Software development for solving multi-stage tasks using dynamic programming with separable breakdown

Fayzullaev Ubaydulla Sagdullaevich[1]
[1] *Tashkent state technical university, 100095, 2 str. University, Tashkent*

E-mail: ubaydulla.fayzullaev@tdtu.uz, ubay86@mail.ru

Abstract

This paper discusses how to solve multi-step processes in the form of a matrix and solve problems through dynamic programming. Typically, in practice, the problem of constructing models of multi-stage processes is, as a rule, complicated by the versatility, uncertainty and nonlinearity of the modeled objects, complete or partial lack of expert experience and analytical description of the dependencies. The transition from one state to another is manifested as a function, and when the study is carried out at the analytical level, additional conditions are required, such as the production of the first, sometimes the second, function. If we express the state of multi-step procedures in the form of a matrix, the problem can be solved using dynamic programming

Keywords: *matrix, dynamic programming, multi-stage, transition.*

1. Introduction

When solving a specific problem of optimization, the researcher must first choose a mathematical method that leads to the final result with the lowest computational value or allows the most information about the desired solution. The choice of a particular method is largely determined by the formulation of the optimal problem, as well as by the mathematical model of the optimization object used.
At present, the following methods are used to solve optimal problems:
• Methods of studying the functions of classical analysis;
• Methods based on the use of unknown Lagrange multipliers;
• Accounting for differences;
• Dynamic programming;
• Maximum principles;
• Linear programming;
• Nonlinear programming
Dynamic programming is very suitable for solving multi-stage process optimization problems, especially in cases where each stage state is characterized by a relatively small number of state variables. However, in cases where a large amount of these variables is present, i.e. with a high size of each phase, it is difficult to apply the dynamic programming method due to the limited speed and memory capacity of the computers [1].

Usually, in practice, the problem of constructing models of multi-stage processes, as a rule, is complicated by the multidimensionality, uncertainty and non-linearity of the modeled objects and their characteristics, the complete or partial lack of expert experience and an analytical description of the dependencies [2 3].

To optimize production management, a number of statistical economic and mathematical models have been developed that adequately describe the capabilities of the enterprise, based on linear, dynamic programming methods, etc. The criteria for optimality are the maximum profit, minimum expenses, maximum production of goods by production.

2. Methodology

The manufactured product depends on the types of raw costs (raw, electric energy, salary, depreciation of a building, the number of machine tools, etc.) that are identical to the cost of the product, then you can imagine the product as a functional ($P\ (t)$) from raw costs ($C\ (t)$), in a certain period of time $0\ < t\ < t_0$ i.e.

$$P(t) = P\big(C1(t), C2(t), ..Ck(t)\big)$$
$$Ck(t)\ Є\ C(t) \qquad (1)$$

where: k-the number of types of raw-expenses.
The purpose of production management is to choose such a management in which the raw-cost was minimal i.e. to find

$$\underset{C\ ki\ (t)Є\ C(t)}{Min\ P(t)} = \underset{C\ ki\ (t)\ Є\ C(t)}{\min} P(\int_0^{t0} C1i(t)dt\ , \int_0^{t0} C2i(t)dt\ , ...$$
$$... \int_0^{t0} Cki(t)dt) \qquad (2)$$

ki- vector of the optimal set $C\ ki\ (t)$ of raw costs during a time
$0 \leq\ t\ \leq\ \ t_0$

The production process - an ordered, regular series of interrelated state multistep procedures, which transforms inputs resources into final products. The transition from one state to another is represented as a function and when the study is carried out at the analytical level, additional conditions are required such as the existence of the first, sometimes second, production of the function. If we represent the state of multi-stage procedures in the form of a matrix, then the problem can be solved using the dynamic programming method and to develop software in a certain algorithmic language. Let the state matrix $S\ (M\ \times\ N)$ be defined, the elements of which represent the costs of the transition from one state to another, then the goal is to choose a path-route, from the starting point ($S\ (0,0)$) to the final ($S(M_{fin}\ , N_{fin})$) with a minimum expense. Those

$$S(0,0)\ \ \Rightarrow_{min}\ \ S(M_{fin}\ , N_{fin}\ ) \qquad (3)$$

And so the task is carried out to the task of dynamic programming. To solve the problems, it is necessary to create transition matrices for horizontal and vertical states.
The transition from one state to another requires the creation of at least two matrices:
-matrix of raw-expenses of transition by horizontal states ($g\ (x1\ \times\ y1)$);
- matrix of raw-expenses of transition by vertical states ($v\ (x2\ \times\ y2)$);
Let the state transition of the matrices in the horizontal and vertical directions be carried out through Δg and Δv steps and we assume $\Delta g\ =\ \Delta v\ =\ 1$, which allows us to preserve the principle of separability. The state level is determined by those states that lie in a horizontal line ($y\ =\ N\Delta g$), or in a vertical line ($x\ =\ M\Delta v$). Research has shown that matrix sizes are interconnected and can be represented as:

$$g\ (y\ \times\ x);\ \ \ v\ (x\ \times\ y)$$

where: $x Є[0;\ M];\ \ \ yЄ[0;\ N-1]$.
A manufacturing product is characterized by at least paired indicators such as: quality and quantity, volume and concentration, height and speed, etc. Then it can be assumed that the coordinates of the state $S\ (x, y)$ depend on the raw-expenses- (τ), and is represented in a parametric form

$$S(x, y) = \begin{cases} x = x(\tau) & \text{for fixed y} \\ y = y(\tau) & \text{for fixed x} \end{cases}$$

In the above case, the functions $x = x(\tau)$ and $y = y(\tau)$ were considered as a linear function of τ and the state transition was determined by numerical values in the form of elements of the matrix $G(x(\tau) \times y(\tau)), V(y(\tau) \times x(\tau))$.
Let the functions $x = x(\tau)$ and $y = y(\tau)$ be nonlinear and continuous in the time interval $0 \le t \le t_0$ and have the first derivative $(dx(\tau)/d\tau)$ and $(dy(\tau)/d\tau)$. Then the state level is determined by the transition from one state to another with a flow rate of horizontal and vertical states equal to the arc length of the curve in the considered interval, expressed by the formula, respectively, i.e.

$$g[yi, yi + \Delta g] = \int_{i}^{i+\Delta g} \sqrt{1 + \left(\frac{dy(\tau)}{d\tau}\right)^2}\, d\tau$$

where: $g[yi, yi + \Delta g]$ elements of matrix $g(y \times x)$; $i = [0; n-1]$.

$$v[xj, xj + \Delta v] = \int_{j}^{j+\Delta v} \sqrt{1 + \left(\frac{dx(\tau)}{d\tau}\right)^2}\, d\tau$$

where: - $v[xj, xj + \Delta v]$ elements of matrix $v(x \times y)$; $j = [0; m]$;
According to the geometric meaning, the length cannot be negative, and this is certainly guaranteed not by the negativity of the integrand $\sqrt{1 + \left(\frac{dx(\tau)}{d\tau}\right)^2} \ge 0$ and $\sqrt{1 + \left(\frac{dy(\tau)}{d\tau}\right)^2} \ge 0$ (under the condition $0 \le t$). Thus, there are no additional conditions regarding how and where to pass the function graph (above the axis, below the axis, etc.) [6].

3. Result and Discussion

It is known that the solution of problems of multi-stage processes by the method of dynamic programming consists in choosing a route minimizing the total cost of the transition from the initial state - $S(0,0)$ to the final state - $S(M_{fin}, N_{fin})$ i.e. global governance choices [3]. Route selection begins with the final state $S(M_{fin}, N_{fin})$ and the optimal path to the near state is determined based on the established criteria. There are four paths:
- put1 - horizontal transition from $S(M_{fin}, N_{fin})$ to $S(M_{fin}, N_{fin} - 1)$- horizontal transition;;
- put21 - diagonal transition from $S(M_{fin}, N_{fin})$ to $S(M_{fin} - 1, N_{fin} - 1)$first horizontally and then vertically;
- put22 - diagonal transition from $S(M_{fin}, N_{fin})$ to $S(M_{fin} - 1, N_{fin} - 1)$- first on a vertical level, and then on a horizontal;
- put3 - horizontal transition from $S(M_{fin}, N_{fin})$ to $S(M_{fin} - 1, N_{fin})$- vertical transition.

Each selected path corresponds to corresponding costs:
- for put1= $g[y_{fin} - 1, x_{fin}]$;
- for put21= $g[y_{fin} - 1, x_{fin}] + v[x_{fin} - 1, y_{fin}]$;
- for put22= $v[x_{fin}, y_{fin}] + g[y_{fin} - 1, x_{fin}]$;
- for put3= $v[x_{fin}, y_{fin}]$;

The choice of a route on the basis of the proposed paths, which is called conditional optimal control [5], may lead to the error of the choice of control, since such a control may not correspond to the global optimal control. To select a route satisfying global optimal control, it is necessary to determine the strategy of the penultimate control i.e. determine condition with related costs:
$S(M_{fin} - 2, N_{fin}) = g[y_{fin}, x_{fin}] + g[y_{fin}, x_{fin} - 1]$,

ISSN: 2005-4238 IJAST

$S(M_{fin}-2\,,\,N_{fin}-1)_g \;=\; g\,[y_{fin},\,x_{fin}]+\,v[x_{fin}\;-1,y_{fin}]$, first horizontally and then vertically level;
$S(M_{fin}-2\,,\,N_{fin}-1)_v \;=\; v\,[y_{fin},\,x_{fin}\,]+\,g\,[y_{fin}-1,\,x_{fin}]$ first on a vertical level, and then on a horizontal;

$$S\big(M_{fin},\,N_{fin}-2\big)=v[x_{fin},y_{fin}]+\,v\,[x_{fin},y_{fin}-1]$$

Based on the costs, we choose the strategy with the lowest costs, which allows you to choose the sequence of paths (put1, put21, put22, put3). After choosing a strategy, the next steps in determining the paths of a multi-stage process are carried out on the basis of the criterions for conditional optimal control.

It is necessary to pay attention to the boundary $S\,(M\;=\;0,N)$, $S\,(M,N\;=\;0)$, $S\,(M\;=\;1,N)$, and $S\,(M,N\;=\;1)$ states. In these states, the choice of strategy cannot be determined, since the strategic states will leave the area under consideration i.e. for the matrices. Therefore, in these cases, the choice of global optimal control is represented as:
- for $S(M=0,N)$ sum of elements of a horizontal matrix -$\sum_i^N$ g [0, x_i];
- for $S(M,N\;=0)$ sum of elements of a vertical matrix -$\sum_i^M$ v [y_i, 0];
- for $S(M=1,N)$ and $S(M,N=1)$ the choice of global optimal control is identical to the choice of conditional optimal control.

Based on the above proposed method, an algorithm and software were developed in the C ++ algorithmic language, and the source data given in [4] were taken as a control example: in this case, $M\;=\;8$, $N\;=\;6$, then the state matrix has the form (for data input memory required in the size of $(M\;\times\;N\;+\;1)$ to the horizontal and $(M\;\times\;N-1)$ to the vertical matrix):

Horizontal states

$g\;(7\times8)=$

12	11	10	9	13	14	17	20
11	9	8	7	9	13	14	18
10	8	8	8	10	12	13	15
12	13	12	10	11	13	15	20
14	13	12	10	9	8	11	15
10	14	13	10	11	13	14	17
20	18	16	15	14	12	15	17

Vertical states

$v\;(9\;\times6)=$

11	10	8	7	9	10
9	8	7	6	8	12
10	10	8	9	9	13
12	11	9	7	10	13
13	10	9	8	11	14

14	11	10	10	12	15
14	12	11	11	13	14
12	10	8	10	12	13
10	9	7	9	12	14

The research showed that the result of the program to achieve the goal $S\ (8, N\ =\ 6)$ is equal to 140 units of expenses, which differs from that defined in [4] from 139 units of expenses. A printout of the result of a software solution is shown in Figure 1.

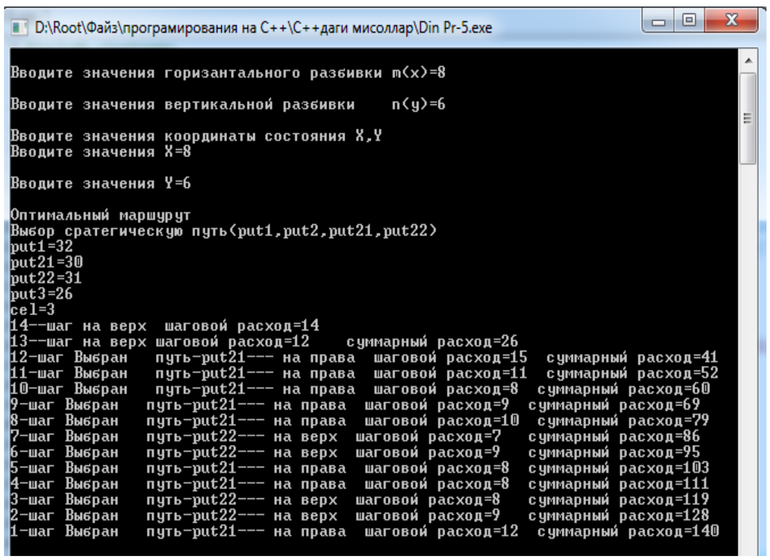

FIGURE. 1. Results

Comparisons of the global optimal control of the proposed program with the global optimal controls proposed in [4] revealed that in step 8 an error was made when choosing a conditional optimal control i.e. put21 was selected to put22.

Conclusion

This article is the first step in solving a multi-step problem. The developed program allows to solve dynamic programming problems using the created auxiliary (horizontal and vertical) matrices.

References

[1] A.G. Trifonov. Statement of the optimization problem and numerical methods for its solution. https://hub.exponenta.ru/post/postanovka-zadachi-optimizatsii -i-chislennye-metody-ee-resheniya356

[2] V.V. Gerasimov, E.K. Kornoushenko. *Diagnosis of dynamic systems defined by structural schemes with non-linear and non-stationary elements*. Automation and telemechanics. - 1990. - No. 4. - S. 133-143.

[3] N.R. Yusupbekov, Kh.Z. Igamberdiev, Sh.M. Gulyamov. *Synthesis of a fuzzy Controller under conditions of uncertainty of the initial information* // Chemical Technology. Control and Management, No. 6 (2007) 44-47.

[4] E.S. Wentzel. *Elements of dynamic programming* // M., Nauka, 1964, p 176.
[5] R. Bellman. *Dynamic programming*. IL, M., 1960.
[6] http://mathprofi.ru