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An $M^{[X]}$/G/1 Priority Retrial Queue With Feedback And Working Breakdowns

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ABSTRACT

In this paper, we analyze the batch arrival of preemptive priority with feedback retrial queueing system with disaster under working breakdown and balking. The system consists of a main server and a substitute server and disasters only occur while the main server is in operation. The occurrence of disasters forces all customers to leave the system and causes the main server to fail. At a failure moment, the major server is sent to be fixed and the server functions at a lower speed during the repair period. Using the supplementary variable technique (SVT), the steady state probability generating functions (PGF) for the system and some system performance measures are discussed.

Key words: Batch Arrival, Priority Queue, Feedback, Working Breakdown.

1. INTRODUCTION

The topic of retrial queues in queueing theory has been interested research topic for the past years. The concept of retrial queues has a great efforts and interest by many researches like Artalejo and Gomez-Corral [1], Kalidass and Ramanath [2] have introduced the concept of the alternate working models when server is in breakdown (called Working Breakdown (WB)). The concept of preemptive priority was researched by Gao [3]. Recently, Liu and Song [4], Rajadurai [5], Kim and Lee [7] developed models in presence of Working Vacations (WV) and Working Breakdowns. Motivated by this paper, we extend the work of Rajadurai et al. [6] andAmmar et al. [8] with the concept of batch arrival retrial queueing system with feedback concept.

2. DESCRIPTION OF THE MODEL

We investigated an $M^{[X]}$/G/1 feedback priority retrial queue with disasters under working breakdown services. The basic components of this model are described as follows:

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(i) **Arrival Process:**There are two types of customers arriving into the system. They are ordinary and priority customers. Assume that both types of customers arrive from outside the system according to independent Poisson processes with rates λ and δ respectively.

(ii) **The Retrial Process:** If an arriving customer finds the server free, they get service immediately. Otherwise the customer finds the server busy, then they join the pool of waiting customers called orbit in accordance with FCFS discipline.

(iii) **The Regular Service Process:** Whenever a new priority customer or regular customer arrived at the server in an idle state, then the server immediately started normal service for the arrivals. The service time of priority customers follows a general distribution and it is denoted by the random variable S_p and the service time of an ordinary customers follows a general distribution and it is denoted by the random variable S_b.

(iv) **The Working Breakdown Process:** However disasters occur in the regular busy server, the server goes for a working breakdown. During working breakdown period, the substitute server works in a lower service rate to an arriving customers $(\mu_w < \mu)$.

(v) **The Feedback Process:** After completion of regular service for each customer, the unsatisfied customer may rejoin into the orbit as a feedback customer for receiving another service with probability p $(0 \le p \le 1)$ or may leave the system with probability $q = 1 - p$.

3. STEADY STATE ANALYSIS OF THE SYSTEM

We developed the steady state difference – differential equations based on SVT.

Let us define some notations in the following: $N(t)$ is the number of customers in the orbit at time t and $C(t)$ is the state of the server at time t.

Then the processes are retrial, priority, regular service and working breakdown service.

The distribution functions are $R(x),\ S_p(x),\ S_b(y)\ and\ S_w(x)$. Laplace stieltjas transforms for the above process are $R^*(\theta),\ S_p^*(\theta),\ S_b^*(\theta)$ and $S_w^*(\theta)$ and the elapsed time at t for the above process are $R^0(t),\ \mathrm{S}_p^0(t),\ \mathrm{S}_b^0(t)$ and $\mathrm{S}_w^0(t)$.

The conditional completion rates are $a(x)dx = \frac{dR(x)}{1-R(x)},\ \mu_p(x)dx = \frac{dS_p(x)}{1-S_p(x)},$

$\mu_b(\mathrm{y})dy = \frac{dS_b(\mathrm{y})}{1-S_b(\mathrm{y})}$ and $\mu_w(\mathrm{x})dx = \frac{dS_w(\mathrm{x})}{1-S_w(\mathrm{x})}.$

Assume that $R(0)=0,\ \mathrm{R}(\infty)=1,\ \mathrm{S}_p(0)=0,\ \mathrm{S}_p(\infty)=1,\ \mathrm{S}_w(0)=0,\ \mathrm{S}_w(\infty)=1$ are continuous at $x=0$ and $S_b(0)=0,\ S_b(\infty)=1$ are continuous at $y=0$.

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Further, we present the random variables for the bivariate Markov process $\{C(t), N(t);\ t \geq 0\}$, where $C(t)$ denotes the server state (0,1, 2, 3, 4, 5) depending on the server is idle, retrial, priority busy,preemptive priority busy, regular busy and working breakdown.

The state probabilities are defined as follows:

$P_0(t) = P\{C(t) = 0,\ N(t) = 0\}$ *and for* $t \geq 0,\ x \geq 0,\ y \geq 0,\ n \geq 0,$

$P_n(x,t)dx = P\{C(t) = 1, N(t) = n,\ x < R^0(t) \leq x + dx\},$

$Q_n(x,t)dx = P\{C(t) = 2, N(t) = n,\ x < S_p^0(t) \leq x + dx\},$

$R_n(x,y,t)\,dxdy = P\{C(t) = 3,\ N(t) = n,\ x < S_p^0(t) \leq x + dx,\ y < S_b^0(t) \leq y + dy\},$

$S_n(y,t)dy = P\{C(t) = 4, N(t) = n,\ y < S_b^0(t) \leq y + dy\},$

$T_n(x,t)dx = P\{C(t) = 5,\ N(t) = n,\ x < S_w^0(t) \leq x + dx\}.$

Then the limiting probabilities are $P_0 = \lim_{t\to\infty} P_0(t),\ P_n(x) = \lim_{t\to\infty} P_n(x,t)$,

$Q_n(x) = \lim_{t\to\infty} Q_n(\mathrm{x},t),\ R_n(x,y) = \lim_{t\to\infty} R_n(\mathrm{x},\mathrm{y},t),\ S_n(y) = \lim_{t\to\infty} S_n(\mathrm{y},t),\ T_n(x) = \lim_{t\to\infty} T_n(\mathrm{x},t).$

The steady state equations: Using the method of SVT, weobtain the following system of equations.

$$(\lambda + \delta + \gamma) P_0 = \gamma P_0 + \int_0^\infty Q_0(x)\mu_p(x)dx + q\int_0^\infty S_0(y)\mu_b(y)dy + \int_0^\infty T_0(x)\mu_w(x)dx + \alpha\int_0^\infty S_n(y)dy \tag{1}$$

$$\frac{dP_n(x)}{dx} + (\lambda + \delta + a(x))P_n(x)dx = 0,\ n \geq 1 \tag{2}$$

$$\frac{dQ_n(x)}{dx} + (\lambda + \mu_p(x))Q_n(x) = \lambda(1-b)Q_n(x) + \lambda b\sum_{k=1}^{n}\chi_k Q_{n-k}(x),\ \mathrm{n} \geq 1 \tag{3}$$

$$\frac{dR_n(x,y)}{dx} + (\lambda + \mu_p(x))R_n(x,y) = \lambda(1-b)R_n(x,y) + \lambda b\sum_{k=1}^{n}\chi_k R_{n-k}(x,y),\ n \geq 1 \tag{4}$$

$$\frac{dS_n(y)}{dy} + (\lambda + \delta + \alpha + \mu_b(y))S_n(y) = \lambda(1-b)S_n(y) + \int_0^\infty \mu_p(x)R_n(x,y)dx + \lambda b\sum_{k=1}^{n}\chi_k S_{n-k}(y),\ n \geq 1 \tag{5}$$

$$\frac{dT_n(x)}{dx} + (\lambda + \gamma + \mu_w(x))T_n(x) = \lambda b\sum_{k=1}^{n}\chi_k T_{n-k}(x) + \lambda(1-b)T_n(x),\ n \geq 1 \tag{6}$$

The steady state boundary conditions at $x = 0$ and $y = 0$ are

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$$P_n(0)=\int_0^\infty \mu_p(x)Q_n(x)dx+p\int_0^\infty \mu_b(y)S_{n-1}(y)dy+q\int_0^\infty \mu_b(y)S_n(y)dy+\int_0^\infty \mu_w(x)T_n(x)dx,\ n\geq 1 \qquad (7)$$

$$Q_n(0)=\delta\int_0^\infty P_n(x)dx,\ n\geq 1 \qquad (8)$$

$$R_n(0,y)=\delta S_n(y),\ n\geq 0 \qquad (9)$$

$$S_n(0)=\int_0^\infty P_{n+1}(x)a(x)dx+\gamma\int_0^\infty T_n(x)dx+\lambda\sum_{k=1}^{n}\chi_k\int_0^\infty P_{n-k+1}(x)dx,\ n\geq 1 \qquad (10)$$

$$T_n(0)=\begin{cases}(\lambda+\delta)P_0, & \text{n} = 0\\ 0 & ,\text{n}\geq 1\end{cases} \qquad (11)$$

The Normalization condition is

$$P_0+\sum_{n=1}^{\infty}\int_0^\infty P_n(x)dx+\sum_{n=0}^{\infty}\left(\int_0^\infty Q_n(x)dx+\int_0^\infty\int_0^\infty R_n(x,y)dxdy+\int_0^\infty S_n(y)dy+\int_0^\infty T_n(x)dx\right)=1 \qquad (12)$$

The steady state solutions: To analyze the developed queueing model, we make useof SVT and PGF method. We define the generating functions for $|z|\leq 1$as follows:

$$P(x,z)=\sum_{n=1}^{\infty}P_n(x)z^n;\ Q(x,z)=\sum_{n=0}^{\infty}Q_n(x)z^n;\ R(x,\text{y},z)=\sum_{n=0}^{\infty}R_n(x,y)z^n$$

$$S(\text{y},z)=\sum_{n=0}^{\infty}S_n(y)z^n;\ T(x,z)=\sum_{n=0}^{\infty}T_n(x)z^n$$

Multiplying the steady state equation and steady state boundary condition (1) to (11) by z^n and summing over n, (n=0,1,2,...)

$$\frac{\partial P(x,\text{z})}{\partial x}+\left(\lambda+\delta+a(x)\right)P(x,z)=0 \qquad (13)$$

$$\frac{\partial Q(x,z)}{\partial x}+\left(\lambda b\left(1-X(z)\right)+\mu_p(x)\right)Q(x,z)=0 \qquad (14)$$

$$\frac{\partial R(x,y,z)}{\partial x}+\left(\mu_p(x)+\lambda b\left(1-X(z)\right)\right)R(x,y,z)=0 \qquad (15)$$

$$\frac{\partial S(\text{y},z)}{\partial x}+\left(\delta+\alpha+\mu_b(y)+\lambda b\left(1-X(z)\right)\right)S(\text{y},z)-\int_0^\infty R(x,y,z)\mu_p(x)dx=0 \qquad (16)$$

$$\frac{\partial T(x,z)}{\partial x}+\left(\gamma+\mu_w(x)+\lambda b\left(1-X(z)\right)\right)T(x,z)=0 \qquad (17)$$

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$$P(0,z)=\int_0^\infty Q(x,z)\mu_p(x)dx+\int_0^\infty (pz+q)S(y,z)\mu_b(y)dy+\int_0^\infty T(x,z)\mu_w(x)dx + \alpha\int_0^\infty S_n(y,z)dy-(\lambda+\delta)P_0 \quad (18)$$

$$Q(0,z)=\delta\int_0^\infty P(x,z)dx \quad (19)$$

$$R(0,y,z)=\delta S(y,z) \quad (20)$$

$$S(0,z)=\frac{1}{z}\int_0^\infty P(x,z)a(x)dx+\gamma\int_0^\infty T(x,z)dx+\frac{\lambda X(z)}{z}\int_0^\infty P(x,z)dx \quad (21)$$

$$T(0,z)=(\lambda+\delta)P_0 \quad (22)$$

Solving the partial differential equations (13) to (17), it follows that

$$P(x,z)=P(0,z)\left[1-R(x)\right]e^{-(\lambda+\delta)x} \quad (23)$$

$$Q(x,z)=Q(0,z)\left[1-S_p(x)\right]e^{-B(z)x} \quad (24)$$

$$R(x,y,z)=R(0,y,z)\left[1-S_p(x)\right]e^{-B(z)x} \quad (25)$$

$$S(y,z)=S(0,z)\left[1-S_b(y)\right]e^{-C(z)y} \quad (26)$$

$$T(x,z)=T(0,z)\left[1-S_w(x)\right]e^{-D(z)x} \quad (27)$$

where,

$B(z)=\lambda b(1-X(z))$; $C(z)=\left[\alpha+\lambda b(1-X(z))+\delta(1-S_p{}^*(B(z)))\right]$; $D(z)=(\gamma+\lambda b(1-X(z)))$

Inserting equation (18) to (22) in (23) to (27) and make some manipulation, finally we get the limiting probability generatingfunctions $P(x,z)$, $Q(x,z)$, $R(x,y,z)$, $S(y,z)$ and $T(x,z)$. Next weare interested in investigating the marginal orbit size distributionsdue to system state of the server in following corollary.

Corollary 1. Under the stability condition $\rho<R^*(\lambda+\delta)$, thePGF of number of customers in the orbit when server being idle,priority busy, preemptive priority busy, normal busy and workingbreakdown are given by

$$P(z)=\frac{z(\lambda+\delta)\overline{R^*}(\lambda+\delta)P_o\left\{W(z)\left((pz+q)S_b^*(C(z))+V(z)\right)+\left(S_w^*(D(z))-1\right)\right\}}{Dr(z)} \quad (28)$$

$$Q(z)=\frac{\left(1-S_p^*(B(z))\right)\begin{pmatrix} zP_0\delta(\lambda+\delta)\overline{R^*}(\lambda+\delta) \\ \left\{W(z)\left((pz+q)S_b^*(C(z))+V(z)\right)+\left(S_w^*(D(z))-1\right)\right\}\end{pmatrix}}{B(z)\times Dr(z)} \quad (29)$$

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$$R(z)=\frac{\begin{array}{l}\delta P_0\left(1-S_b^*(C(z))\right)\left(1-S_p^*(B(z))\right)(\lambda+\delta)\times\\ \left\{\left(S_w^*(D(z))-1\right)\left(R^*(\lambda+\delta)+\lambda X(z)\overline{R^*}(\lambda+\delta)\right)+zW(z)\left(1-\delta\overline{R^*}(\lambda+\delta)S_p^*(B(z))\right)\right\}\end{array}}{\left(B(z)\times C(z)\right)\times Dr(z)} \quad (30)$$

$$S(z)=\frac{\begin{array}{l}(\lambda+\delta)P_0\left(1-S_b^*(C(z))\right)\times\\ \left\{\left(S_w^*(D(z))-1\right)\left(R^*(\lambda+\delta)+\lambda X(z)\overline{R^*}(\lambda+\delta)\right)+zW(z)\left(1-\delta\overline{R^*}(\lambda+\delta)S_p^*(B(z))\right)\right\}\end{array}}{C(z)\times Dr(z)} \quad (31)$$

$$T(z)=\left\{\frac{(\lambda+\delta)P_0W(z)}{\gamma}\right\} \quad (32)$$

Where,

$$Dr(z)=\left\{\begin{array}{l}z-\left(R^*(\lambda+\delta)+\lambda X(z)\overline{R^*}(\lambda+\delta)\right)\left((pz+q)S_b^*(C(z))+V(z)\right)\\ -z\delta\overline{R^*}(\lambda+\delta)S_p^*(B(z))\end{array}\right\}$$

$$W(z)=\gamma\left(\frac{1-S_w^*(D(z))}{D(z)}\right);\; V(z)=\frac{\alpha\left(1-S_b^*(C(z))\right)}{C(z)};$$

$$C(z)=\left[\alpha+\lambda b(1-X(z))+\delta\left(1-S_p^*(B(z))\right)\right];\; D(z)=\left(\gamma+\lambda b(1-X(z))\right)$$

Applying the normalizing condition $P_0+P(1)+Q(1)+R(1)+S(1)+T(1)=1$ and using the equations by setting z = 1 in (28) to (32), we get the value of P_0.

$$P_0=\frac{1-\overline{R^*}(\lambda+\delta)(\lambda E(X)+\delta)-\rho}{\eta}$$

$$\rho=\left(R^*(\lambda+\delta)+\lambda\overline{R^*}(\lambda+\delta)\right)\left(\begin{array}{l}S_b^{*'}(\alpha)(-\lambda bE(X))\left(1+\delta E(S_p)\right)\\ +pS_b^{*'}(\alpha)+V'(1)\end{array}\right)+\delta\lambda bE(X)E(S_p)\overline{R^*}(\lambda+\delta)$$

$$\begin{aligned}\eta=&\left(1-\overline{R^*}(\lambda+\delta)(\lambda E(X)+\delta)-\rho\right)\left(1+(\lambda+\delta)\left(1-S_w^*(\gamma)\right)/\gamma\right)\\ &+(\lambda+\delta)\left\{1+\delta E(S_p)\right\}\left(1-S_w^*(\gamma)\right)\\ &\times\left\{\begin{array}{l}\overline{R^*}(\lambda+\delta)\left(\dfrac{\lambda bE(X)}{\gamma}+pS_b^*(\alpha)+V'(1)-\lambda bE(X)S_b^{*'}(\alpha)\left(1+\delta E(S_p)\right)\right)+\\ \overline{S_b^*}(\alpha)\left\{\begin{array}{l}1-\delta\overline{R^*}(\lambda+\delta)\lambda bE(X)E(S_p)-\overline{R^*}(\lambda+\delta)(\lambda E(X)+\delta)\\ +\dfrac{\lambda bE(X)}{\gamma}\left(1-\delta\overline{R^*}(\lambda+\delta)\right)\end{array}\right\}\end{array}\right\}\end{aligned}$$

Corollary 2. If the system satisfies the stability condition,

(i) The PGF of number of customers in the system is

$$K(z)=P_0+P(z)+z\left(Q(z)+R(z)+S(z)+T(z)\right)$$

(ii) The PGF of the number of customers in the orbit is

$$H(z)=P_0+P(z)+Q(z)+R(z)+S(z)+T(z)$$

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4. SYSTEM PERFORMANCE MEASURES

From equations (28) - (32), by setting z = 1 and applying L-Hospital's rule whenever necessary, then we get the following probabilities for retrial, priority busy, preemptive priority busy, regularbusy, working breakdowns are,

$$P(1)=\frac{(\lambda+\delta)P_0\overline{R^*}(\lambda+\delta)\times\left(\left(1-S_w^*(\gamma)\right)\left\{\frac{\lambda bE(X)}{\gamma}-\lambda bE(X)S_b^{*'}(\alpha)\left(1+\delta E\left(S_p\right)\right)+pS_b^*(\alpha)+V'(1)\right\}\right)}{Dr(1)}$$

$$Q(1)=\frac{\delta(\lambda+\delta)P_0\overline{R^*}(\lambda+\delta)\times\left(E\left(S_p\right)\left(1-S_w^*(\gamma)\right)\left\{\frac{\lambda bE(X)}{\gamma}+pS_b^*(\alpha)+V'(1)-\lambda bE(X)S_b^{*'}(\alpha)\left(1+\delta E\left(S_p\right)\right)\right\}\right)}{Dr(1)}$$

$$R(1)=\frac{\delta(\lambda+\delta)P_0E\left(S_p\right)\overline{S_b^*}(\alpha)\left(1-S_w^*(\gamma)\right)\times\left\{\frac{\lambda bE(X)}{\gamma}\left[1-\delta\overline{R^*}(\lambda+\delta)\right]+1-\delta\overline{R^*}(\lambda+\delta)\left(\lambda bE(X)E\left(S_p\right)\right)-\overline{R^*}(\lambda+\delta)\left(\lambda E(X)+\delta\right)\right\}}{Dr(1)}$$

$$S(1)=\frac{(\lambda+\delta)P_0\overline{S_b^*}(\alpha)\left(1-S_w^*(\gamma)\right)\times\left\{\frac{\lambda bE(X)}{\gamma}\left[1-\delta\overline{R^*}(\lambda+\delta)\right]+1-\delta\overline{R^*}(\lambda+\delta)\left(\lambda bE(X)E\left(S_p\right)\right)-\overline{R^*}(\lambda+\delta)\left(\lambda E(X)+\delta\right)\right\}}{Dr(1)}$$

$$T(1)=\left\{\frac{(\lambda+\delta)P_0\left(1-S_w^*(\gamma)\right)}{\gamma}\right\}$$

where

$$Dr(1)=1-\delta\overline{R^*}(\lambda+\delta)\lambda bE(X)E\left(S_p\right)-\overline{R^*}(\lambda+\delta)\left(\lambda E(X)+\delta\right)-\left(R^*(\lambda+\delta)+\lambda\overline{R^*}(\lambda+\delta)\right)\begin{pmatrix}S_b^{*'}(\alpha)\left(-\lambda bE(X)\right)\left(1+\delta E\left(S_p\right)\right)\\+pS_b^*(\alpha)+V'(1)\end{pmatrix}$$

The expected number of customers in the orbit L_q is

$$L_q=H'(1)=\lim_{z\to1}\frac{d}{dz}H(z)$$

$(i.e).,\ \mathrm{L}_q=P_0+P'(1)++Q'(1)+R'(1)+S'(1)+T'(1)$

where,

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$$P'(1)=\frac{(\lambda+\delta)P_0\overline{R^*}(\lambda+\delta)I''(1)}{J''(1)}$$

$$Q'(1)=\delta\times\frac{-3\lambda bE(X)E(S_p)I''(1)-3E(S_p^2)(\lambda bE(X))^2I'(1)+2(-\lambda bE(X(X-1)))E(S_p)I'(1)}{3(-\lambda bE(X))J''(1)+3(-\lambda bE(X(X-1)))J'(1)}$$

$$R'(1)=\frac{-\delta\left[M''(1)E(S_p)\lambda bE(X)+M'(1)E(S_p^2)(\lambda bE(X))^2+M'(1)E(S_p)(\lambda bE(X(X-1)))\right]}{\alpha(-\lambda bE(X(X-1)))J'(1)+2\alpha(-\lambda bE(X))J'(1)}$$

$$S'(1)=\frac{(\lambda+\delta)P_0M''(1)}{\alpha J''(1)+2J'(1)C'(1)}$$

$$T'(1)=P_0W'(1)\frac{(\lambda+\delta)}{\gamma}$$

Where

$$I'(1)=\left(1-S_w^*(\gamma)\right)\left(S_b^{*'}(\alpha)\left(-\lambda bE(X)\left(1+\delta E(S_p)\right)\right)+pS_b^*(\alpha)+V'(1)\right)+W'(1)-\lambda bE(X)S_w^{*'}(\gamma)$$

$$I''(1)=\left(1-S_w^*(\gamma)\right)\left[\begin{array}{l}2pS_b^{*'}(\alpha)\left(-\lambda bE(X)\left(1+\delta E(S_p)\right)\right)+S_b^{*'}(\alpha)C''(1)+V''(1)+S_b^{*''}(\alpha)\left(\lambda bE(X)\left(1+\delta E(S_p)\right)\right)^2\\+2\left(S_b^{*'}(\alpha)\left(-\lambda bE(X)\left(1+\delta E(S_p)\right)\right)+pS_b^*(\alpha)+V'(1)\right)\end{array}\right]$$
$$+2W'(1)\left(S_b^{*'}(\alpha)\left(-\lambda bE(X)\left(1+\delta E(S_p)\right)\right)+pS_b^*(\alpha)+V'(1)\right)$$
$$+W''(1)+S_w^{*'}(\gamma)D''(1)+2S_w^{*'}(\gamma)(-\lambda bE(X))+2W'(1)$$

$$J'(1)=1-\delta\overline{R^*}(\lambda+\delta)E(S_p)\lambda bE(X)-\lambda\overline{R^*}(\lambda+\delta)E(X)$$
$$-\left(R^*(\lambda+\delta)+\lambda\overline{R^*}(\lambda+\delta)\right)\left(S_b^{*'}(\alpha)\left(-\lambda bE(X)\left(1+\delta E(S_p)\right)\right)+pS_b^*(\alpha)+V'(1)\right)$$

$$J''(1)=-\delta\overline{R^*}(\lambda+\delta)\left(\lambda bE(X)E(S_p)+E(S_p^2)(\lambda bE(X))^2+\lambda E(X)E(S_p)-\lambda bE(X)E(S_p)E(X(X-1))\right)$$
$$-\lambda\overline{R^*}(\lambda+\delta)E(X(X-1))-\left(R^*(\lambda+\delta)+\lambda\overline{R^*}(\lambda+\delta)\right)\left(\begin{array}{l}-2pS_b^{*'}(\alpha)\left(\lambda bE(X)\left(1+\delta E(S_p)\right)\right)+V''(1)\\+S_b^{*''}(\alpha)\left[\lambda bE(X)\left(1+\delta E(S_p)\right)\right]^2+S_b^{*'}(\alpha)C''(1)\end{array}\right)$$
$$-2\left(\lambda E(X)\overline{R^*}(\lambda+\delta)\right)\left(-S_b^{*'}(\alpha)\left(\lambda bE(X)\left(1+\delta E(S_p)\right)\right)+pS_b^*(\alpha)+V'(1)\right)$$

$$M'(1)=\left(1-S_b^*(\alpha)\right)\left\{\left(1-S_w^*(\gamma)\right)\left[\left(\frac{\lambda bE(X)}{\gamma}\right)\left(1-\delta\overline{R^*}(\lambda+\delta)\right)+1-\delta\overline{R^*}(\lambda+\delta)\left(\lambda bE(X)E(S_p)\right)-\overline{R^*}(\lambda+\delta)(\lambda E(X)+\delta)\right]\right\}$$

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$$M^{''}(1)=\left(1-S_b^{*}(\alpha)\right)\left\{\begin{array}{l}-2\delta\overline{R^{*}}(\lambda+\delta)E\left(S_p\right)\lambda bE(X)\left[\left(1-S_W^{*}(\gamma)\right)+W^{'}(1)\right]+\left(1-\delta\overline{R^{*}}(\lambda+\delta)\right)\left[2W^{'}(1)+W^{''}(1)\right]\\+2E(X)\overline{R^{*}}(\lambda+\delta)S_W^{*'}(\gamma)\left(-\lambda bE(X)\right)+\left(S_W^{*}(\gamma)-1\right)\left(\lambda\overline{R^{*}}(\lambda+\delta)E\left(X(X-1)\right)\right)\\-\delta\overline{R^{*}}(\lambda+\delta)\left(1-S_W^{*}(\gamma)\right)\left[E\left(S_p^2\right)\left(\lambda bE(X)\right)^2+E\left(S_p\right)\lambda bE\left(X(X-1)\right)\right]\\+\left(R^{*}(\lambda+\delta)+\lambda\overline{R^{*}}(\lambda+\delta)\right)\left[S_W^{*''}(\gamma)\left(\lambda bE(X)\right)^2+S_W^{*'}(\gamma)\left(-\lambda bE\left(X(X-1)\right)\right)\right]\end{array}\right\}$$

$$+S_b^{*'}(\alpha)C^{'}(1)\left(1-S_W^{*}(\gamma)\right)\left[\delta\overline{R^{*}}(\lambda+\delta)\lambda bE(X)E\left(S_p\right)-\left(1-\delta\overline{R^{*}}(\lambda+\delta)\right)\right]$$

$$-S_b^{*'}(\alpha)C^{'}(1)\left(R^{*}(\lambda+\delta)+\lambda\overline{R^{*}}(\lambda+\delta)S_W^{*}(\gamma)\left(-\lambda bE(X)\right)\right)$$

$$-S_b^{*'}(\alpha)C^{'}(1)W^{'}(1)\left(1-\delta\overline{R^{*}}(\lambda+\delta)\right)-S_b^{*'}(\alpha)C^{'}(1)\left(S_W^{*}(\gamma)-1\right)\lambda E(X)\overline{R^{*}}(\lambda+\delta)$$

$$B^{'}(1)=-\lambda bE(X);\ B^{''}(1)=-\lambda bE\left(X(X-1)\right);\ C^{'}(1)=-\lambda bE(X)\left(1+\delta E\left(S_p\right)\right);$$

$$C^{''}(1)=-\lambda bE\left(X(X-1)\right)-\delta E\left(S_p\right)E\left(X(X-1)\right)-\delta E\left(S_p^2\right)\left(\lambda bE(X)\right)^2;$$

$$D^{'}(1)=-\lambda bE(X);\ D^{''}(1)=-\lambda bE\left(X(X-1)\right);\ V^{'}(1)=\frac{\lambda bE(X)\left(1+\delta E\left(S_p\right)\right)}{\alpha}\left(\alpha S_b^{*'}(\alpha)+1-S_b^{*}(\alpha)\right)$$

$$V^{''}(1)=\frac{1}{\alpha^2}\left[2\left(\lambda bE(X)\left(1+\delta E\left(S_p\right)\right)\right)^2\left(\alpha S_b^{*'}(\alpha)+1-S_b^{*}(\alpha)\right)\right]$$

$$-\alpha\left[\alpha S_b^{*''}(\alpha)\left(\lambda bE(X)\left(1+\delta E\left(S_p\right)\right)\right)^2+\alpha S_b^{*'}(\alpha)C^{''}(1)+\left(1-S_b^{*}(\alpha)\right)C^{''}(1)\right];$$

$$W^{'}(1)=\frac{\lambda bE(X)}{\gamma}\left[\gamma S_W^{*'}(\gamma)+1-S_W^{*}(\gamma)\right];$$

$$W^{''}(1)=\frac{1}{\gamma^2}\left\{\begin{array}{l}2\lambda bE(X)\left[\gamma S_W^{*'}(\gamma)\lambda bE(X)+\lambda bE(X)\left(1-S_W^{*}(\gamma)\right)\right]\\+\gamma\left[\begin{array}{l}S_W^{*'}(\gamma)\left(\lambda bE(X)\right)^2+\left(1-S_W^{*}(\gamma)\right)\left(\lambda bE\left(X(X-1)\right)\right)\\+\gamma S_W^{*'}(\gamma)\left(\lambda bE\left(X(X-1)\right)\right)-\gamma\left(\lambda bE(X)\right)^2S_W^{*''}(\gamma)-\left(\lambda bE(X)\right)^2S_W^{*}(\gamma)\end{array}\right]\end{array}\right\}$$

The expected number of customers in the system $L_s = K^{'}(1) = L_q + \rho.$

The average times are $W_s = \frac{L_s}{\lambda}$ and $W_q = \frac{L_q}{\lambda}.$

5. SPECIAL CASES

We present the following five special cases of our model:

Case 1: No batch arrival, No feedback and No balking

In this case, we put p = b = 0, our model reduced to an M/G/1 preemptive priority retrial queueing system with disaster under working breakdownservices. The result coincides with the result of Sherif I. Ammar and Rajadurai(2019).

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Case 2: No batch arrival, No priority arrival, No feedback and No balking

Let $\delta = p = b = 0$, our model reduced to M/G/1 retrial queueing systemwith disaster and working breakdowns.

Case 3: No disaster, No feedback, No Working Breakdown and No Balking

Let $(\gamma, \alpha, p, b) \rightarrow (0,0,0,0)$, our model reduced to $M^{[X]}$/G/1 preemptivepriority retrial queueing system.

Case 4: No priority arrival, No Batch arrival, No feedback, No Working Breakdown, No Disaster and No Balking

Let $(\delta, p, \gamma, \alpha, b) \rightarrow (0,0,0,0,0)$, our model can be reduced to M/G/1 queue with general retrial times.This result coincides with the result of Gao et al. (2014).

Case 5: No batch arrival, No priority arrival, No Retrial, No Working Breakdown, No Feedback, No Balking and No disaster

Let $R^*(\lambda) \rightarrow 1$ and $(\delta, p, \gamma, \alpha, \mathrm{b}) \rightarrow (0,0,0,0,0)$, our model can be reducedto M/G/1 queueing system.This result coincides with the result of Gomez-Corral.

5. CONCLUSION

In this work, we have studied a Performance Analysis of Feedbackand Preemptive Priority Retrial Queueing System with Disaster under Working Breakdown Services and Balking. Applying the PGFapproach and the supplementary variable technique, the PGFsforthe number of customers in the system and its orbit when it is free,priority busy, preemptive priority busy, regular busy, on lower speed service are derived. Various system performance measures and theexplicit expressions for the average queue length of orbit and system have been obtained. This model has potential application inWireless Sensor Networks (WSNs).

REFERENCES

[1] Artalejo, J. R. and Corral, A.G. Retrial Queueing Systems. Springer, Berlin, Germany, 2008.

[2] Kalidass, K.,Ramanath, K; A queue with working breakdowns, Computers and Industrial Eng. 2018, **63**, 779-783.

[3] Gao, S. A preemptive priority retrial queue with two classes of customers and general retrial times. Oper. Res. 2015, **15**, 233-251.

[4] Liu, Z.; Song, Y. The $M^{[X]}$/M/1 queue with working breakdown. RairoOper. Res. Rech. Opr. 2014, **48**, 399-413.

[5] Rajadurai, P. Sensitivity analysis of an M/G/1 retrial queueing system with disaster under working vacations and working breakdowns. RairoOper. Res. 2018, **52**, 35-54.

[6] Rajadurai,P.; Priyavarshini, A.S.; Sivapriya, V; Narasimhan, D. Priority Retrial Queue with Re-Service andWorking Breakdown Services, Test Engineering and Management. 2020, **82**, 2682-2684.

[7] B.K. Kim and D.H. Lee, The M/G/1 queue with disasters and working breakdowns. Appl. Math. Model. 38 (2014) 1788–1798.

[8] SI Ammar and P.Rajadurai, Performance analysis of preemptive priority retrial queueing system with disaster under working breakdown services, Symmetry 11 (3), 419.