International Journal of Advanced Science and Technology
Vol. 28, No. 16, (2019), pp. 1012 - 1020

Solving Fuzzy Multi-Objective Linear Programming Problems with Non Linear Membership Function Using Artificial Neural Networks

Khaled Elsharkawy
October High Institute for Engineering & Technology-
Dept. of Basic science,
6th of October, Egypt.
khaled.elsharkawy@ohins.edu.eg

Abstract

Fuzzy multi-objective linear programming problems (FMLOPP) have many applications in the field of science and engineering. In this paper, (FMOLPP) we studied the Cost factors to be fuzzy with non- linear membership function, then we compare the time of solution with the algebraic approach which proposed to solve the above problem using the technique proposed and between the solution using trained feed forward artificial neural networks (FFANN).

Keywords: *Multi-Objective Linear Programming, Fuzzy Multi-Objective Linear Programming Problems, Feed Forward Artificial Neural Networks, Decision Making, Multi-Objective Mixed Non-Linear Programming Problem.*

1. Introduction

Among mathematical programming models, linear models are particularly interesting from a practical point of view. Since effective solution techniques for these models are well developed they are widely applied in practice. One should also expect a similar situation in the rapidly developing fuzzy programming based on the concept of Bellman and Zadeh [1] for decision making under fuzzy conditions. Fuzzy linear models can also be a great importance in practice. Linear programming problems with fuzzy variables are of almost in many fields as those related in artificial intelligence, operation research and management.

There are many literatures and papers that deal with the solution of (FMOLPP). Among the many algorithms suggested for these problems are those due to [2 - 6]. In [2] Sakawa and Yono presented an interactive decision making method for multi-objective nonlinear programming problems with fuzzy parameters. These fuzzy parameters were in the objective functions and constraints and have been characterized by fuzzy numbers where the concept of α- Pareto optimality has been introduced.

Stochastic uncertainty relates to the uncertainty of occurrences of phenomena or events. Its characteristics, lie in that descriptions of information are crisp and well defined; however, they vary in their frequency of occurrence. The systems with this type of uncertainty are called stochastic systems, which can be solved by stochastic optimization techniques using probability theory.

In some other situations, the decision-maker does not think about the frequently used probability distribution which is always appropriate, especially when the information is vague. It may be related to human language and behavior, imprecise/ ambiguous system data. Such types of uncertainty are called fuzziness. It cannot be formulated and solved effectively by traditional mathematics-based optimization techniques and probability based stochastic optimization approaches. The idea of

ISSN: 2005-4238 IJAST

fuzzy set was first proposed by Zadeh (1965), as a mean of handling uncertainty that is due to imprecision rather than to randomness.

In this paper an (FFANN) is trained to solve the (FMOLPP) and comparing the solution with the algebraic solution of weighting sum approach method.

2. Literature Review

In 1970 the concept of fuzzy decision and the decision model under fuzzy environments were proposed by Bellman and Zadeh. Zimmerman (1978) [7] first considered (MOLP) problems with fuzzy goals. Tanaka and Asai (1984) [8] introduced fuzzy linear programming problem in fuzzy environment. There are several methods in the literature for solving multi-objective linear programming models, by adopting fuzzy programming approaches.

In 1989, Sakawa and Yano [3] introduced the concept of multi-objective programming and (M) - Pareto optimality based on the α – level sets of the fuzzy numbers. Sakawa, et al. (1989) presented an interactive decision making method for multi-objective nonlinear programming problems with fuzzy parameters. They presented several interactive decision making methods not only in objective spaces but also in membership spaces to derive the satisfying solution for the decision maker efficiently from an (M-)–Pareto optimal solution set for multi-objective linear, linear fractional and nonlinear programming problems as a generalization of their previous results. Lai-Hawng (1992) [9] considered (MOLP) problem with all parameters, having a triangular possibility distribution. They used an auxiliary model and it was solved by (MOLP) methods. Sakawa et al. (1994) [10] Presented an interactive fuzzy satisfying method for large-scale (FMOLPP) problems with the block angular structure. Saad (1995) [11] suggested a procedure for solving (FMOLPP) problems and some basic stability notions have been characterized for (FMOLPP) problems. Sakawa et al. (1996) [12] focused on large-scale (FMOLPP) problems with the block angular structure. The (FMOLPP) problem has been transformed to its crisp equivalent, using possibility programming. The crisp (MOLP) problems, has been solved using the global criterion method and the distance functions method is proposed by M.G. Iskander (2008) [13].

There have been a number of studies on applications of the (FMOLPP) problems to regional planning issues, such as regional environmental management, water resource management, and agricultural development planning etc. Slowinski (1986,1987) [14,15] proposed an interactive (FMOLPP) method and applied it to water supply planning problems. Kindler (1992) [16] proposed a fuzzy linear programming formulation for water resource planning problems. In agricultural development planning, Czyzak (1989) [17] applied a fuzzy linear programming method for solving multi-criteria agricultural planning problems under uncertainty. Pickens and Hof (1991) [18] applied fuzzy goal programming to forestry management and planning under uncertainty. Recently, David Peidro et al (2009) [19] proposed fuzzy optimization for supply chain planning under supply, demand and process uncertainties. In this paper, we have proposed a (FMOLPP) problem in which technological coefficient and resources are fuzzy. Using Bellman and Zadeh's fuzzy decision-making process, the (FMOLPP) problem is converted into an equivalent crisp non-linear programming problem. The non-linear programming problem is solved by fuzzy decisive set method.

The rest of this paper is organized as follows. Section 3, (MOLP) problem and its solutions are discussed. Fuzzy model of (MOLP) is given in section 4. In section 5, solution methodology and algorithm for (FMOLPP) using α -Pareto optimal are analyzed. In section 6 the example illustrated is solved using trained feed forward artificial neural network (FFANN) and mathematical method. In section 7 the advantages and disadvantages of using artificial neural network is presented giving the fact that the developed method can be successfully applied and conclusion is drawn in Section 8.

3. Multi- Objective Linear Programming (MOLP) Problem

Multi-objective Linear Programming (MOLP) Problems is an interest area of research, since most real-life problems have a set of conflict objectives. A mathematical model of the (MOLP) problem can be written as follows in model (1):

$$Max: \; Z_1(x) = C_1 x$$

$$Max \; Z_2(x) = C_2 x$$

$$Max \; Z_k(x) = C_k x$$

$$Subject\ to \; x \in X = \{x \in R^n / Ax = b, \; x \ge o\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where x, is (n – dimensional) vector of decision variables; $Z_1(x), Z_2(x) ..... Z_k(x)$ are k-distinct linear objective function of the decision vector; $C_1, C_2 ..... C_k$ are (n – dimensional) cost factor vectors : A is m× n constraint matrix, b is m – dimensional constant vector.

Definition 3.1 (Complete Optimal Solution):

The point $x^* \in X$ is said to be a complete optimal solution of the (MOLP) problem (1), if $Z_i(x^*) \ge Z_i(x)$, i=1,2….k for all $x \in X$.

In general, when the objective functions conflict with one another, a complete optimal solution may not exist and hence, a new concept of optimality, called Pareto optimality, is considered.

Definition 3.2 (Pareto Optimal Solution):

The point $x^* \in X$ is said to be a Pareto optimal solution if there does not exist $x \in X$, such that if $Z_i(x) \ge Z_i(x^*)$ for all i and $Z_j(x) > Z_j(x^*)$ for at least one j.

4. Fuzzy Multi- Objective Linear Programming (FMOLPP) Problem

In model (1), all coefficients of A, b and C are crisp numbers. However, in the real-world decision problems, a decision maker does not always know the exact values of the coefficients taking part in the problem, and that vagueness in the coefficients may not be a probabilistic type. In this situation, the decision maker can model inexactness by means of fuzzy parameter. In this section we consider a (FMOLPP) problem with fuzzy technological coefficients and fuzzy resources. A mathematical model of the (FMOLPP) problem can be written as follows in model (2):

ISSN: 2005-4238 IJAST

$$Max: Z_1(x) = C_1^{\approx} x, Z_2(x) = C_2^{\approx} x, Z_k(x) = C_k^{\approx} x$$
$$subject\ to\ x \in X = \{x \in E^n / Ax = b, x \geq 0\} \ldots\ldots\ldots\ldots\ldots(2)$$

Where x is an n – dimensional vector tor of decision variables. $Z_1(x).....Z_k(x)$ are k- distinct linear objective function of the decision vector x, $C_1^{\approx}, C_2^{\approx}, C_k^{\approx}$ are n – dimensional cost factor fuzzy vectors, A is an mxn constraint matrix, b is an m– dimensional constant vector (resources).

The membership function of the fuzzy matrix $C^{\approx}$ *is*

$$\mu_{c^{\approx}}(c) = \begin{cases} 0 & c \leq p_1 \\ 1 - (\frac{c - p_2}{p_1 - p_2})^2 & p_1 \leq c \leq p_2 \\ 1 & p_2 \leq c \leq p_3 \\ 1 - (\frac{c - p_3}{p_4 - p_3})^2 & p_3 \leq c \leq p_4 \\ 0 & c \geq p_4 \end{cases}$$

Where $c \in R$ *and* $p_i \succ 0$ *(tolerance level), for i=1,.....m.*

5- Solution Methodology and Algorithm

5.1 The algorithm of the fuzzy solution set method:

In this section, we describe an algorithm for solving (FMOLPP). This algorithm can be summarized as follows:

1- Set a certain degree $\alpha = \alpha^* \in [0,1]$

2- Determine the points $p = (p_1, p_2, p_3, p_4)$ for each fuzzy number in the formulated problem $(FMOLPP)_C$.

3- Convert $(FMOLPP)_C$ in the form of problem $(\alpha - MOMNLP)_C$, i.e. de-fuzzify the problem.

4- Formulate the non-fuzzy problem corresponding to $(\alpha - MOMNLP)_C$ in order to solve it using weighting sum approach method as shown in (3) or $\in$ - constrained method or any other mathematical method

$(\alpha - MOMNLP)_C$:

Max $Z(x, \lambda^{\approx}) = [Z_1(x, \lambda_1^{\approx}), Z_2(x, \lambda_2^{\approx})]$

Subject to

$$\begin{Bmatrix} x \in X, Z_1(x, \lambda_1^{\approx}) = (1 + \lambda_{11})x_1 + x_2, Z_2(x, \lambda_2^{\approx}) = -x_1 + (1 + \lambda_{22})x_2 \\ L_{11} \leq \lambda_{11} \leq L_{12}, L_{21} \leq \lambda_{22} \leq L_{22}\ where\ L_{ij}\ are\ \lim its\ of\ \alpha - cut \ldots\ldots\ldots\ldots\ldots(3) \end{Bmatrix}$$

Using weighting Sum approach and de-fuzzify the problem (4)

$(\alpha - MOMNLP)_C$:

$$P(w): \begin{array}{l} \max Z(x,\lambda) = [w_1 Z_1(x,\lambda_1) + w_2 Z_2(x,\lambda_2)] \\ x \in X \\ Z_1 = (1+\lambda_{11})x_1 + x_2, Z_2 = (1+\lambda_{22})x_2 - x_1 \\ L_{11} \le \lambda_{11} \le L_{12}, L_{21} \le \lambda_{22} \le L_{22} \; where \; L_{ij} \; are \lim its \; of \; \alpha - cut \end{array}$$

Perform the transformations $y_{11} = \lambda_{11} x_1, y_{11} = \lambda_{11} x_1$

Consequently the problem becomes:

$$\begin{array}{l} P(w): \max \; w_1(y_{11} + x_1 + x_2) + w_2(y_{22} + x_2 - x_1) \\ \textit{Subject to} \\ x \in X \\ L_{11}x_1 \le y_{11} \le L_{12}x_1, L_{21}x_2 \le y_{22} \le L_{22}x_2 \\ x_1, x_2, y_{11}, y_{22} \ge 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4) \end{array}$$

5.2 Brief algorithm to train (FFANN) and weighting sub-objective functions [20]:

Steps:

1- Identify the sub-objectives.
2- Randomly choose s+1 ($s < k$) of sub-objectives and assign rating values from 0 to 1 where k is the total number of objective functions.
3- Let the DM weights the s+1 sub-objectives and assign rating values from 0 to 1.
4- Isolate the presented s+1 alternative from the set of total alternatives.
5- By using the trained (FFANN) function, find the numerical rating and the ranking of these s+1 alternatives .
6- Present the s+1 sub- objectives to the DM with their numerical ratings which were obtained from the (FFANN). If the DM accepts the (FFANN) weighting and rating of these s+1 sub-objectives go to 10.
7- If the DM wishes to change any of his previous and recent responses, incorporate the new responses.
8- Use all of the DM recent and previous responses and the ratings of the normalized functions to train the (FFANN) functions and derive its parameters.
9- Go to step 5 to train the new set of sub-objectives.
10- Stop.

Demonstrating a network, weight matrix (5) as shown in figure (1) and feed forward calculations (6) for this network which we shall give a numerical example with the same given topology using sigmoid function as shown in figure (2) in the process of training and calculations.

$$w = \begin{pmatrix} w_1 & w_3 & 0 & h_1 \\ w_2 & w_4 & 0 & h_2 \\ w_5 & w_6 & 0 & h \end{pmatrix}$$

$$y_1 = \frac{1}{1 + \exp(w_1 x_1 + w_3 x_2 + h_1)}$$

$$y_2 = \frac{1}{1 + \exp(w_2 x_1 + w_4 x_2 + h_2)}$$

$$z_n^* = \frac{1}{1 + \exp(w_5 y_1 + w_6 y_2 + h)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

ISSN: 2005-4238 IJAST

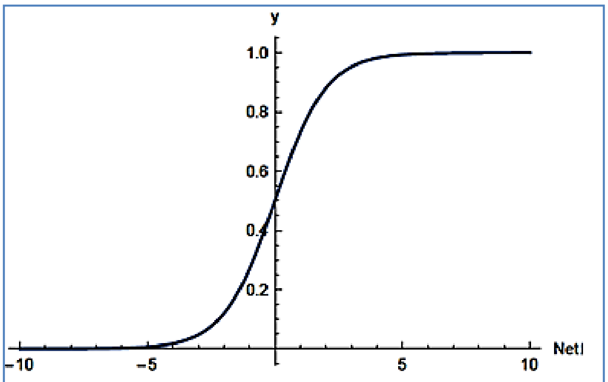

Fig.1: Graph of the function [y=1/1+Exp(- Net)]

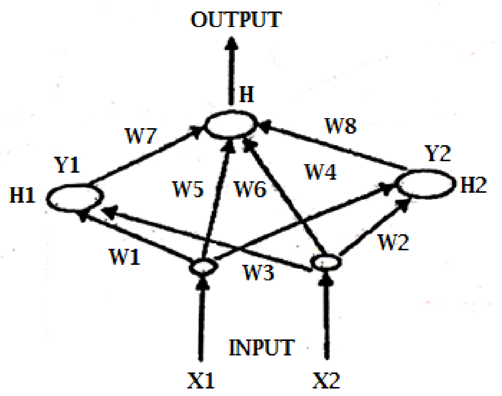

Figure (2): Sigmoid function of neural network training

6- Numerical Example

$(FMOLPP)_C$:

$$\max Z(x,\tilde{\lambda}) = [Z_1(x,\tilde{\lambda}_1), Z_2(x,\tilde{\lambda}_2)]$$

Subject to $x \in X$

$$\{x \in R^2 \,/\, 2x_1 \le 7, 4x_2 \le 9, x_1 \ge 1, x_2 \ge 1\}$$

$$Z_1(x,\tilde{\lambda}_1) = (1+\lambda_{11})x_1 + x_2, Z_2(x,\tilde{\lambda}_2) = -x_1 + (1+\lambda_{22})x_2$$

Let the fuzzy parameters are characterized by the following fuzzy numbers

$$\{\tilde{\lambda}_{11} = (0,1,3,5) \text{ and } \tilde{\lambda}_{22} = (0,1,4,6)\}$$

Assume the membership function in the form given before and let α = 0.36

We get $0.2 \le \lambda_{11} \le 4.6, 0.2 \le \lambda_{22} \le 5.6$

The non-fuzzy $(\alpha - MOMNLP)_C$ **can be written as follows:**

$(\alpha - MOMNLP)_\lambda$:

$$Max\ Z(x,\lambda) = [Z_1(x,\lambda_1), Z_2(x,\lambda_2)]$$

$$Subject\ to \begin{cases} 0.2 \le \lambda_{11} \le 4.6, 0.2 \le \lambda_{22} \le 5.6 \\ x \in X \end{cases}$$

ISSN: 2005-4238 IJAST

Using weighting sum approach method then the problem becomes:

$$P(w): \max Z(x,\lambda\ ) = [w_1 Z_1(x,\lambda_1\ ) + w_2 Z_2(x,\lambda_2\ )]$$
$$x \in X$$
$$Z_1 = (1+\lambda_{11})x_1 + x_2, Z_2 = (1+\lambda_{22})x_2 - x_1$$
$$0.2 \le \lambda_{11} \le 4.6, 0.2 \le \lambda_{22} \le 5.6$$
$$Let\ y_{11} = \lambda_1 x_1, y_{22} = \lambda_2 x_2$$
$$0.2x_1 \le y_{11} \le 4.6x_1, 0.2x_2 \le y_{22} \le 5.6x_2$$
$$x_1, x_2, y_{11}, y_{22} \ge 0.$$

By choosing different values for w_1 *and* w_2 we can solve the problem algebraically also we can give these Values to the (FFANN) to train it then solve the non-fuzzy problem

Note: ($z_n^*(DM)$) is the solution of DM given to the (FFANN) as shown in tables (1) and (2)

Results:

Table (1): Training data of the example example where ($z_n^*(DM)$) is the solution of DM given to the (FFANN)

Alternative	w_1	w_2	$z_n^*(DM)$	Alternative	w_1	w_2	$z_n^*(DM)$
A	0	1	1.000	H	0.4	0.6	0.8077
B	1	0	0.6338	K	0.3	0.7	0.8558
C	0.5	0.5	0.7590	L	0.2	0.8	0.9038
D	0.9	0.1	0.6060	M	0.1	0.9	0.9519
E	0.8	0.2	0.6160	R	0.75	0.25	0.6395
F	0.7	0.3	0.6636	V	0.65	0.35	0.6876
G	0.6	0.4	0.7116	X	0.35	0.65	0.8318

Table (2): Test process and error system of example

Alternative	w_1	w_2	z_{DM} (algebraically)	z_{neural}^*	Algebraic rank	Neural rank	Error $E = (z_n^* - z_{DM})^2$
1	0.15	0.85	0.9279	0.93	1	1	0.00000441
2	0.85	0.15	0.5926	0.609	5	5	0.000268
3	0.67	0.33	0.678	0.6721	4	4	0.00000441
4	0.55	0.45	0.7356	0.722	3	3	0.000184
5	0.45	0.55	0.7837	0.7749	2	2	0.00007744

Results:

R.M.S Error= 0.000086752

Number of iterations=18505

Time =3.5 minutes

Good patterns=100%

Target error=0.001

$$\text{Wight Matrix W=} \begin{bmatrix} 2.401299 & 2.660406 & 0 & 2.487703 \\ 2.10193 & -2.696737 & 0 & -1.403044 \\ -0.497733 & 4.556748 & 0 & 0.872183 \end{bmatrix}$$

$$y_1 = \frac{1}{1+\exp(-2.401299w_1 - 2.660406w_2 - 2.487703)}$$

$$y_2 = \frac{1}{1+\exp(-2.10193w_1 + 2.696737w_2 + 1.403044)}$$

$$z_n^* = \frac{1}{1+\exp(0.497733y_1 - 4.556748y_2 - 0.872183)}$$

7-Advantages and Disadvantages of Using (FFANN)

Disadvantages	Advantages
1- The process of training may take some time due to many factors as bad data, bad learning parameters, bad number of layers. 2- (FFANN) solves the problems of each case separately i.e: the neural function obtained from the training data of any example cannot be used to obtain the result of other example. 3- It may give bad ranking from the beginning.	1- Building up utility function indicates the DM preferences and the efficient solutions. 2- Reducing errors of the DM . 3- Relating the mathematical information with the DM choices using the artificial intelligence while the DM ranges the choices according to his preference. 4- Its benefit appears as the number of objective functions increases and the number of alternative choice increase.

8- Conclusion

In this paper, we introduce linearization procedure to overcome the computational difficulty with non- linearity in the non-fuzzy problems, also we introduce an (FFANN) approach in weighting criteria and its sub-criteria in (FMOLPP) problems. We demonstrated that (FMOLPP) problems could be solved by (FFANN) without having prior information regarding the properties or structure of the DM utility function. The approach proved to be very effective for representing both linear and complex non-linear utility functions while asking few questions of the DM.

References

[1] Bellman R.E., Zadeh L.A.," Decision making in a fuzzy environment", Management Science ,vol. 17,(1970), pp.141-164.

[2] M. Sakawa and H., Yano, "Interactive decision making for multi-objective linear fractional programming problems with fuzzy parameters". Cybernetics Systems, vol.16, (1985), pp.377-394.

[3] M. Sakawa and H., Yano, "Interactive fuzzy satisficing method for multi-objective nonlinear programming problems with fuzzy parameters", Fuzzy Sets and Systems,vol.30,(1989),pp. 221-238.

[4] Qingqing Zhou, Marin Purvis and Nikola Kasabov "A membership function selection method for fuzzy Neural networks", vol.14,(1992), pp.1-6

[5] Ecker J. G, and Hegner N.S, "Generating the efficient set for multiple objective linear programs, Transactions of twenty-fourth conference of Army Mathematical", Aro report 79-1,(1980), pp. 439-449.

ISSN: 2005-4238 IJAST

[6] Ecker J. G, and Hegner N.S and Kouada I.A," Generating all maximal efficient faces for multiple objective linear programs", Journal of optimization Theory and Applications, vol.30, (1980), pp.353-381..

[7] H.J. Zimmermann , "Fuzzy programming and linear programming with several objective functions", Fuzzy sets and System vol. 1,(1978), pp.45- 55.

[8] H. Tanaka and K., Asai, " Fuzzy linear programming problems with fuzzy numbers", Fuzzy Sets and Systems, vol.13, (1984), pp.1-10.

[9] Lai Y.J. and Hwang C.L., "Fuzzy multiple objective decision making", Springer-velag, Berlin Heidelberg (1994).

[10] 10-M. Sakawa and K., Sawada, "An interactive fuzzy satisficing method for large-scale multi-objective linear programming problems with block angular structure", Fuzzy Sets and Systems, vol.67,(1994),pp. 5-17.

[11] O.M. Saad , "Stability on multi-objective linear programming problems with fuzzy parameters", Fuzzy Sets and Systems,vol. 74,(1995),pp. 207-215.

[12] M. Sakawa Inuiguchi, K., Sawada, "A fuzzy satisficing method for large-scale multi-objective linear programming problems with block angular structure", Fuzzy Sets and Systems, vol. 78,(1996),pp. 279- 288.

[13] 13-V. Iskander, "A computational comparison between two evaluation criteria in fuzzy multi-objective linear programs using possibility programming", Computers and Mathematics with Applications,vol.55,(2008),pp. 2506-2511

[14] 14-R. Slowinski, "A multi-criteria fuzzy linear programming method for water supply system development planning", Fuzzy Sets and Systems, vol.19,(1986),pp. 217-237.

[15] 15-R. Slowinski, "An interactive method for multi-objective linear programming with fuzzy parameters and its application to water supply planning". In: Kacprzyk, J. and Orlovski, S.A., Editors (1987).

[16] 16- J. Kindler,Rationalizing, "water requirements with aid of fuzzy allocation model". J.Water Res. Plng. and Mgmt., ASCE,vol. 118,(1992),pp. 308-323

[17] 17-P. Czyza, "Multicriteria agricultural problem solving under uncertainty", Foundations of Control Engineering, vol.14,(1989),pp. 61-80.

[18] 18- J.B. Pickens and Hof J.G.," Fuzzy goal programming in forestry: an application with special solution problems", Fuzzy Sets and Systems,vol. 39,(1991),pp. 239-246.

[19] 19- Peidro Mula Poler R. and José-Luis Verdegay, "Fuzzy optimization for supply chainplanning under supply", demand and process uncertainties Fuzzy Sets and Systems,vol. 160,(2009),pp. 2640-2657.

[20] 20- Benham Malakoti. Ying Q. Zhou , "A feed forward artificial neural network for solving MOLP probulems", Management science, vol.40, No.11,(1994), PP. 1524-1559.