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Some Graphs with A Given Achromatic Number

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Abstract

Achromatic coloring of a graph is a proper vertex coloring in which every distinct pair of color classes is adjacent by at least one edge. The largest number of colors that can be used to color a graph under such coloring is the achromatic number, denoted by $\psi(G)$. In this paper we try to attach some graphs to a complete graph K_n so that the achromatic number of the resultant graph is $n + 1$.

***AMS Subject Classification**: 05C15*
Keywords: *Complete coloring, Achromatic number, Star graph, Wheel graph, Corona product.*

1. Introduction

We consider a simple undirected graph $G = (V, E)$, where V is the set of vertices and E is the set of edges of the given graph G. Let $|V| = n$ and $|E| = m$ be the cardinalities of vertices and edges of the graph G. A vertex coloring of G is a mapping $f: V \to C$, where C is the set of distinct colors. It is a proper vertex coloring if for all $\forall\, uv \in E(G),\ f(u) \neq f(v)$. The minimum number of colors that can be used for the proper vertex coloring of G is called the chromatic number of a graph G and is denoted by $\chi(G)$. A complete 'n' coloring of a graph G is an assignment of n colors, $1, 2, \ldots, n$ to the vertices of G such that all the adjacent vertices are assigned different colors and every pair of the distinct colors appears on the end vertices of some edge [8]. The achromatic number $\psi(G)$ is the greatest number of colors in a complete coloring of the graph G.

F. Harary and S.T. Hedetniemi introduced the concept of achromatic coloring of a graph in the article 'The Achromatic number of a graph' [3] in 1970. Pavol Hell and D.J. Miller proved that with a given achromatic number, there are only finitely many irreducible graphs and described all graphs with achromatic number less than four. They deduced certain bounds on the achromatic number in terms of the number of the vertices of irreducible graphs [5]. In the paper 'Achromatic numbers and graph operations' [4] Pavol Hell and D.J. Miller investigated the achromatic number of the disjoint union of graphs and achromatic number of the categorial product of graphs. They obtained the upper and lower bounds for the union of graphs and the lower bound for the product of graphs. They also showed the achromatic number of the product of graphs is bounded above. G. MacGillivray and A. Rodriguez [8] determined the achromatic number of disjoined union of paths of given length. Keith J. Edwards [1] gave a necessary and sufficient condition of a large graph which is a disjoint union of paths and cycles to have a complete coloring of k colors.

In this paper we consider the graph G which is obtained by attaching the graph G_j to the complete graph K_n with a bridge so that achromatic number of the graph G is $n + 1$. We try to find the minimum number of vertices in a path P_m and cycle C_p to be attached to K_n so that the resulting graph will have achromatic number $n + 1$. We also obtained the minimum number of paths to be attached to K_n hence the resulting graph has achromatic number $n + 1$. In this paper we attach star graph, wheel graph and complete graph to K_n and obtained the achromatic number of the new graph as $n + 1$. The achromatic number of corona product of a complete graph K_n and K_2 complement is also obtained as $n + 1$.

2. Paths attached to complete graph

Let G_j be a path P_m with minimum number of vertices which when attached to K_n will result in a graph G with achromatic number $n + 1$. Now we have the following theorem.

2.1. Lemma

The achromatic number of the resultant graph G is $n + 1$ if G is obtained by attaching a path P_m to a single vertex of K_n, $n \in \mathbb{N}$, where m is given by

$$m = \begin{cases} \dfrac{3n-1}{2}, & n\ odd \\ \dfrac{3n-2}{2}, & n\ even \end{cases} \tag{1}$$

Proof. Consider the complete graph K_n. Let $u_1, u_2, \dots, u_n$ be the vertices of K_n. All the vertices of K_n can be colored completely by n distinct colors from the color class $C = \{1, 2, \dots, n\}$. Thus, achromatic number of K_n is $\psi(K_n) = n$. Let G be the graph obtained by attaching path P_m to a single vertex of K_n. Let $v_1, v_2, \dots, v_m$ be the vertices of P_m. We assume that $u_n v_1$ is the bridge connecting K_n and P_m. Now assume that $\psi(G) = n + 1$, to prove (1) we consider two cases.

Case 1: When n is odd

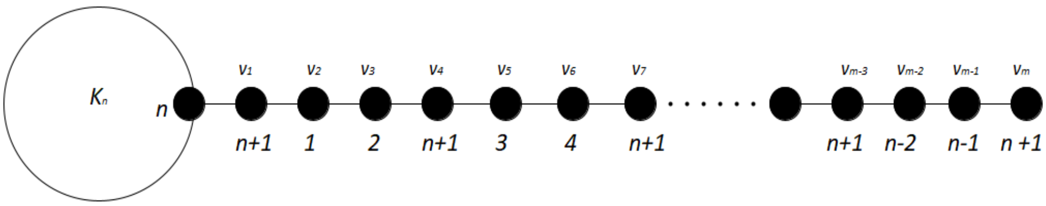

Figure 1: Process of coloring a path attached to K_n, when n is odd

Let us take n to be successive odd integers, where $n \geq 3$, say $n = 3, 5, \dots$. We use induction to prove the result. Consider K_3. Let u_1, u_2, u_3 be the vertices of K_3. Now $\psi(K_3) = 3$ and let G be the graph obtained by attaching a path P_m to a vertex u_3 of K_3, so that $\psi(G) = 4$.

Let $C = \{1, 2, 3, 4\}$ be the color class. The vertices u_1, u_2, u_3 be colored by the colors $1, 2, 3$ respectively. Let the vertex v_1 in P_m be attached to K_3 using a bridge. Color the vertex v_1 using the color four. Now we have the pair of colors $(3, 4)$. By the definition of complete coloring, the color pairs $(1, 4)$ and $(2, 4)$ has to be occurred in the end vertices of any of the edges of G. Assign the colors $1, 2, 4$ to the vertices v_2, v_3 and v_4 of P_m respectively. It is clear that

$$4 = \frac{3 \times 3 - 1}{2}$$

Consider K_{n-2} and assume that the theorem holds for $n = n - 2$. Let m_{n-2} denote the minimum number of vertices of the path attached to K_{n-2}. Hence,

$$m_{n-2} = \frac{3(n-2) - 1}{2}$$

To check if the result is true for K_n, when n is odd. Consider the process of achromatic coloring of graph G when a path P_m is attached to K_n, n is odd, at the vertex u_n. Color $v_1$using the color $n + 1$ followed by the colors $1, 2$ to the vertices v_2, v_3 respectively. The vertex v_4 is colored using the color $n + 1$. Color the vertices v_5 and v_6 by the colors 3,4 respectively. Continue the process. The number of vertices in P_m is obtained by the sum of number of vertices which are paired with vertices assigned with color $n + 1$ in P_m and number of occurrence of vertices colored $n + 1$ in P_m. That is,

$$m = n - 1 + \frac{n-1}{2} + 1 = \frac{3n-1}{2}$$

Now we have to show that m is minimum. Let us suppose m is not the minimum. That is, there exist $m' < m$ such that $P_{m'}$ when attached to K_n the achromatic number is $n + 1$. Using the process of achromatic coloring m' should have at least three vertices more than m_{n-1}

$$m' \geq \frac{3(n-1)-1}{2} + 3$$
$$= \frac{3n-6-1+6}{2} + 3$$
$$= \frac{3n-1}{2} = m$$
$$\Rightarrow m' \geq m$$

which is a contradiction.

Case 2: When n is even

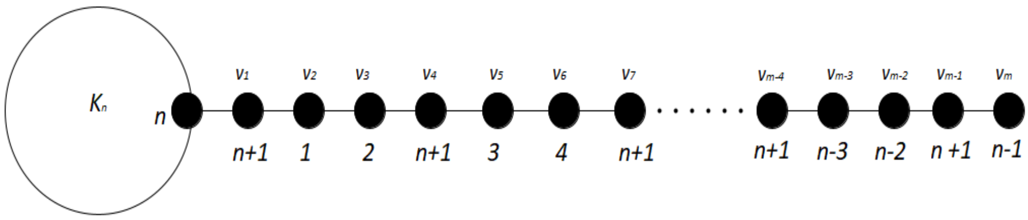

Figure 2: Process of coloring P_m when n is even

Let n be successive even integers, where $n \geq 2$, say $n = 2, 4, \ldots$ We use induction to prove the result. Consider K_4. Let u_1, u_2, u_3, u_4 be the vertices of K_4. Now $\psi(K_4) = 4$ and let G be the graph obtained by attaching a path P_m to a vertex u_4 of K_4 so that $\psi(G) = 5$. Let $C = \{1, 2, 3, 4, 5\}$ be the color class. The vertices u_1, u_2, u_3, u_4 be colored by the color $1, 2, 3, 4$ respectively. Let the vertex v_1 in P_m be attached to K_4 using a bridge. Color the vertex $v_1$using the color 5. Now we have the color pair $(4, 5)$. By the definition of complete coloring, the color pairs $(1, 5), (2, 5)$ and $(3, 5)$ has to be occurred in the end vertices of any of the edges of G. Assign the colors $1, 2, 5, 3$ to the vertices v_2, v_3, v_4 and v_5 respectively. Thus, the achromatic number of G is obtained as five. It is clear that

$$5 = \frac{3 \times 4 - 2}{2}$$

No path with vertices less than five can have achromatic number five when attached to K_4. Now consider K_{n-2} and let the theorem holds for $n = n - 2$. Let m_{n-2} denote the minimum number of vertices of path attached to K_{n-2} so that achromatic number is $n + 1$. Hence,

$$m_{n-2} = \frac{3(n-2)-2}{2}$$

To check if the result is true for K_n, when n is even. Consider the construction of attachment of paths. Let P_m be the path attached to the vertex u_n of K_n. Color v_1 by the color $n + 1$, followed by the colors $1, 2$ to the vertices v_2, v_3 respectively. Color v_4 by the color $n + 1$. Then color v_5 and v_6 by the colors

3, 4 respectively. Continue this process. The number of vertices in P_m is obtained by the sum of number of vertices which are paired with vertex assigned with color $n+1$ and number of occurrence of vertices assigned color $n + 1$. That is

$$m = n - 1 + \frac{n}{2} = \frac{3n-2}{2}$$

Now to show that m is minimum. Let us suppose contrary. That is, there exist $m' < m$, such that $P_{m'}$ when attached to K_n has achromatic number $n+1$, using achromatic coloring m' should have at least three vertices more than m_{n-1}.

$$m' \geq \frac{3(n-2)-2}{2} + 3$$
$$= \frac{3n-1}{2} + 3$$
$$> \frac{3n-2}{2} = m$$
$$\Rightarrow m' \geq m$$

which is a contradiction.
Hence by the principle of mathematical induction, theorem hold for all n.

2.2. Theorem

Let G_i be the graph obtained by attaching the path $P_{m_i^{(j)}}$ to i vertices of K_n so that achromatic number is $n + 1$, where j denotes the index of the path, $j \leq i \leq n$. Let sum of the vertices of the path attached to i vertices of K_n such that $m_i^{(j)}$ is minimum is given by

$$\sum_{j=1}^{i} m_i^{(j)} = v(i)$$

then $v(1) \geq v(2) \geq \ldots \geq v(n)$

Proof. Consider the graphs G_1, G_2 where G_1 is obtained by attaching the path $P_{m_1^{(1)}}$ to $K_n$and G_2 is obtained by attaching the paths $P_{m_1^{(2)}}$ and $P_{m_2^{(2)}}$ to K_n so that the achromatic numbers of both G_1 and G_2 are $n + 1$. We have to show that

$$m_1^{(1)} \geq m_1^{(2)} + m_2^{(2)}$$

Let $u_1, u_2, \ldots, u_n$be the vertices of K_n. Attach path $P_{m_1^{(1)}}$ to the vertex u_n and $P_{m_1^{(2)}}$, $P_{m_2^{(2)}}$ to the vertices u_n, u_{n-1} respectively. Let the first vertex of the paths $P_{m_1^{(1)}}$, $P_{m_1^{(2)}}$ and $P_{m_2^{(2)}}$be colored using the color $n + 1$. Now, we have the color pair $(n, n+1)$ in G_1 and the pairs $(n, n+1)$, $(n-1, n+1)$ in G_2. The pair $(n-1, n+1)$ is absent in the coloring of G_1. To get complete coloring in G_1, we need to introduce the color pairs $(n-1, n+1)$ somewhere in the path $P_{m_1^{(1)}}$ of G_1. This increases the number of vertices in $P_{m_1^{(1)}}$. But since the color pair $(n-1, n+1)$ has already occurred in G_2 this need not repeat in the coloring of paths $P_{m_1^{(2)}}$ and $P_{m_2^{(2)}}$ of G_2. So, the color $n-1$ need not be used in the coloring of the paths $P_{m_1^{(2)}}$ and $P_{m_2^{(2)}}$ and. But the color $n-1$ will be used in the vertex coloring of path $P_{m_1^{(1)}}$ of G_1. Thus, we can write,

$$m_1^{(1)} \geq m_1^{(2)} + m_2^{(2)} \qquad (2)$$

Similarly, consider G_3 and G_2, let G_3 be obtained by attaching the path $P_{m_1^{(3)}}$, $P_{m_2^{(3)}}$ and $P_{m_3^{(3)}}$ to the vertices u_n , u_{n-1} and u_{n-2} respectively so that achromatic number of G_3 is $n+1$. As above the first vertex of $P_{m_1^{(3)}}$, $P_{m_2^{(3)}}$ and $P_{m_3^{(3)}}$ is colored $n + 1$ and we have the color pair $(n, n + 1), (n - 1, n + 1)$ and $(n - 2, n + 1)$ in G_3. Since the pair $(n - 2, n + 1)$ has already appeared, this need not repeat in the coloring of the paths $P_{m_1^{(3)}}$, $P_{m_2^{(3)}}$ and $P_{m_3^{(3)}}$ of G_3. But the color $n - 2$ should appear somewhere in $P_{m_1^{(2)}}$ or $P_{m_2^{(2)}}$ of G_2 and as above we have,

$$m_1^{(2)} + m_2^{(2)} \geq m_1^{(3)} + m_2^{(3)} + m_3^{(3)}$$

or

$$m_1^{(1)} \geq m_1^{(2)} + m_2^{(2)} \geq m_1^{(3)} + m_2^{(3)} + m_3^{(3)}$$

i.e.,

$$v(1) \geq v(2) \geq v(3)$$

On proceeding like this we have $v(1) \geq v(2) \geq \ldots \geq v(n)$

2.3. Corollary

Let K_n be complete graph and G be the graph obtained by attaching P_1 to all the n vertices of K_n. Then $\psi(G) = n + 1$.

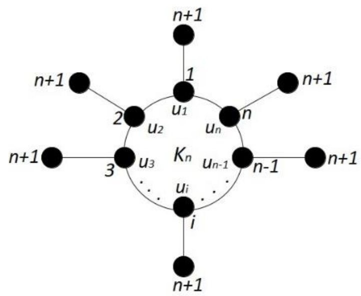

Figure 3: P_1 attached to all the n vertices of K_n

Proof. Let $u_1, u_2, \ldots, u_n$ be the vertices of K_n, we have $\psi(K_n) = n$. Use the color $n + 1$ to color the vertex of P_1. Since P_1 is attached to every vertex of K_n, we have the complete coloring of new graph with $n + 1$ colors. Now we have to show that $n + 1$ is the maximum number of colors which can be used to color the graph . If possible, assume that there exists $s \in \mathbb{N}, s > 1$ such that G can be colored by $n + s$ colors by the process of achromatic coloring. Suppose we give different colors like $n + 1, n + 2, \ldots, n + s$ to the vertices of P_1 which are attached to $u_1, u_2, \ldots, u_n$ of K_n respectively. Then the pairs $(i, n + 1), (i, n + 2), \ldots, (i, n + s)$, where $i \in \{1, 2, \ldots, n\}$ cannot occur anywhere in G. Thus, the complete pairing of colors is not possible. Which gives a contradiction. Hence the result.

3. Cycle attached to a complete graph

3.1. Lemma

The achromatic number of the graph G is $n + 1$ if G is obtained by attaching cycle C_p to a single vertex of K_n, $n \in \mathbb{N}$ where p is given by

$$p = \begin{cases} \dfrac{3n-1}{2}, & n\ odd \\ \dfrac{3n-4}{2}, & n\ even \end{cases}$$

Proof. Let 'p' be the number of vertices in a cycle C_p attached to a single vertex of K_n. Let G be the graph obtained by attaching a cycle C_p to K_n so that achromatic number is $n+1$. That is $\psi(G) = n + 1$. The result and proof of lemma 2.1 can be extended to the case of cycle also.
Let $v_1, v_2, \ldots, v_m$ be the vertices of P_m as in lemma 2.1.
Case 1: When n is even.
Since the vertices v_1 and v_m of the path P_m is colored different, both the vertices of the path P_m attached to K_n can be made adjacent which gives a cycle. Also, the coloring is complete with the same number of vertices as in lemma 2.1. This value of p is minimum, otherwise it would contradict the lemma 2.1.

$$\therefore p = m \geq \frac{3n-1}{2}$$

Case 2: When n is odd.
We have the vertices v_1 and v_m of the path P_m attached to K_n is colored same with the color $n+1$. The vertex v_{m-1} is colored $n-1$. By deleting the vertex v_m of P_m attached to K_n and by making v_1 and v_{m-1} adjacent gives a cycle. Also, the coloring is complete with one vertex less than the number of vertices in lemma 2.1. This value of p is minimum, otherwise it would contradict the lemma 2.1.

$$\therefore p = m - 1 \geq \left(\frac{3n-1}{2}\right) - 1 = \frac{3n-4}{2}$$

Hence the result hold.

3.2. Corollary

For a complete graph $K_n, n \geq 3$ on attaching C_3 to $\left\lceil \frac{n}{3} \right\rceil$ vertices of K_n, the achromatic number of the resultant graph is $n + 1$.

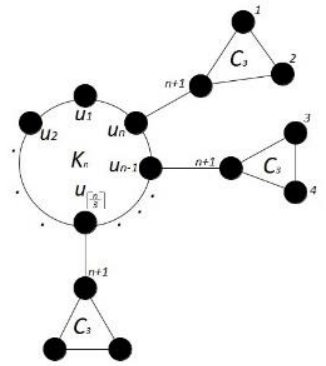

Figure 4: C_3 attached to $\left\lceil \frac{n}{3} \right\rceil$ vertices of K_n

4. Graphs attached to complete graph

4.1. Star graph

Star graph S_n is the complete bipartite graph $K_{1,n}$ with one root vertex and n leaves. Attaching the star graph S_{n-1} to the central root vertex of K_n the achromatic number is obtained as $n+1$.

4.1.1. Proposition

Let G be a graph obtained by attaching the central root vertex of the star graph S_{n-1} to a vertex of the complete graph K_n. Then, $\psi(G) = n + 1$.

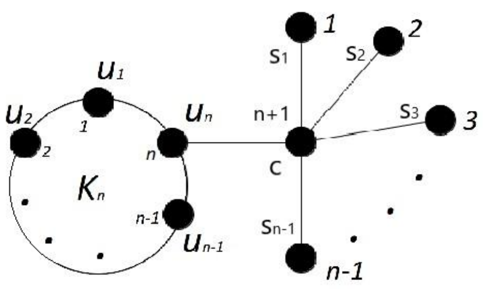

Figure 5: Star graph S_{n-1} attached to a vertex *of* K_n

Proof. Let $u_1, u_2, \ldots, u_n$ be the vertices of K_n. Let $s_1, s_2, \ldots, s_{n-1}$ be the vertices of S_{n-1} and let c be the central vertex of S_{n-1}. Let the vertex u_i of K_n is colored by the color i where $i \in \{1, 2, \ldots, n\}$. Without loss of generality suppose that S_{n-1} is attached to the vertex $u_n$of K_n by a bridge to the central vertex c of S_{n-1}.
Now color the vertex c by the color $n + 1$ and we have the color pair $(n, n + 1)$. Now color the vertex $s_j, j \in \{1,2, \ldots, n - 1\}$ by the color j and so that the color pairs $(1, n + 1), (2, n + 1), \ldots, (n - 1, n + 1)$. Thus G can be colored by the process of complete coloring with $n + 1$. Now we have to show that the maximum number of colors that can be used for the complete coloring of G is $n + 1$. Suppose contrary, G can be colored using $n + s,\ s \geq 2$ colors using complete coloring. Consider the case when $s = 2$. Use the color $n + 2$ to color the vertex c, but the pair of colors $(n + 1, n + 2)$ cannot be seen in the complete coloring of graph. If we color any of the vertex of S_{n-1}, say s_j, $j \in \{1, 2, \ldots, n - 1\}$ by the color $n + 2$ and the vertex c by the color $n + 1$, then the color pair $(j, n + 1)$ will not occur in the coloring of G. If we color any two of the vertices of S_{n-1}, say s_j and s_k, $j, k \in \{1,2, \ldots, n - 1\}$ by the color $n + 1$ and $n + 2$ respectively. Then the color pair $(n + 1, n + 2)$ cannot be seen in the coloring of G since the vertices s_j and s_k are not adjacent in S_{n-1}.
Hence the vertices of G cannot be colored with $n + 2$ colors by preserving the conditions of achromatic coloring Also G cannot be colored with $n + s$ colors, where $s \geq 2$. Thus $\psi(G) = n + 1$.

4.2. Wheel graph

Wheel graph W_n, $n \geq 3$ is a graph with $n + 1$ vertices obtained by connecting all the vertices of a cycle C_n to a single central vertex c. We consider K_n attached to a wheel graph $W_{n-1}$and examined the achromatic number.

4.2.1. Proposition

Let G be a graph obtained by attaching the central vertex of the wheel graph W_{n-1} to a vertex of the complete graph K_n. Then, $\psi(G) = n + 1$.

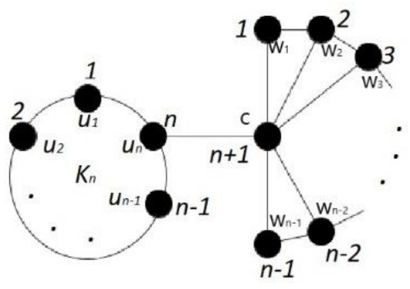

Figure 6: Wheel graph W_{n-1} attached to a vertex of K_n

Proof. Let $u_1, u_2, \ldots, u_n$ be the vertices of K_n. Let $w_1, w_2, \ldots, w_{n-1}$ be the vertices which belongs to the cycle of the wheel graph W_{n-1} and let c be the central vertex of W_{n-1}. Suppose W_{n-1} is attached to the vertex u_n of K_n. Color the vertices u_i, $i \in \{1, 2, \ldots, n\}$ by the color i, $i \in \{1, 2, \ldots, n\}$. Now color the vertices $w_j, j \in \{1, 2, \ldots, n-1\}$ of W_n by the color $j, j \in \{1, 2, \ldots, n-1\}$ and the vertex c, the central vertex of W_{n-1}, by the color $n+1$. Now graph G has all the pair of colors $(1, n+1), (2, n+1), \ldots, (n-1, n+1)$ to color its vertices. Hence G can be colored by the process of achromatic coloring using $n+1$ colors. Now it remains to show that $n+1$ is the maximum number of colors that can be used for the complete coloring of G. Suppose we color the vertices of G by the process of achromatic coloring using $n+k,\ k \geq 2$ colors. Then the color $n+k,\ k \geq 2$ will occur either in K_n or in W_n. Suppose that $u_i, i \in \{1,2,\ldots,n\}$ is colored by the color $n+k$ then the pairs $(i, n+k)$ will not appear in G. If $w_j, j \in \{1,2,..,n-1\}$ is colored by the color $n+k$ then the pairs $(i, n+k)$ will not appear in the graph. And if the vertex c is colored by the color $n+k$ then the pair $(n+1, n+k)$ will be absent in G. Hence G cannot be colored using $n+k$ colors by the process of achromatic coloring. Hence G can be colored using $n+1$ colors and $\psi(G) = n+1$.

4.3. Complete graph

4.3.1. Proposition

Let G be a graph obtained by attaching the vertices of two complete graphs K_n by a bridge. Then, $\psi(G) = n+1$.

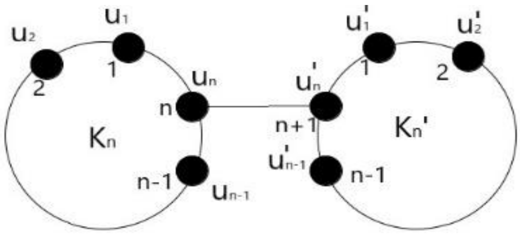

Figure 7: $K_n$attached to a vertex of K_n

Proof. Suppose that K_n and K_n' be two complete graphs in G. Let $u_1, u_2, \ldots, u_n$ be the vertices of K_n $u_1', u_2', \ldots, u_n'$ be vertices of K_n'. Ley $u_n u_n'$ be the bridge joining K_n and K_n'. Color the vertices u_i of K_n by the color i where $i \in \{1, 2, \ldots, n\}$. Now color the vertices u_j' of K_n' by the color j where $j \in \{1, 2, \ldots, n-1\}$ and the vertex u_j' by the color $n+1$. Hence, we have the pair of colors $(1, n+1), (2, n+1), \ldots, (n-1, n+1)$. Therefore G can be colored by the process of achromatic coloring using $n+1$ colors. Now it remains to show that $n+1$ is the maximum number of colors that can be used for the complete coloring of G. Suppose contrary, G can be colored using $n+2$ colors by the process of complete coloring. If we color u_n' by the color $n+2$ then the color pair $(n+1, n+2)$ will be absent in the graph G. If we color any of the vertices
$u_j', j \epsilon \{1, 2, \ldots, n-1\}$ by the color $n+2$ then the pair $(j, n+2)$ will be absent in G. Hence G can never be colored by $n+2$ colors and therefore we have $\psi(G) = n+1$.

4.4. Corona product of K_n and K_2^c

Corona product: The corona product of the graphs G and H denoted by $G \circ H$ is obtained by taking one copy of G and $|V(G)|$ copies of H and making the i^{th} vertex of G adjacent to every vertex of i^{th} copy of H, where $1 \leq i \leq |V(G)|$.

4.4.1. Proposition

The corona product of a complete graph K_n and the complement of complete graph K_2 has achromatic number $n+1$.

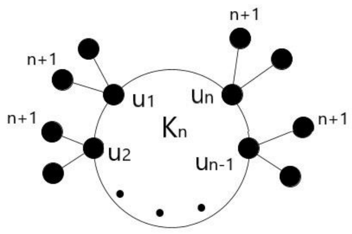

Figure 8: Corona product of a complete K_n and the compliment of K_2

Proof. Let $G = K_n \circ K_2^c$. Let $u_1, u_2, \ldots, u_n$ be the vertices of K_n. Let c_1, c_2 be the vertices of K_2^c. Color the vertices u_i, $i \in \{1, 2, \ldots, n\}$ by the color i. Also color the vertices c_1 or c_2 or both by the color $n + 1$. In this process of coloring we have all the ${}^{n+1}C_2$ pairs preserving the conditions of achromatic coloring. Hence G can be colored by $n + 1$ colors under achromatic coloring. Now to show that $\psi(G) = n + 1$. Suppose G can be colored using $n + 2$ colors by the process of complete coloring. Since coloring of K_n is complete without any repetitions, we color the vertices of K_2^c using the color $n + 2$. Let c_1 be colored $n + 1$ and c_2 be colored $n + 2$, then we have the color pairs $\{(n + 1,1), (n + 1,2), \ldots, (n + 1, n)\}$ and the pairs $\{(n + 2, 1), (n + 2, 2), \ldots, (n + 2, n)\}$. But the pair $(n + 1, n + 2)$ does not occur in the coloring of vertices of G. This is a contradiction to our assumption that G can be colored using $n + 2$ colors by the process of complete coloring. Hence, we can write G cannot be colored $n + s, \forall s \geq 2$ colors by preserving the conditions of complete coloring. Hence, $\psi(G) = n + 1$

5. Conclusion

In this paper, we have studied about some graphs which when attached to a complete graph, the achromatic number is one more than the number of vertices of the given complete graph. The study can be extended to find the general graph G_j and its properties which when attached to K_n will give the achromatic number of the resultant graph to be $n + 1$.

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