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Stability Control of Ship in Missile Firing Mission Using Model Predictive Control

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Abstract

Warships have a mission to defend themselves from enemy attacks. One of the weapons possessed is an air defense missile. When firing a missile, the ship receives an impact force. To restore the stability of the vessel due to the impact force, it is necessary to control the steering angle of the ship (rudder) in order to keep the yaw angle (or heading angle) in the desired condition. This work aims to implement a Model Predictive Control (MPC) control system to control the yaw angle of the ship during a shooting period. The results show that the MPC controller is able to drive the ship's heading angle to reach the reference angle. From two scenarios, the best performance is achieved when prediction horizon equals 25 based on Root Mean Square Error (RMSE).

Keywords: *model predictive control; rudder angle; impact force; heading angle*

1 Introduction:

One of the missions of warships is to be able to defend the territory guarded against the enemy. Based on the basic mission, the warships are also equipped with several weapons and one of them is an air defense missile. In addition, the ship is also equipped with a radar sensor that can detect targets.

The warships require control of the ship's heading when a firing missile. The controller designed should be able to overcome the interference that affects the system. The interference that occurs in this research is the disturbance that occurs due to the firing of the missile, i.e. the impact force [1]. Therefore, the Model Predictive Control (MPC) algorithm is used to solve this problem. MPC is a control system designed to minimize a certain objective function, and is used to complete the various control processes required while the system is working [2], [3]. MPC can be used to control unstable processes, as well as a process that has a large delay time [4], [5].

Several studies have been done in the literature that use MPC [6]. Li and Sun [7] discussed the application of DCMPC algorithm to overcome the problem of environmental disturbance so that the ship can be controlled according to the constraints set [7]. The paper written by Subchan and Asfihani [8] explained about the ship's control in the presence of a disturbance using the MPC algorithm. The simulation results show that the controller is able to reach the appropriate level of yaw angle with the existing constraints. Munadhif [9] designs a control for a ship in shooting missions taking into account four degrees of freedom, and cannon is fired in the form of a cannon's launcher. The result of this research is the ability of FGS-PID (Fuzzy Gain Scheduling-Proportional Integral Derivative) controller to overcome the stability problem of the ship when firing the cannon with wave disturbance, and FGS-PID controller has better performance compared to the ordinary PID controller [9].

Based on the above description, in this research, the MPC method is applied to the ship control system with two degrees of freedom i.e. sway and yaw, with rudder angle as the control input and yaw angle as the output, and firing missile as a constraint. Furthermore, the closed-loop system to measure the performance of the obtained controller is simulated.

2 Models And Prelımınarıes

This section discusses the mathematical model. The first part presents the dynamical model of ship. The modeling of impact force caused by firing a missile is proposed at the second part. The ship and the model of impact force are combined at the third part. The final section explains the controller synthesis using model predictive control.

A. Dynamical Model of a Ship

The general dynamical model of ship at the time of maneuver is the well-known Davidson-Schiff model, which is written as follows [10]:

$$M\dot{\mathrm{v}} + N\, u_0 \mathrm{v} = \mathrm{b}\, \delta_R$$

where M, N are matrices of dimensional hydrodynamic coefficients, v=$[v\ r]^T$, v is the sway displacement, r is the yaw rate, δ_R is the rudder angle and the remaining variables are given by:

$$M = \begin{bmatrix} m - Y_{\dot{v}} & m\, x_G - Y_{\dot{r}} \\ m\, x_G - N_{\dot{v}} & I_z - N_{\dot{r}} \end{bmatrix},$$

$$N\, u_0 = \begin{bmatrix} -Y_v & mu_o - Y_r \\ -N_v & mx_G u_0 - N_r \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The inertial matrix M is not symmetric, i.e. $M \neq M^T$. Then the dimensional hydrodynamic coefficient is transformed to the non-dimensional form using the formula in [10]. The linear dynamical system can be obtained as follow

$$M\dot{\mathrm{v}} + N\, u_0 \mathrm{v} = \mathrm{b}\, \delta_R \quad (1)$$

where $A_1 = M'^{-1}N', B_1 = M'^{-1}\mathbf{b}$ and M', $N'$are the matrices of non-dimensional hydrodynamic coefficients. By defining $\dot{\psi} = r$, the state space model can be constructed. The state vector is $\mathbf{x} = [v, r, \psi]^T$ and the input vector is $\mathbf{u} = \delta_R$.

$$\dot{\mathrm{x}} = A_c + c\mathrm{x} + B_c \mathrm{u}$$

where

$$A_c = \begin{bmatrix} A_{1,1} & A_{1,2} & 0 \\ A_{2,1} & A_{2,2} & 0 \\ 0 & 1 & 0 \end{bmatrix}, B_c = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$$

B. Impact Force Mathematical Model

When a ship fires a missile, there will be a force on the ship. Impact force is the force on a ship caused by firing a missile.

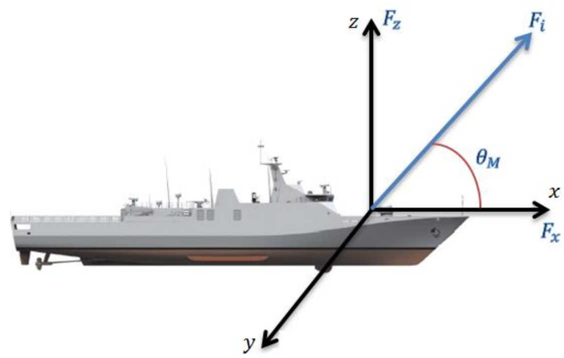

Fig. 1. Projection of firing force wrt x axis and z axis

Based on Fig. 1, the equation can be defined:

$$F_x = F_i \cos\theta_M,\ \ F_z = F_i \sin\theta_M \qquad (2)$$

where $F_i = m_M \times a_M$, m_M is the mass of missile and a_M is the acceleration of missile.

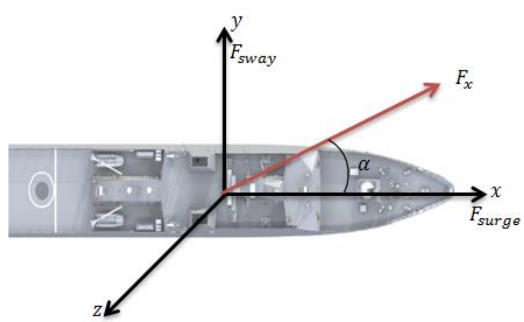

Fig. 2. Projection of firing force wrt x axis and y axis

Based on Fig. 2, the equation can be written:

$$F_{surge} = F_x \cos\alpha_M, F_{sway} = F_x \sin\alpha_M \qquad (3)$$

It follows that the moment of impact force can be written as follows:

$$N = F_i \cos\theta_M \sin\alpha_M \times d$$

where d is the distance of launcher to the center of the ship mass (x_G). The impact matrix that corresponds to the space state is as follows:

$$I = \begin{bmatrix} Y_{impact} \\ N_{impact} \\ 0 \end{bmatrix} = \begin{bmatrix} -m_M a_M \cos\theta_M \sin\alpha_M \\ -m_M a_M \cos\theta_M \sin\alpha_M \ d \\ 0 \end{bmatrix}$$

Furthermore, by using the Prime System I [10], the non-dimensional form of impact force matrix can be defined as follows:

$$I' = \begin{bmatrix} \frac{-m_M a_M \cos\theta_M \sin\alpha_M}{\frac{\rho}{2}U^2 L^2} \\ -\frac{m_M a_M \cos\theta_M \sin\alpha_M\, d}{\frac{\rho}{2}U^2 L^3} \\ 0 \end{bmatrix} \quad (4)$$

The acceleration of missile uses Augmented Proportional Navigation (APN) guidance law. APN is the effective control of missile to achieve a target [11], [12].

C. Mathematical Model of Ship Dynamics with Impact Force

At the launch of the ship's missile having an impact force, the mathematical model of ship dynamics becomes:

$$\dot{\mathrm{x}} = A_c\mathbf{x}(t) + B_c\mathbf{u}(t) + M'^{-1}I' \quad (5)$$

where I' is based on Eq. (4) and M' is matrix of non-dimensional hydrodynamic coefficients.

D. Model Predictive Control

The Model Predictive Control (MPC) is a method to synthesize a controller for discrete linear systems with disturbance as follows [8]:

$$\mathbf{x}(k+1|k) = A\mathbf{x}(k|k) + B\mathbf{u}(k|k) + \mathbf{w}(k|k) \quad (6)$$

while the output equation is of the form

$$\mathbf{y}(k|k) = C\mathbf{x}(k|k) \quad (7)$$

where $\mathbf{x}(k|k) = \mathbf{x}(k)$, $\mathbf{x}(k)$ is the state variable at time k, $\mathbf{u}(k)$ is the input system at time k, $\mathbf{y}(k)$ is the output system at time k, A is the discrete form of A_c, B is the discrete form of B_c, C is the output matrix, $\mathbf{x}(k+1|k)$ is the predicted state variable at time $k+1$.

The steps in designing a control using MPC is given bellow.

1. Determine the input increment using the following equation [5], [13]:

$$\Delta\mathbf{u}(k) = \mathbf{u}(k) - \mathbf{u}(k-1) \quad (8)$$

In MPC, there are some design parameters, including prediction horizon $\boldsymbol{N_p}$ and control horizon $\boldsymbol{N_c}$. In this work, it is assumed that $\boldsymbol{N_p} = \boldsymbol{N_c}$.

$$\mathbf{x}(k+1|k) = A\mathbf{x}(k|k) + B\mathbf{u}(k|k)$$

$$= A\mathbf{x}(k|k) + B\mathbf{u}(k-1|k) + B\Delta\mathbf{u}(k|k)$$

$$\mathbf{x}(k+2|k) = A\mathbf{x}(k+1|k) + B\mathbf{u}(k+1|k)$$

$$= A\mathbf{x}(k+1|k) + B\mathbf{u}(k-1|k) +$$

$$B\Delta\mathbf{u}(k|k) + B\Delta\mathbf{u}(k+1|k) \quad (9)$$

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$$\mathbf{x}(k+N_p|k) = A\mathbf{x}(k+N_p-1|k) + B\mathbf{u}(k-1|k) +$$

$$B\Delta\mathbf{u}(k|k) + \cdots + B\Delta\mathbf{u}(k+N_p-1|k)$$

In addition, output prediction can be written into the

following form:

$$\mathbf{y}(k|k) = C\mathbf{x}(k|k)$$

$$\mathbf{y}(k+1|k) = C\mathbf{x}(k+1|k)$$

$$\vdots$$

$$\mathbf{y}(k+N_p|k) = C\mathbf{x}(k+N_p|k) \qquad (10)$$

2. Formulation of Objective Functions

The general form of objective function is given by

$$\boldsymbol{J} = \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{N_p}} \|\boldsymbol{y_d}(k+i) - \boldsymbol{y}(k+i)\|^{\boldsymbol{2}}_{\boldsymbol{Q_i}} + \sum_{\boldsymbol{i}=\boldsymbol{0}}^{\boldsymbol{N_c-1}} \|\boldsymbol{\Delta u}(k+i)\|^{\boldsymbol{2}}_{\boldsymbol{R_i}}$$

(11)

where

$$\|\boldsymbol{y_d}(k+i) - \boldsymbol{y}(k+i)\|^{2}_{\boldsymbol{Q_i}} = \left(\boldsymbol{y_d}(k+i) - \boldsymbol{y}(k+i)\right)^{\boldsymbol{T}} \boldsymbol{Q_i}\left(\boldsymbol{y_d}(k+i) - \boldsymbol{y}(k+i)\right)$$

$$\|\boldsymbol{\Delta u}(k+i)\|^{\boldsymbol{2}}_{\boldsymbol{R_i}} = \boldsymbol{\Delta u}(k+i)^{\mathrm{T}} R_i \boldsymbol{\Delta u}(k+i)$$

By using matrix notation, the objective function can be written as

$$\boldsymbol{J} = (Y_d - Y)^T Q (Y_d - Y) + (\Delta U)^T R\, \Delta U \qquad (12)$$

where

$$Y_d = \begin{bmatrix} \boldsymbol{y_d}(k+1|k) \\ \boldsymbol{y_d}(k+2|k) \\ \boldsymbol{y_d}(k+3|k) \\ \vdots \\ \boldsymbol{y_d}(k+N_p|k) \end{bmatrix}, \ \Delta U = \begin{bmatrix} \Delta U(k+1|k) \\ \Delta U(k+2|k) \\ \Delta U(k+3|k) \\ \vdots \\ \Delta U(k+N_p|k) \end{bmatrix}$$

3. Formulation of constraints on input increment

The input increment constraint is given as follows:

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$$\Delta U^{min} \leq \Delta U \leq \Delta U^{max} \tag{13}$$

From (13), the equation can be defined as follow:

$M_1 \Delta U \leq N_1$,

where $M_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $N_1 = \begin{bmatrix} -\Delta U^{min} \\ \Delta U^{max} \end{bmatrix}$

4. Constraint formulation on control variables

$$U = C_1 U(k-1|k) + C_2 \Delta U \tag{14}$$

where $C_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{N_p \times 1}$, $C_1 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix}_{N_p \times N_p}$

The control variable constraints are given as follows

$$U^{min} \leq U \leq U^{max} \tag{15}$$

By substituting Eq. (14) to (15), then

$M_2 \Delta U \leq N_2$,

where $M_1 = \begin{bmatrix} -C_2 \\ C_2 \end{bmatrix}$ and $N_1 = \begin{bmatrix} -\Delta U^{min} + C_1 U(k-1) \\ \Delta U^{max} - C_1 U(k-1) \end{bmatrix}$

5. Formulation of constraints on state variables

Based on specification of the ship, the ship has yaw rate constraints, i.e. the maximum yaw rate is r^{max} and the minimum yaw rate is r^{min}. The state constraint can be rewritten in the following form:

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_{1j} \\ x_{2j} \\ x_{3j} \end{bmatrix} \leq \begin{bmatrix} r^{min} \\ r^{max} \end{bmatrix} \tag{16}$$

By assuming $D = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $x_j = \begin{bmatrix} x_{1j} \\ x_{2j} \\ x_{3j} \end{bmatrix}$ and $E_1 = \begin{bmatrix} r^{min} \\ r^{max} \end{bmatrix}$, then Eq. (16) can be rewritten

$$D\, x_j \leq E_1 \tag{17}$$

where $j = 1,2,3, \cdots, N_p$.

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3 Sımulatıon Results

This section contains the simulation of closed-loop system. In particular, the influence of a missile launcher (turret) angle (α_M) and the influence of prediction horizon value (N_p) to the system's performance are observed.

Table 1. Comparison of RMSE with various N_p

Case	α_M	N_p	RMSE (10^{-22})
1	15^0	15	5.61760477767938
		17	5.61758908261499
		20	5.61757576043805
		25	5.61757177697635
2	30^0	15	5.61714084117546
		17	5.61712515021549
		20	5.61711182952431
		25	5.61710784644527
3	45^0	15	5.61986836461624
		17	5.61985264952543
		20	5.61983932009882
		25	5.61983533476869

A. Effect of turret angle (α_M)

In this simulation, the various turret angle values are considered, i.e. $\alpha_M = \{15^0, 30^0, 45^0\}$. The value of Q and R is made fixed i.e. $Q = 100$ and $R = 1$. The comparison of the ship's heading angle due to some different values of impact force can be seen in Fig. 3.

According to the simulation the greatest impact force is produced by turret angle $\alpha_M = 45^0$.

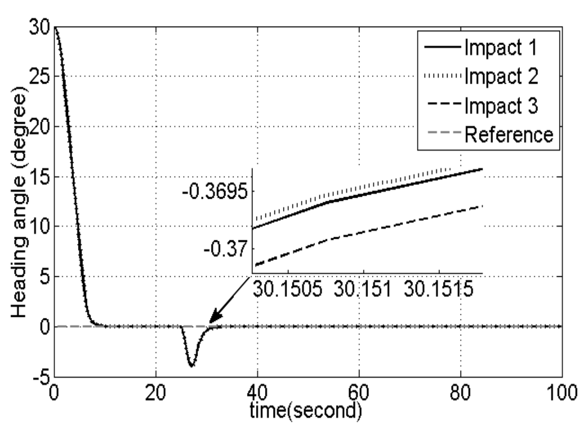

Fig. 3. Heading angle of the ship with various of impact force

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B. The Influence of Prediction Horizon Value (Np) in MPC

This simulation observes the effect of changing the prediction horizon values. The following prediction horizon values is used $\boldsymbol{N_p} = \{15, 17, 20, 25\}$. The weighting matrices Q and R are fixed i.e. Q = 100 and R = 1. The simulation results of heading angle due to some different the prediction horizon value are shown in Fig. 4. The comparison of the RMSE output by varying the prediction horizon value ($\boldsymbol{N_p}$) can be seen in Table I.

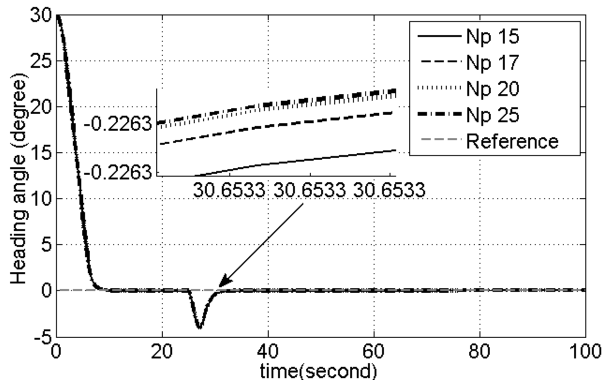

Fig. 4. Heading angle with various of prediction horizon N_p

Simulation results show that when the value of prediction horizon ($\boldsymbol{N_p}$) is higher, then the ship goes faster to the set point (or reference point).

4 Conclusıons

Based on the simulation results presented in this paper, firstly the MPC method can be applied to control the heading angle of a warship when firing missile. This can be seen from the simulation of the yaw angle control that is approaching the reference angle after firing the missile.

Secondly the simulation results show that by using weight coefficient *Q = 100, R = 1*, and influenced by the greatest impact force ($\alpha_M = 45^0$), according to the RMSE, the prediction horizon $\boldsymbol{N_p} = 25$ produces better predictions than values $\boldsymbol{N_p} = 15$, $\boldsymbol{N_p} = 17$ and $\boldsymbol{N_p} = 20$.

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