i := \alpha 0 + i * \Delta\alpha$, $\beta_j := \beta 0 + j * \Delta\beta$
$\boldsymbol{x_{i,j}} := a * \cos(\beta_j) - \alpha_i * \sin(\beta_j)$ $\boldsymbol{y_{i,j}} := a * \sin(\beta_j) + \alpha_i * \cos(\beta_j)$

,

$\boldsymbol{z_{i,j}} := b * \beta_j$

a. Side View b. Top view c. Isometric view

Figure 6 Pseudo developable helicoid

Helical design in architecture

Fig. 6 Double Helix

Eco-Tower, Taiwan [7].

structure [2].

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Through the centuries helicoids have been applied in architecture mostly for staircase construction as to its minimal space requirement [2]. Nevertheless, helical ruled surfaces are being adopted not only as a single element, but as an overall arrangement of elements or a single structure as seen in **Fig. 6**. In this case the design of the “ecotower” helps take advantage of the environmental site conditions enhancing the overall sustainability [7]. The use of this concept gives place to an innovative variation of structures. This is another application of helicoids through twisting, in which the geometry is assimilated with torsion as seen above. There is a central vertical axis that the architectural piece "winds" around and either toward or away from. This special type of similarity symmetry expresses a theme of continuity, which is repeated through every

Another structural design application consists in using the generatrix as the main structural element giving plastisicity to the design and intensifying visual performance as seen in **Fig. 7**. This type of design has proved to have excellent performance in public spaces [8].

Fig. 7 Meixi Urban helix,

China [8].

Conclusions

It has been possible to conclude that helicoidal surfaces have evolved over the years, producing an evident development. Additionally, the forms have brought originality and expressiveness to the structures through generations, creating a visual impact over the years ranging from large cities to rural settlements.By analyzing and comparing, it is noticed that the developable, pseudodevelopable general type and pseudodevelopable helicoids, when being generated, leave a free space between the generatrix and the horizontal plane, which makes the overall structure design easier. By contrast, the right and oblique helicoids start from the generatrix making it more appropriate for separate mobilization elements such as stairs. Moreover, it is important to mention the developable type due to its developnes, the property that can be very useful for manufacturing because it lets obtain the surface from the plane product. The appearance of developable helicoid also gives a voluminous shape capturing the attention of the viewer. This can be seen in Fig. 1. where stairs are generated with a right helicoid but the suface under the stairs gives the illusion of a developable helicoid. As an addition developable surfaces as such are cost efficient and widely used in manufacturing and engineering and of importance for CAD/CAM design.

The further investigation should be done to clearly show the advantages of each ruled helical surfaces presented above for engineering and architectural design, manufacturing and eficient material usage.

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