Analysis of MGs with Strongly Independent and Weakly Independent Places Using Sign Incidence Matrix and Their Conversion intoDirected graphs

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Abstract: In this paper in the beginning introduction to PT net (Place –transition net) is given. Basic notations and formal definition are given. Finally a MG (Marked Graph) with strongly independent and weakly independent places is taken and analyzed using sign incidence matrix for finding set of places which are both S & T.(Siphon and Trap) Also the MG is converted into directed graphs by three methods. Then the directed graphs are analyzed.

Introduction:PT net are dynamic bipartites graphs. MGs are special class of PTnet. Finding of S & T are important for checking of the liveness of PT net.Conversion of PT net into directed graphs are important which is also used in this paper.

This paper is organised as follows.Section I contains basic defiintions of PT net and Graph theory. Section II containsEnumeration of S & T which are Subsets of Places of MGs.Section III contains converion of MGs into directed graphs and Section IV Contains Conclusions and References.

I-Basic Definitions on PT net and Graph theory

1.1 Definition : PT netA PT net is a directed bipartite graph composed of four parts, a set of places P, a set of transitions T, an input function I, an output function O. These two functions connect places and transitions. A PT net N is denoted by N = {P, T, I, O, M_0} Where M_0 is the initial marking (no of tokens present in each place in the beginning).[1,2]

1.2 Definition S & T

A subset of places denoted as S in PT net N is called a siphon if *S $\subset$ S *, i,e every transition having an output place in S has an input place in S.

where

*S denotes the set of input places for all transitions.

S* denotes the set of output places for all transitions.

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A non-empty subset of places Q in a PT net N is called a trap if $Q^* \subset {}^*Q$ i,e every transition having an input place in Q has an output place in Q.[3,4,5]

1.3Definition:A MG is a PT net C = (P, T, I, O) such that for each $p_i \in P$, $|I(p_i)| = |\{ t_j / p_i \in O(t_j)\}| = 1$ and $|O(p_i)| = |\{ t_j / p_i \in I(t_j)\}| = 1$. [4][5]. An example is given in Figure

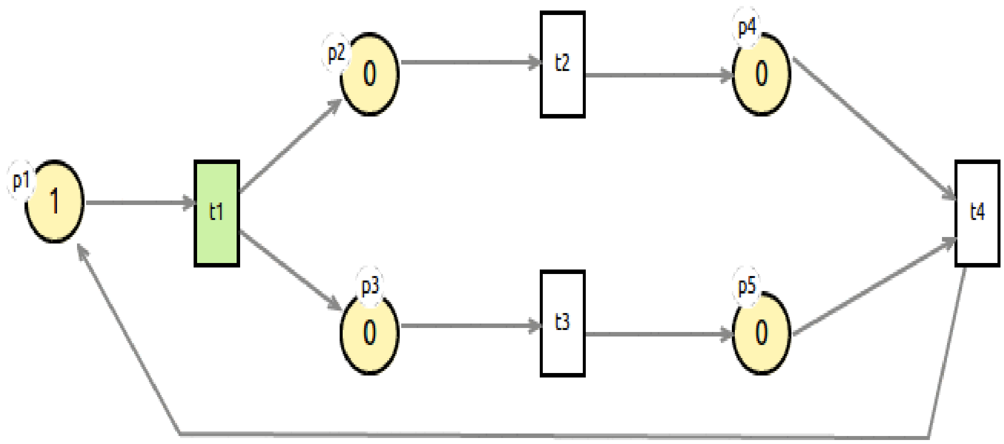

Figure Example for MG

Here the algorithm is presented to find all subsets of places which are both S & T for MGs given in .To apply the Algorithm the sign incidence matrix is required, and that is also defined..[3,4,5]

1.4 Definition : Sign Incidence Matrix

For a PT net N with n-transitions and m-places, the sign incidence matrix $A = [a_{ij}]$ is an n x m matrix whose entry is given as follows.

a_{ij}= + if place j is an output place of transition i.

a_{ij} = - if it is an input place of transition i.

a_{ij} = ± if it is both input and output places of transition i (I.e. transition i and place j form a self loop)

a_{ij} = 0 otherwise..[3,4,5]

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1.5 Definition: Addition: The addition denoted by $\oplus$ is a commutative binary operation on the set of four elements. B= {+,-, 0, ±} defined as follows.

$+ \oplus - = \pm$

$x \oplus x = x$, For every $x \in B$

$\pm \oplus x = \pm$, For every $x \in B$

$0 \oplus x = x$, For every $x \in B$.[3,4,5]

1.6 Theorem: A subset of k-places Z = {p_1, p_2 ...... p_k) in a MG N is both S & T if and only if the addition of k-column vectors of the sign incidence matrix of N, $A_1 \oplus A_2 \oplus \ldots \oplus A_k$ contains either zero entry or ± entry where A_j denote the column vector corresponding to place P_j, j = 1,2,....k. .[3,4,5]

1.7 Definition: A + entry is said to be neutralized by adding a - entry to get a ± entry..[3,4,5]

1.8 Definition: A directed graph D consists of a non-empty finite set V (D) of elements called vertices, and finite family A (D) of ordered pairs of elements of V (D) called arcs. V (D) is called the vertex set and A (D) the arc family of D. .[7,8]

1.9 Definition: Euler Directed graph: In a directed graph G a closed directed walk which traverses every edge of G exactly once is called a directed Euler line. A directed graph containing a directed Euler line is called Euler directed graph..[7,8]

1.10 Theorem: A directed graph G is an Euler directed graph if and only if G is connected and is balanced i.e. $d^-(v) = d^+(v)$ for every vertex v in G..[7,8]

1.11 Algorithm for Finding S & T using Sign Incidence Matrix

Input: Sign incidence matrix A of order m x n.

Step 1: Select A_j, the first column in the sign incidence matrix A, whose corresponding 'place is denoted as $PLACE_j$.

Set recursion level r to 1

Set $V_{j\,r} = A_j$

Set $PLACE_{jr} = PLACE_j$

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Step 2: If V_{jr} has a ± entry at i^{th} row then $PLACE_j$ is a self loop with transition t_j. Go to step 5.

Step 3: If V_{ij} has a+ entry in the k^{th} row find a column in A, which contains a - entry at the k^{th} row.

(a) If no such column in $A_\backslash$exists, Go to step 5.

(b) If such A, exists add it to V_{jr} to obtain $V_{j(r+1),} = V_{jr} \oplus A_s$ containing a ± entry at k^{th} row. Then $PLACE_{j(r+1)} = PLACE_{jr} \cup PLACE_s$.

(c) Repeat this step for all possible neutralizing columns A_s. This gives a new set of $V_{j(r+1)}$'s and $PLACE_{j(r+!)}$'s.

Step 4: Increment r by 1. Repeat step 3 until there are no more + entries in each

$V_{jr} = A_1 \oplus A_2 \oplus ... A_{jr}$ or no neutralizing column can be defined.

Step 5: Any V_{jr} without + entries and without-entries (i.e., all the entries are either zero or ±) represents S & T. i.e., the places in $PLACE_{jr}$ form both S & T.

Step 6:

Delete A_j

j=j+1

Go to step 1.

Output: All sets which are both S &T...[3,4,5]

1.12 Methods to Convert PT net into Directed graphs

A PT net can be converted into a directed graph in three ways.

Method 1: Here the transitions are changed in to vertices and places in to edges and connections as in the MG.

Method 2: Here the transitions are changed into edges and places into vertices.

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Method 3: Here both transitions and place are changed in to vertices and arcs between them as edges.

II-Enumeration of S &T which are Subsets of Places of MGs

2.1 Consider the following MG taken from [8].First the sign inciidence matrix is formed and then the algorithm 1.11 applied.

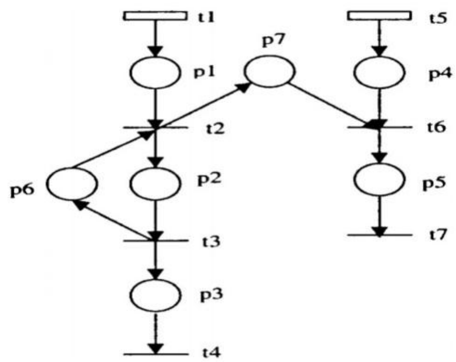

Figure 1

$$A=\begin{array}{c} \\ t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \end{array}\begin{array}{c} \begin{matrix} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 \end{matrix} \\ \begin{bmatrix} + & 0 & 0 & 0 & 0 & 0 & 0 \\ - & + & 0 & 0 & 0 & - & + \\ 0 & - & + & 0 & 0 & + & 0 \\ 0 & 0 & - & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & + & 0 & 0 & 0 \\ 0 & 0 & 0 & - & + & 0 & - \\ 0 & 0 & 0 & 0 & - & 0 & 0 \end{bmatrix} \end{array}$$

Step 1:

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$$A_2 = \begin{bmatrix} 0 \\ + \\ - \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{PLACE}_2 = \{p_2\}$$

For r = 1;

$$V_{21} = A_2 = \begin{bmatrix} 0 \\ + \\ - \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{PLACE}_{21} = \{p_2\}$$

Step 2: V_{21} has no ± entry. Therefore p_2 does not form self-loop with any transition t_j.

Steps 3, 4, 5 & 6 V_{21} has + entry at 2nd row. The neutralizing columns are p_6, p_1.

$$V_{22}^{(1)} = V_{21} \oplus A_1 = \begin{bmatrix} 0 \\ + \\ - \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \oplus \begin{bmatrix} + \\ - \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} + \\ \pm \\ - \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{PLACE}_{22}^{(1)} = \{p_2, p_1\}$$

$V_{22}^{(1)}$ has – in 3rd row. The neutralizing column is p_3, p_6

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$$V_{23}^{(1)} = V_{22}^{(1)} \oplus A_3 = \begin{bmatrix} + \\ \pm \\ - \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \oplus \begin{bmatrix} 0 \\ 0 \\ + \\ - \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} + \\ \pm \\ \pm \\ - \\ 0 \\ 0 \\ 0 \end{bmatrix} \mathrm{PLACE}_{23}^{(1)} = \{p_2, p_1, p_3\}$$

$$V_{23}^{(2)} = V_{22}^{(1)} \oplus A_6 = \begin{bmatrix} + \\ \pm \\ - \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \oplus \begin{bmatrix} 0 \\ - \\ + \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} + \\ \pm \\ \pm \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \mathrm{PLACE}_{23}^{(2)} = \{p_2, p_1, p_6\}$$

$$V_{22}^{(2)} = V_{21} \oplus A_6 = \begin{bmatrix} 0 \\ + \\ - \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \oplus \begin{bmatrix} 0 \\ - \\ + \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \pm \\ \pm \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \mathrm{PLACE}_{22}^{(2)} = \{p_2, p_6\}$$

The entries of $V_{22}^{(2)}$ are zeros and $\pm$ ‘s only.

Output: All the sets which are both S & T are $\{p_2, p_6\}$.

2.2 Consider the next MG taken from [8]

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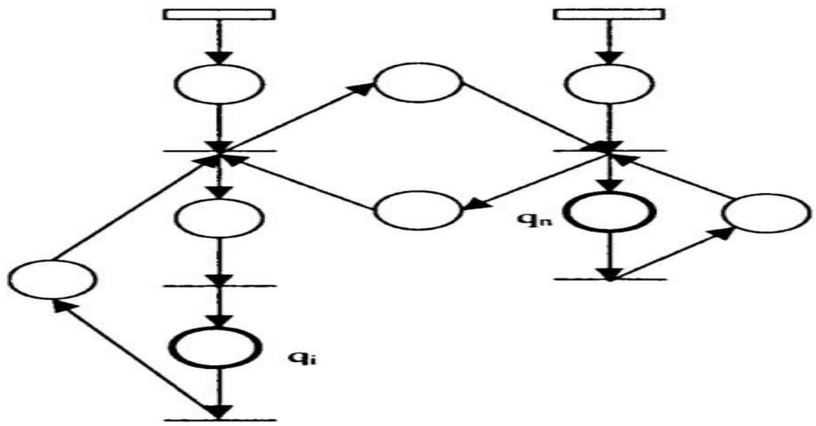

Figure 2

The sign incidence matrix is

$$A = \begin{array}{c} \\ t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \end{array} \begin{array}{c} \begin{array}{ccccccccc} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 & p_9 \end{array} \\ \begin{bmatrix} + & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ - & + & 0 & 0 & 0 & - & + & - & 0 \\ 0 & - & + & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 & + & 0 & 0 & 0 \\ 0 & 0 & 0 & + & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & - & + & 0 & - & + & - \\ 0 & 0 & 0 & 0 & - & 0 & 0 & 0 & + \end{bmatrix} \end{array}$$

Proceeding like this for all columns, the distinct set of places form the both S & T are $\{p_2, p_6, p_3\}$, $\{p_2, p_8, p_7, p_3, p_6\}$, $\{p_3, p_2, p_8, p_9, p_5, p_6\}$, $\{p_9, p_5\}$, $\{p_7, p_6, p_3, p_2, p_5, p_9\}$, $\{p_8, p_7\}$, $\{p_8, p_9, p_5, p_7\}$.

Output: All sets which are both S & T are $\{p_2, p_6, p_3\}$, $\{p_2, p_8, p_7, p_3, p_6\}$, $\{p_3, p_2, p_8, p_9, p_5, p_6\}$, $\{p_9, p_5\}$, $\{p_7, p_6, p_3, p_2, p_5, p_9\}$, $\{p_8, p_7\}$, $\{p_8, p_9, p_5, p_7\}$.

2.3 Consider the next MG taken from [8]

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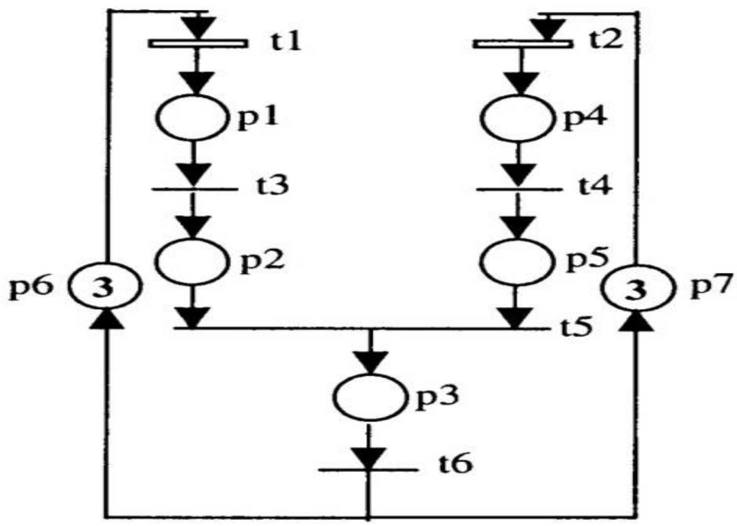

Figure 3

The sign incidence matrix is

$$A = \begin{array}{c} \\ t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \end{array} \begin{array}{c} \begin{array}{ccccccc} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 \end{array} \\ \begin{bmatrix} + & 0 & 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & + & 0 & 0 & - \\ - & + & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & - & + & 0 & 0 \\ 0 & - & + & 0 & - & 0 & 0 \\ 0 & 0 & - & 0 & 0 & + & + \end{bmatrix} \end{array}$$

Proceeding like this for all columns, the distinct set of places form the both S & T are $\{p_3, p_5, p_4, p_7\}$, $\{p_1, p_6, p_3, p_4, p_5, p_7, p_2\}$, $\{p_1, p_6, p_3, p_2\}$.

Output: All sets which are both S & T are $\{p_3, p_5, p_4, p_7\}$, $\{p_1, p_6, p_3, p_4, p_5, p_7, p_2\}$, $\{p_1, p_6, p_3, p_2\}$.[4][5].. [10][11][12] are the similar papers in this area.

3.1Directed graphs of the MGs given in Figure 1

Now the above MG is converted into directed graphs by the methods given in 1.12

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METHOD – 1

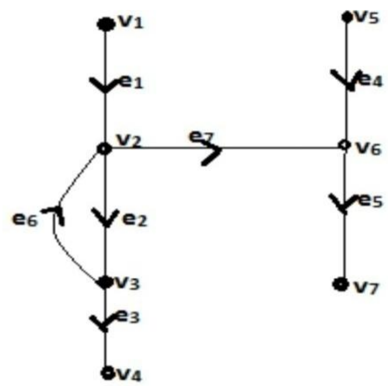

Figure 4

METHOD – 2

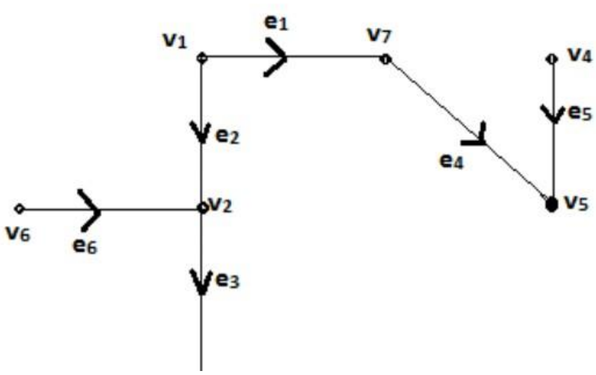

Figure 5

METHOD – 3

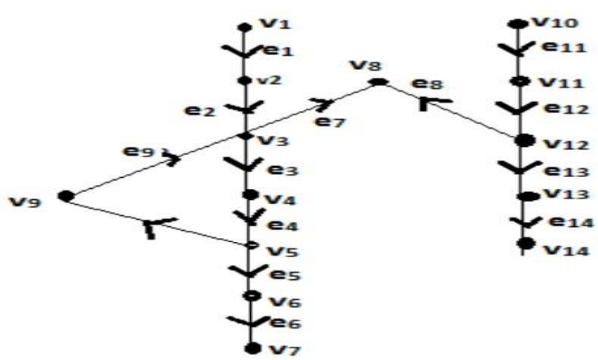

Figure 6

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3.2 Analysis of the directed graphs of the MG

S. no	Directed graph Figure number	Number of vertices	Number of Edges	Directed Euler Graph	Chromatic number	No of Directed Circuits
1	4	7	7	No	3	1
2	5	7	6	No	3	0
3	6	14	14	No	3	1

3.3 Directed graphs of the MGs given in Figure 2

Method 1

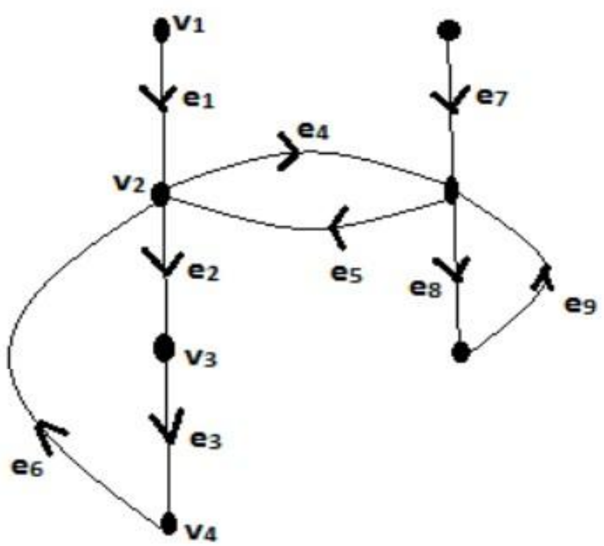

Figure 7

Method 2

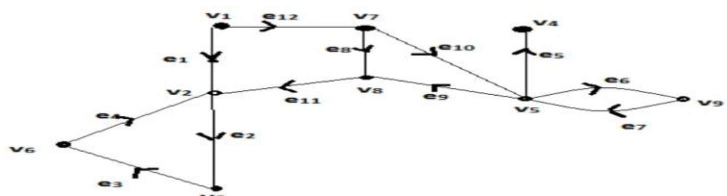

Figure 8

Method 3

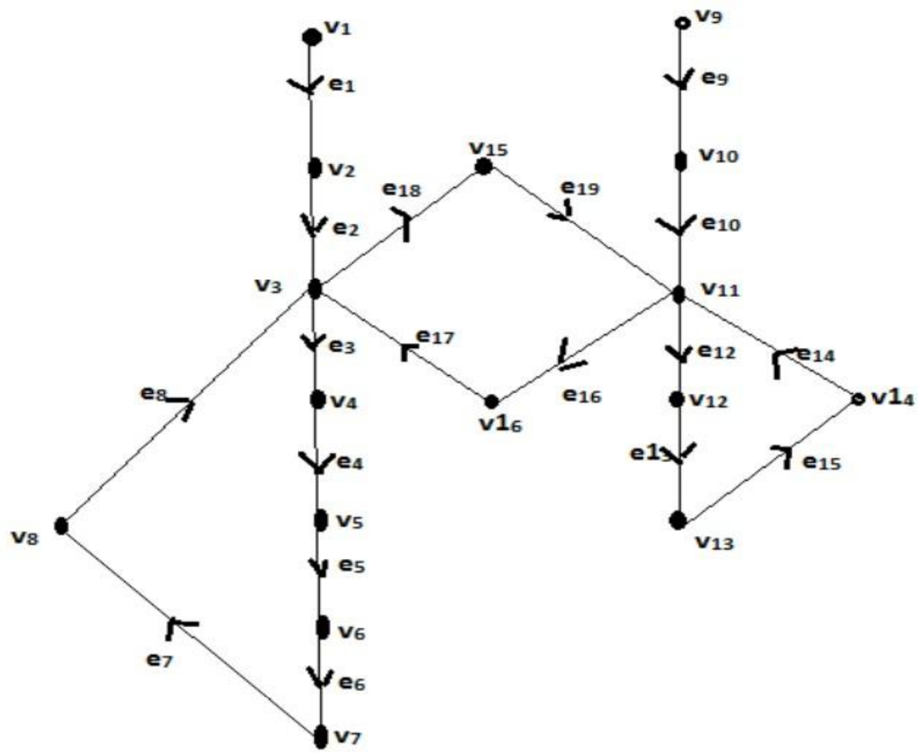

Figure 9

3.4Analysis of the directed graphs of the MG

S. no	Directed graph Figure number	Number of vertices	Number of Edges	Directed Euler Graph	Chromatic number	No of Directed Circuits
1	7	7	9	No	3	3
2	8	9	12	No	3	4
3	9	16	19	No	3	3

3.5 Directed graphs of the MGs given in Figure 3

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Method 1

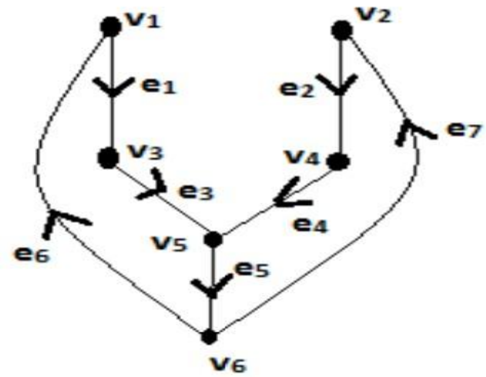

Figure 10

Method 2

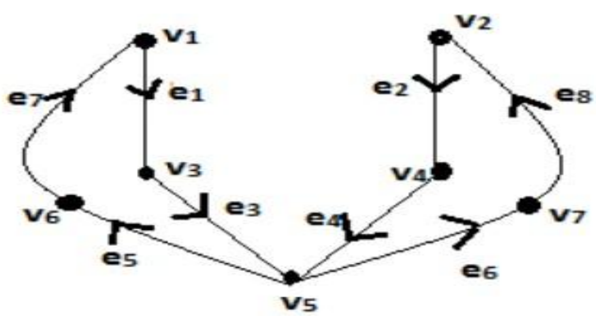

Figure 11

Method 3

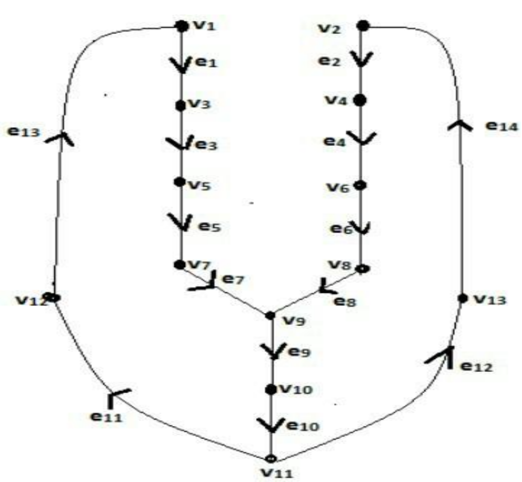

Figure 12

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3.6 Analysis of the directed graphs of the MG3

S. no	Directed graph Figure number	Number of vertices	Number of Edges	Directed Euler Graph	Chromatic number	No of Directed Circuits
1	10	7	7	No	4	2
2	11	8	8	yes	2	2
3	12	13	14	No	3	2

IV-Conclusion and References.

Conclusion: Theset of places which form both S & T are reported for the three MGs.Also the converted directed graphs and their analysis also reported.

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