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External Equitable Domination in Graphs

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Abstract

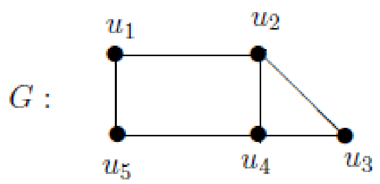
This paper aims at the study of a new concept called external equitability. Properties like independence, excellence, domination etc are coupled with external equitability and a detailed study is made.

1. Introduction

The concept of equitability has been studied in external equitable domination. Prof. E. Sampathkumar introduced degree equitability in graphs. A subset S of the vertex set of a graph is said to be degree equitable if the degrees of any two vertices of S differ by at most one. Arumugam et. al. [2] studied degree equitable sets and degree equtable proper coloring of vertices of a graph. K.M.Dharmalingam defined degree equitability and out degree equitability studied dominating sets which are (i) degree equitable and (ii) out degree equitable. A subset S of the vertex set V of a graph G is said to be externally equitable, if for any $x, y \in V - S$, $||N(x) \cap S| - |N(y) \cap S|| \leq 1$. Independence, Domination are studied with respect to this new concept.

Definition 1.1 *Let $G = (V, E)$ be a simple graph. A subset D of V is called externally equitable dominating set of G if D is a dominating set of G and for every $v_1, v_2 \in V - D$, $\|N(v_1) \cap D| - |N(v_2) \cap D\| \leq 1$. An externally equitable dominating set is also called complementary equitable dominating set. Since V is an externally equitable dominating set of G, the existence of externally equitable dominating set is guaranteed in any graph. The minimum (maximum) cardinality of a minimal externally equitable dominating set of G is called externally equitable domination number (upper externally equitable domination number) of G and is denoted by $\gamma^{ee}(G)$ $(\Gamma^{ee}(G))$.*

Example 1.2



$D = \{u_2, u_4\}$ is a externally equitable dominating set. It can be checked that

$\gamma^{ee}(G) = 2$.

Proposition 1.3

1. $\gamma^{ee}(K_n) = 1$.
2. $\gamma^{ee}(K_{1,n}) = 1$.
3. $\gamma^{ee}(W_n) = 1$.
4. $\gamma^{ee}(P_n) = \left\lceil \frac{n}{3} \right\rceil = \gamma(P_n)$.
5. $\gamma^{ee}(C_n) = \left\lceil \frac{n}{3} \right\rceil = \gamma(C_n)$.
6. $\gamma^{ee}(D_{r,s}) = 2$

Proposition 1.4 $\gamma^{ee}(P) = 4$, *where* P *is the Petersen graph.*

Proof :

The minimum dominating set of the Petersen graph are $S_1 = \{u_1, u_3, u_7\}$, $S_2 = \{u_1, u_4, u_{10}\}$, $S_3 = \{u_2, u_4, u_8\}$, $S_4 = \{u_2, u_5, u_6\}$, $S_5 = \{u_3, u_5, u_9\}$,$S_6 = \{u_6, u_7, u_4\}$, $S_7 = \{u_6, u_{10}, u_3\}$,$S_8 = \{u_7, u_8, u_5\}$, $S_9 = \{u_9, u_8, u_1\}$, $S_8 = \{u_9, u_{10}, u_2\}$. None of these sets are externally equitable. Therefore $\gamma^{ee}(P) \geq 4$. Since $\{u_1, u_6, u_8, u_9\}$ is an externally equitable dominating set of P, $\gamma^{ee}(P) \leq 4$. Therefore $\gamma^{ee}(P) = 4$.■

Proposition 1.5 *Let* $H_{m,n}$ *be the Harary graph with* $m > 2r$ *and* $n > m$. *Then* $\gamma^{ee}(H_{m,n}) = \lceil \frac{n}{2r+1} \rceil$

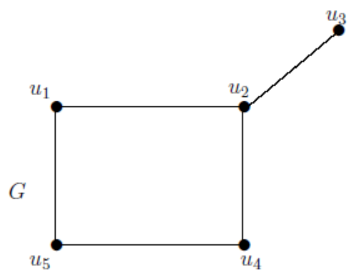
Proof :

Let $m = 2r$, $n > m$.

Let $t = \lceil \frac{n}{2r+1} \rceil$. Let $V(H_{m,n}) = \{u_1, u_2, \dots, u_n\}$. Let $D = \{u_1, u_{2r+2}, \dots, u_{t(2r+1)+1}\}$. It can be easily checked that D is a externally equitable dominating set of $H_{2r,n}$. Therefore $\gamma^{ee}(H_{m,n}) \leq t$. But $\gamma(H_{2r,n}) = t$ and $\gamma(H_{2r,n}) \leq \gamma^{ee}(H_{2r,n})$. Therefore $\gamma^{ee}(H_{2r,n}) = t = \lceil \frac{n}{2r+1} \rceil$.■

Remark 1.6 *Externally equitable dominating property is not* super hereditary. That is, a super set of an externally equitable dominating set need not be an externally equitable dominating set, even though it is a dominating set.

Example 1.7



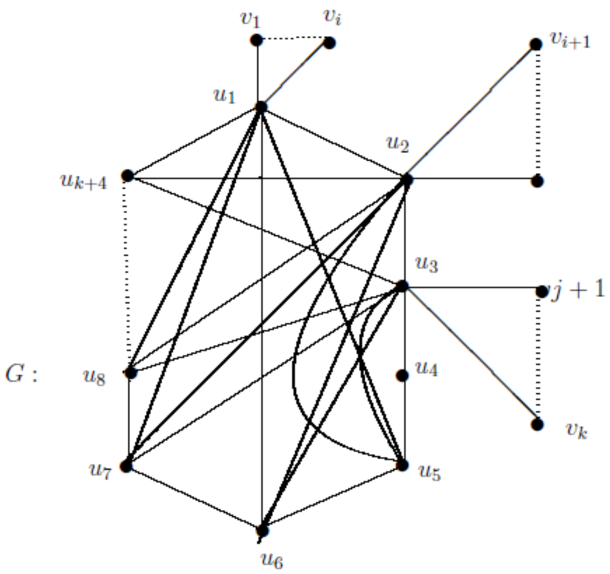
$\{u_1, u_2\}$ is a equitable dominating set of G. But $\{u_1, u_2, u_5\}$ is not a equitable dominating set of G since $|N(u_3) \cap \{u_1, u_2, u_5\}| = 1$. $|N(u_4) \cap \{u_1, u_2, u_5\}| = 3$.

Remark 1.8 *If G has a full degree vertex or $deg(u) \leq 2$, for every $u \in V(G)$, then $\gamma^{ee}(G) = \gamma(G)$. Further, if $\gamma(G) \leq 2$, then $\gamma^{ee}(G) = \gamma(G)$. Also, $\gamma(G) \leq \gamma^{ee}(G)$.*

Remark 1.9 *There exists regular graphs G such that $\gamma(G) < \gamma^{ee}(G)$.*
For: consider the Petersen graph P which is 3 regular. $\gamma(P) = 3$ and $\gamma^{ee}(P) = 4$.■

Proposition 1.10 *Given a positive integer k , there exist a graph G with $|\gamma^{ee}(G) - \gamma(G)| = k$.*

Proof :



Consider C_{k+4}. Let $V(C_{k+4}) = \{u_1, u_2, \ldots, u_{k+4}\}$. Join u_i, $5 \leq i \leq k+4$ with u_1, u_2 and u_3. Add vertices $v_1, v_2, v_3, \ldots, v_k$ such that each of them is a

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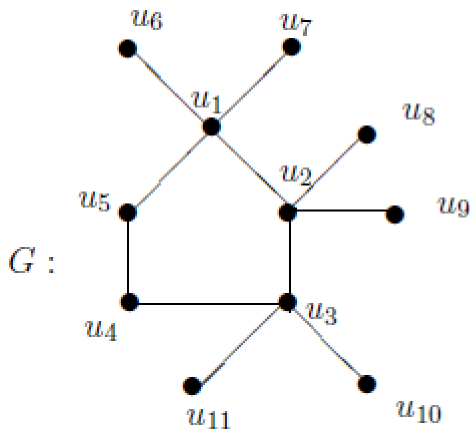
pendant vertex and each is adjacent exactly to one of u_1, u_2 and u_3. Let G be the resulting graph . Clearly $\{u_1, u_2, u_3\}$ is a minimum dominating set of G and $\{u_1, u_2, u_3, u_5, u_6, \ldots, u_{k+4}\}$ and $\{u_1, u_2, u_3, u_4, v_1, v_2, \ldots, v_k\}$ are externally equitable dominating sets of G. Hence $\gamma(G) = 3$ and $\gamma^{ee}(G) = k + 3$. ■

Illustration 1.11



It can be easily checked that $\gamma(G) = 3$ and $\gamma^{ee}(G) = 4$. Also, $D = \{u_1, u_2, u_3\}$ and $D_1 = \{u_1, u_2, u_3, u_5\}$ are $\gamma -$ set and $\gamma^{ee} -$ set respectively.

Theorem 1.12 *An externally equitable dominating set D of $V(G)$ is $1 -$ minimal if and only if the following conditions are satisfied for any $u \in D$,*

(i) u is an isolate of G.

(ii) u has a private neighbour in $V - D$ with respect to D.

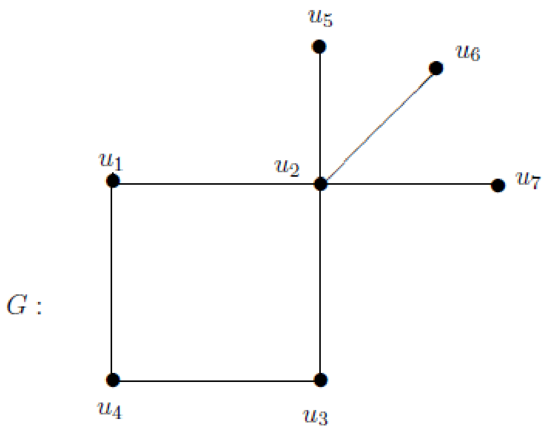
(iii)Let $U = \{x, y \in V - D : |N(x) \cap D| > |N(y) \cap D|$ and u is adjacent with y but not with $x\}$. Then $U \neq \phi$.

Proof :

Let D be a minimal externally equitable dominating set of G. If $u \in D$ satisfies $(i) or (ii) or (iii)$ we are through. Suppose none of the conditions is satisfied. Therefore $U = \phi$. Therefore those vertices $x, y \in V - D$ with $|N(x) \cap D| - |N(y) \cap D| = 1$ will be such that either u is adjacent with both x and y or u is not adjacent with any of x, y or u is not adjacent with y but adjacent with x. Therefore $D - \{u\}$ is an externally equitable dominating set of G, a contradiction . Therefore either $(i) or (ii) or (iii)$ is satisfied for any $u \in D$. Suppose an externally equitable dominating set D satisfies the hypothesis of the theorem. Let $u \in D$. If u satisfies (i) or (ii) then $D - \{u\}$ is not even a dominating set. Suppose u satisfies (iii). Then $D - \{u\}$ is not externally equitable since $|N(x) \cap (D - \{u\})| - |N(y) \cap (D - \{u\})| \geq 2$. Therefore D is one minimal.■

Remark 1.13 *The complement of a minimal externally equitable dominating set* *need not be an externally equitable dominating set.*

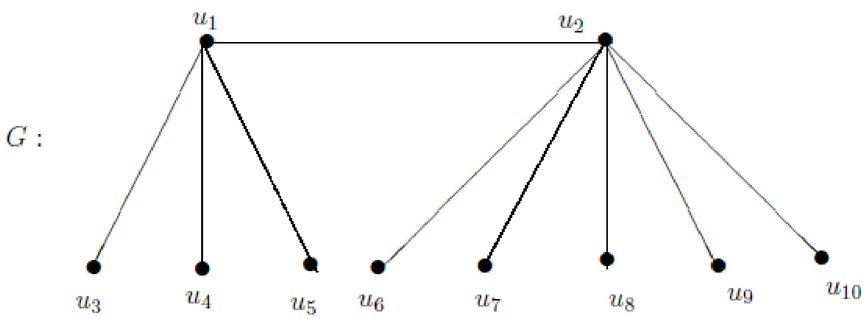
For example, let



$D = \{u_2, u_4\}$ is a minimum externally equitable dominating set of G.

$V - D = \{u_1, u_3, u_5, u_6, u_7\}$. This is a dominating set but not externally equitable.

Remark 1.14 *A maximal β_0^{ee}- set of G need not be a minimal externally equitable dominating set of G. For: let*



$S = \{u_3, u_6\}$ is a $\beta_0^{ee} -$ set of G but S is not even a dominating set of G.

Remark 1.15 *A maximal externally equitable independent set of G which is also dominating is a minimal externally equitable dominating set of G.*

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