International Journal of Advanced Science and Technology
Vol. 29, No. 7s, (2020), pp. 5144 – 5147

An Introduction to Nano Ideal Graph of a Graph

R. Manoharan[1], S. Vijayalekshmi[2] & J. Sebastian Lawrence[3]
[1] *Department of Mathematics, Sathyabama Institute of Science and Technology, Chennai-600119, India.*
[2] *Department of Physics, Jeppiaar Institute of Technology, Sriperumpudur - 631604, India.*
[3] *Department of Mathematics(S&H), Saveetha School of Engineering, Chennai-600077, India.*

Abstract:
Nano ideal graph of a simple , undirected graph G is introduced. As the size of the Nano ideal graph of a graph G is smaller than the graph G, we attempt to approach some of the properties like isomorphism between two graphs, connectedness and coloring properties of graphs via Nano ideal graph of the graph G. A typical u-v path of the graph G becomes an uv edge in Nano ideal graph of a graph, analysis of isomorphism between trees become simple when using Nano ideal graphs.

KeyWords: *Graph, Ideal graph, Nano ideal graph. AMS(2010): 05C38, 05C60*

1. Introduction and Preliminaries:

In this paper, we introduce Nano ideal graph of a graph which is of smaller size than the actual graph. Graphs considered here are finite, simple and undirected. The symbols V (G) and E(G) will denote the vertex set and edge set of a graph G. Terms not defined here are used in the sense of Harary [2] and Gary Chartrand [1]. Two Graphs G1 and G2 are isomorphic if there exists a one-to-one correspondence f from V (G1) to V (G2) such that uv $\in$ E(G1) if and only if f (u)f (v) $\in$ E(G2). By a coloring of a graph, we mean an assignment of colors to the vertices of G such that adjacent vertices are colored differently. The smallest number of colors in any coloring of a graph G is called the chromatic number of G and is denoted by $\chi(G)$. If it is possible to color G from a set of k colors, then G is said to be k-colorable. A coloring that uses k-colors is called a k-coloring. Manoharan *et.al* introduced ideal graph of a graph and approached graph properties via its ideal graph.

Definition 1.1[3] For any graph G, the ideal graph I(G) of G is formed as follows:

(i) Any cycles and the edges which are lying in the cycles of G will remain same in $I_d(G)$.
(ii) Every u-v path with maximum possible length such that all the internal vertices of degree two of G will be considered as an edge uv in $I_d(G)$.

Definition 1.2 [3] The vertices of the ideal graph $I_d(G)$ are called strong vertices of the graph G and the vertices, which are not in the ideal graph Id(G) are called weak vertices of the graph G. The vanishing number of an edge uv of the ideal graph of a graph Gis defined as the number of internal vertices of the u-v path in the graph G. The vanishing number of an edge e of an ideal graph by $v_0(e)$.The vanishing number of the ideal graph of a graph is denoted by vid and is defined as the sum of all vanishing numbers of the edges of Id(G) or the number of weak vertices of the graph G. The ideal number of a graph Gis defined as the number of vertices in the ideal graph of the graph G or the number of strong vertices of the graph. It is denoted by p_{id}.

2. Nano ideal Graph of a graph:

In this section, we introduce Nano ideal graph of a graph and approach graph properties via Nano ideal graphs. Also we discuss how the results differ while using Nano ideal graph instead of ideal graph of a graph.

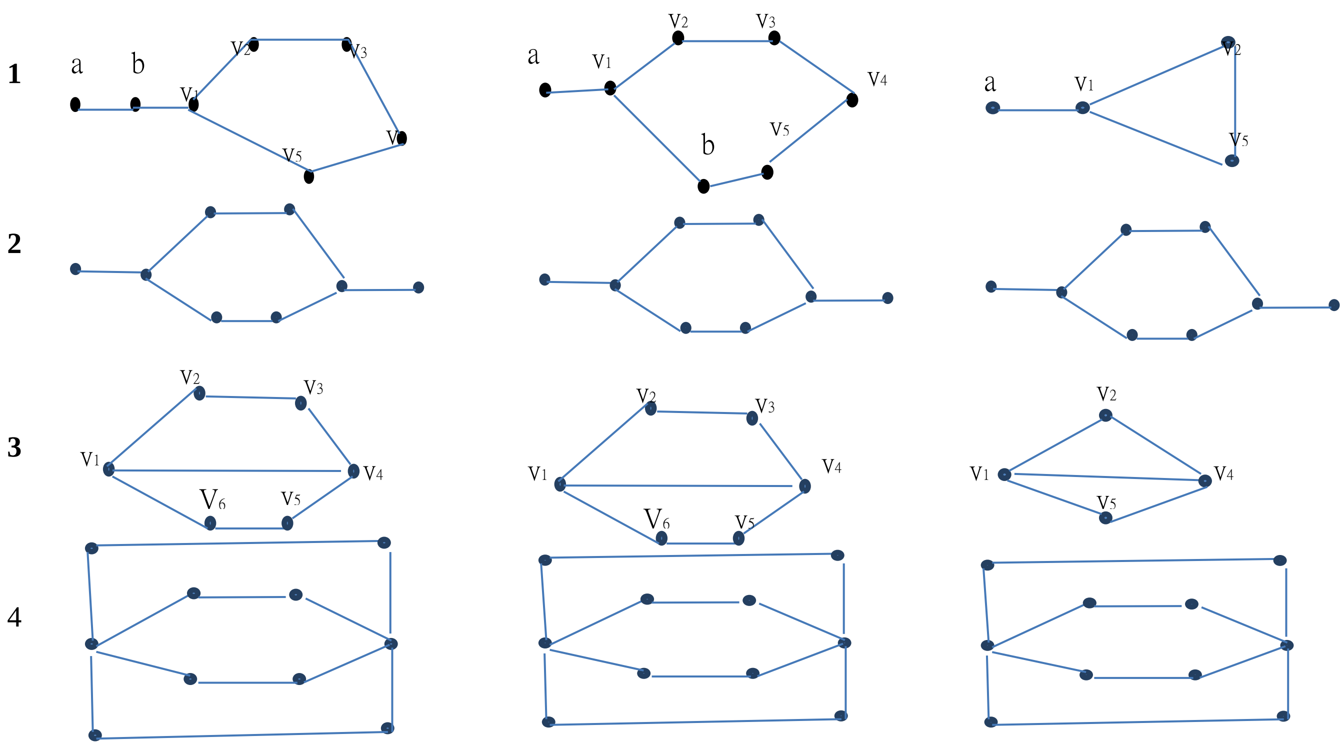
Definition 2.1: For any graph G, the Nano ideal graph of the graph G which is denoted by $N_d(G)$ is the ideal graph of G in which cycles with 'n' vertices (n >3) $v_1v_2\ldots\ldots v_nv_1$ such that $\deg(v_i) = 2$ for all

ISSN: 2005-4238 IJAST

$i = 2,3,\ldots,n-1$ will be considered as $v_1v_2v_nv_1$, that is the path v_2- v_n in G is considered as an edge v_2v_n in $N_d(G)$.

Examples 2.2:

G **$I_d(G)$** **$N_d(G)$**



Definition 2.3: The vertices of the Nano ideal graph Nd(G)are called N-strong vertices of the graph G and the vertices, which are not in the Nano ideal graph Nd(G) are called N-weak vertices of the graph G. The vanishing number of an edge uv of Nd(G) is defined as the number of internal vertices of the u-v path in the graph G [3]. We denote the vanishing number of an edge e of Nd(G) by v0(e). The vanishing number of the Nano ideal graph of a graph is denoted by vnd and is defined as the sum of all vanishing numbers of the edges of Nd(G) or the number of N- weak vertices of the graph G. The Nano ideal number of a graph G is defined as the number of vertices in the Nano ideal graph of the graph G or the number of N-strong vertices of the graph. It is denoted by p_{nd}.

The following proposition is obvious from the above definitions.

Proposition 2.4: Let G be a graph and $p = |V(G)|$. The following properties are true.

(i) $p = p_{nd} + v_{nd}$.
(ii) $p \geq p_{id} \geq p_{nd}$.
(iii) $p = p_{nd}$ if and only if $G = N_d(G)$.
(iv) $I_d(G)= N_d(G)$ if and only if G has no cycles.
(v) $G = I_d(G)= N_d(G)$ if and only $\delta \geq 3$, where δ is minimum of degress of vertices of G.

Remark 2.5

(i) $N_d(P_n) = P_2$ and $N_d(C_n) = C_3$ for every $n \geq 3$..
(ii) $N_d(W_n) = W_n$ and $N_d(K_n) = K_n$ for all n.
(iii) $N_d(K_{m,n}) = K_{m,n}$ except for $K_{1,2}$ & $K_{2,2}$ since $N_d(K_{1,2})= P_2$ and $N_d(K_{2,2})= C_3$.
(iv) $N_d(N_d(G))= N_d(G)$ for any graph G.

ISSN: 2005-4238 IJAST

Proposition 2.6: A vertex v of a graph G is a N-strong vertex if and only if $deg(v) \leq 1$ or $deg(v) \geq 3$.

Proposition 2.7: The following results are true for any graph G.
(i) If a vertex v of a graph G is an N- weak vertex, then deg(v) = 2.
(ii) Every weak vertex is an N-weak vertex but not the converse.
(iii) Every N-strong vertex is a strong vertex but not the converse.

Remark 2.8 Converse of the results discussed in the above are not true. For, consider G = $K_{2,3}$. Then there two vertices are of degree 2 but they are not N-weak vertices. The graphs given in the examples for Nano ideal graph contains the examples that indicates converse of (ii) and (iii) not true.

Proposition 2.9: A graph G is connected if and only if $N_d(G)$ is connected.
Proof: It is proved that " A graph G is connected if and only if $I_d(G)$ is connected"[2]. This proof is similar to the proof of the above theorem.

Corollary: A graph G and $I_d(G)$ have same number of connected components.

Lemma 2.10 ([1]) If a graph G is isomorphic to a graph G' under a function f, then
(i) G and G' have same degree sequence
(ii) if G contains a k-cycle for some integer $k \geq 3$, so does G' and
(iii) f G contains a u-v path of length k, then G' contains a $f(u)-f(v)$ path of length k.

Theorem 2.11: A graph G is isomorphic to a graph G′ implies Nd(G) is isomorphic to Nd(G′) but not the converse.

Proof: follows from the above Lemma. Take G = C_4 and G' = C_5. Here $N_d(G) = N_d(G') = C_3$ where the actual graphs are not isomorphic. Already we had a characterization for isomorphism of graphs via $I_d(G)$. But $N_d(G)$ has fewer vertices than $I_d(G)$ which motivate us to express a new characterization for isomorphism of graphs via $N_d(G)$.
Theorem 2.12: A graph G is isomorphic to the graph G′ if and only if Nd(G) is isomorphic to Nd(G′) and the isomorphic edges have same vanishing number.
In Graph theory we have "A graph G is 2-colorable if and only if G contains no odd cycles". This result is useful to prove the following results.

Theorem 2.13: [3] A graph G is 2-colorable if and only if $I_d(G)$ is 2-colorable.
We cannot just replace $I_d(G)$ by $N_d(G)$ as we did it for characterization of isomorphism of graphs. Because G = C_4 is 2-colorable but $N_d(G)= N_d(C_4) =C_3$ is not 2-colorable. But luckily we have the following results.

Proposition 2.14: If $N_d(G)$ of any graph G has no odd cycles then G has no odd cycles.
Proof: Let $N_d(G)$ of the graph G has no odd cycles. Then G has no C_3. That means the cycles in G remains same in $N_d(G)$. Hence G has no odd cycles.

Theorem 2.15: For any graph G, if $N_d(G)$ is two colorable implies G is two colorable.
Proof: Let G be a graph such that $N_d(G)$ is two colorable. Therefore, $N_d(G)$ has no odd cycles. By the previous proposition , G also does not contain any odd cycles. Hence G is 2-colorable.

Theorem 2.16: A graph G is k-colorable with $k \geq 3$ if and only if $N_d(G)$ is k-colorable. Proof Let G is k-colorable with $k \geq 3$. Since degrees of N-strong vertices remain same in $N_d(G)$, $N_d(G)$ is also k-colorable. Assume $N_d(G)$ is k-colorable. Since degrees of N-strong vertices remain same in G and $N_d(G)$ and degree of weak vertices are only two which is less k (since $k \geq 3$), G is k-colorable.

ISSN: 2005-4238 IJAST

Corollary 2.17 For any graph G, $\delta(G) \leq \chi(G) \leq \chi(N_d(G))$.

References

1. Gary Chartrand and Ping Zhang, Introduction to Graph Theory, Tata McGraw-Hill Publishing Company Limited, 2006.
2. F. Harary, Graph Theory, Addison Wesley, Reading Mass., 1972.
3. R. Manoharan, R. Vasuki and R. Manisekaran, Ideal graph of a graph, International Journal of Mathematical Combinatorics, Vol.3, (2011),11-16.

ISSN: 2005-4238 IJAST