International Journal of Advanced Science and Technology
Vol. 29, No. 8, (2020), pp. 1120-1126

A Decision Model for Location-Routing Problem with Distance Consideration

Joni Mulyasri[1], Opim Salim Sitompul[2], Sutarman[3], Syahril Efendi[2]
[1]Graduate School of Mathematics, Universitas Sumatera Utara, Medan, Indonesia
[2]Faculty of Computer Science and Information Technology, Universitas Sumatera Utara, Medan, Indonesia
[3]Department of Mathematics, Universitas Sumatera Utara, Medan, Indonesia

Abstract

In a logistic delivery system, it is essential to determine optimally the sites of facilities that will give outcome not only the valuable for a company but also the needs to serve customers suitably. Commonly, this location-routing decision problem is to reduce the total operating costs by choosing a subset of candidate facilities and creating a set of delivery routes that meet some conditions. In this paper there would be non-accessed route in the constraint. We develop an integer programming model to illustrate the problem. A better feasible neighbourhood search is tackled for solving the model.
Keywords: Logistic system, Routing problem, Integer programming

1. Introduction

The location-routing problem for a logistic system considered in this paper is to determine where to establish the facilities and how to assign customers to the chosen facilities. The decision can be acquired utilizing a model, which assume that customers are served individually on out-and-back routes. Yet, when customers have demands that are less-than-vehicle capacity and therefore can take service from routes making numerous stops, the assumption of individual routes will not precisely seize the transportation cost. Hence, it is required to integrate location-allocation and routing determinations which could give more precise and profitable solutions.

Location-Routing Problems (LRPs) merge two major planning tasks in logistics, viz., to determine the sites of facilities and the routing of vehicles to visit customers. It is familiar that making these types of choices individually of one another may cause highly suboptimal planning results, even if the location choices must be made for the long term [1]. Thus the location-routing problem (LRP) can be expressed as a mathematical optimization problem where at least the following two types of decisions must be made interdependently; which facilities should be selected and which route should be passed by the vehicles ([2], [3]).
The major goal of the LRP is to reduce the total operations costs with the following constraints:

(i) Customer demands are satisfied without surpassing vehicle or facility capacities.
(ii) The quantity of vehicles, the route lengths and the route durations do not surpass the specified limits and
(iii) Each route begins and ends at the same facility.

In this paper, we consider an LRP with distance and forbidden route constraints.

Forbidden route involving pairs of edges occur frequently ("No left turn") and can happen dynamically caused by rush hour constraints, lane closures, construction, etc. Longer forbidden sub paths are uncommon, but can happen, for instance if heavy traffic makes it difficult to turn left soon after entering a multi-lane roadway from the right. If we are routing a single vehicle it is more natural to find an alternative route from the point of failure when a forbidden path is found.

ISSN: 2005-4238 IJAST
Copyright ⓒ 2020 SERSC

Location-routing problems are obviously connected to both the classic allocation problem and the vehicle routing problem. In fact, both final problems are special cases of the LRP. If we need all customers to be directly related to a depot, the LRP becomes a standard location problem. If, alternatively, we fix the depot locations, the LRP shrinks to a VRP. From a practical perspective, location-routing forms part of *distribution management*, while from a mathematical perspective, it can normally be demonstrated as a *combinatorial optimization* problem. We note that this is an NP-hard problem, as it encompasses two NP-hard problems (facility location and vehicle routing). Since numerous problem versions occur, we cannot reproduce all the formulations here. In the first instance, the reader is referred to ([4], [5]) for excellent reviews of various formulations.

Most of the research so far has concentrated on heuristic techniques since LRPs mix two NP-hard problems. The heuristics normally decompose the problem into its three components, facility location, customer allocation to facilities and vehicle routing, and solve a series of familiar problems such as *p*-median, location-allocation and vehicle routing. Exact techniques have been established for a small number of LRP models that are derived from two-index flow formulations for the vehicle routing problem (VRP). Laporte and Nobert [7] solve a single depot model by a constraint relaxation technique.

Laporte [6] creates an equivalent model and extends the model to the case where the number of vehicles utilized is a variable in the model. Laporte et al. [7] solve a multi-depot capacitated LRP utilizing a constraint relaxation technique. In their work, the largest problem solved to optimality has eight candidate facilities and 20 customers. Laporte et al. [8] utilize a branch and-bound procedure to solve asymmetric LRPs that contain as many as three candidate facilities and 80 customers. An adaptive large neighbourhood search algorithm was suggested by [9] and [10] for two echelon vehicle routing and the LRP. [11] utilize a heuristic algorithm, called hybrid evolutionary search algorithm, to solve LRP with mix fleet size.

Success in developing exact techniques for solving larger instances of LRPs is expected to arise from leveraging the advances in exact techniques for solving VRPs and other difficult combinatorial optimization problems. Motivated by the success of set partitioning formulations for a variety of transportation problems, such as the VRP with time windows (e.g. [12]), the pickup and delivery problem with time windows (e.g. [13]) and the crew scheduling problem (e.g. [14]), we inspect the effectiveness of set partitioning formulations and branch-and-price algorithms in the context of creating exact algorithms for LRPs. The two major influences of this paper are to introduce a new formulation for the LRP with distance constraints and we discover an alternative set of constraints that dramatically enhances the linear programming (LP) relaxation bound. Guerra et al. [15] suggest a heuristic algorithm for solving LRP in a logistic system. [16] consider the LRP utilizing a combination of facility location and vehicle routing problems. The major purpose of their paper is to create LRP with less constraints and variables. [17] describe a VRP combined in LRP for solving a catering problem.

2. Problem formulation

In this section, we introduce a new set-partitioning-based formulation of the LRP with distance constraints. The purpose of the LRP with distance constraints is to choose a set of locations and to build a set of correlated delivery routes in such a way as to decrease facility costs plus routing costs. The set of routes must be such that each customer is visited exactly once by one route and that the length of each route does not surpass the maximum distance.

3. Initial model

Let I be the set of customer locations and J be the set of candidate facility locations. We describe the graph $G = (N, A)$, where $N = I \cup J$ is the set of nodes and $A = N \times N$ is the set of arcs. We let d_{ij} for all $(i, j) \in A$ be the distance between nodes i and j. The distances fulfil the triangle inequality. For applications in which the distance constraint uses to the length of the route to the last customer instead of the length of the return trip to the depot, we set d_{ij} to 0 for all (i, j) with $i \in I$ and $j \in J$. We describe a feasible route k related with facility j as a simple circuit that starts at facility j, visits one or more customer nodes and returns to facility j and that has a total distance of at most the maximum distance, denoted M. Then, we

Copyright ⓒ 2020 SERSC

let P_j signify the set of all possible routes related with the facility j for all $j \in J$. The cost of a route $k \in P_j$ is the total of the costs of the arcs in the route. The cost of an arc $(i, j) \in A$ is related to the distance d_{ij} to reflect distance related operating costs.

3.1 Parameters

$$a_{ijk} = \begin{cases} 1, & \text{if route k associated with facility } j \text{ visits customer } i, \forall i \in I, \forall j \in J, \forall k \in P_j \\ 0, & \text{otherwise} \end{cases}$$

c_{jk} cost of route k associated with facility $j, \forall j \in$ J, $\forall k \in P_j$

f_j fixed cost associated with selecting facility $j, \forall j \in J$

$\propto$ object w $\propto$ eighted factor

3.2 Decision Variables

$$X_j = \begin{cases} 1, & \text{if facility } j \text{ is selected, } \forall j \in J \\ 0, & \text{otherwise} \end{cases}$$

$$Y_{jk} = \begin{cases} 1, & \text{if route } k \text{ associated with facility } j \text{ is selected, } \forall j \in J, \forall k \in P_j \\ 0, & \text{otherwise} \end{cases}$$

(LRP-DC) Minimize $$\alpha . \sum_{j \in J} f_j X_j + \sum_{j \in J} \sum_{k \in P_j} c_{jk} Y_{jk} \quad (1)$$

s.t. $$\sum_{j \in J} \sum_{k \in P_j} a_{ijk} Y_{jk} = 1 \quad \forall i \in I$$

(2)

$$X_j - Y_{jk} \geq 0 \quad \forall j \in J, \forall k \in P_j \quad (3)$$

$$X_j \in \{0,1\} \quad \forall j \in J \quad (4)$$

$$Y_{jk} \in \{0,1\} \quad \forall j \in J, \forall k \in P_j$$

(5)

The objective function (1) pursues to reduce the weighted total of the facility costs and the routing costs. Constraints (2) are the set partitioning constraints that involve each customer i be served by exactly one of the selected routes. Constraints (3) demand that facility j be chosen if a route k related with facility j is chosen. Constraints (4) and (5) are standard binary restrictions. The LRP with distance constraints is NP-hard. By putting very large costs on the arcs connecting two customer nodes, we acquire a special case of the model in which the chosen routes include exactly one customer.

As presented, the formulation LRP-DC potentially includes an exponential number of variables y_{jk} and an exponential number of constraints (3). Hence, for cases of practical size, listing all the possible routes and solving the resulting integer program is improbable to be effective. Instead, we will utilize viable neighbourhood search for solving the model.

The model for vehicle routing with forbidden route can be written as:
(VRP-FR)

$$\min \sum_{(i,j) \in A; (i,j) \notin X} c_{ij} x_{ij} \quad (6)$$

s.t. $$\sum_{(s,j) \in \delta^+(i)} x_{ij} = 1 \quad \forall i \in C, i \notin X \quad (7)$$

$$\sum_{(s,j) \in \delta^-(i)} x_{ij} = \sum_{(s,j) \in \delta^+(i)} x_{ij} \quad i \notin X,$$

$\forall i \in V$ (8)

$$\sum_{(s,j) \in \delta^-(i)} (T_{ji}^r + t_{ji}^r x_{ij}) \leq \sum_{(s,j) \in \delta^+(i)} T_{ji}^r \quad i \notin X, \forall r \in R, \forall i \in C \quad (9)$$

$$a_i^r x_{ij} \leq T_{ji}^r \leq b_i^r x_{ij} \quad (i,j) \notin X, \forall r \in R, \forall (i,j) \in A \quad (10)$$

ISSN: 2005-4238 IJAST
Copyright ⓒ 2020 SERSC

$T_{ij}^{r} \geq 0$ $\quad$ $(i,j) \notin X, \forall r \in R, \ \forall (i,j) \in A$ (11)

$x_{ij} \in \{0,1\}$ $\quad$ $(i,j) \notin X, \forall (i,j) \in A$ (12)

The combination of the two models we can get a model for LRP with Distance and Forbidden Route constraints.

4. The Non- Basic Variables Solution Approach

Consider a mixed integer linear programming (MILP) problem with the following form

$$\text{Minimize } P = \boldsymbol{c}^T \boldsymbol{x} \tag{12}$$

$$\text{Subject to } A\boldsymbol{x} \leq \boldsymbol{b} \tag{13}$$

$$\boldsymbol{x} \geq \boldsymbol{0} \tag{14}$$

$$x_j \text{ integer for some } j \in J \qquad (J \text{ is index set}) \tag{15}$$

A component of the optimal basic feasible vector $(\boldsymbol{x_B})_k$, to MILP solved as continuous can be written as

$$(\boldsymbol{x_B})_k = \beta_k - \alpha_{k1}(\boldsymbol{x_N})_1 - \cdots - \alpha_{kj}(\boldsymbol{x_N})_j - \cdots - \alpha_{k,n-m}(\boldsymbol{x_N})_{n-m} \tag{16}$$

Note that, this expression can be discovered in the final tableau of Simplex procedure. If $(\boldsymbol{x_B})_k$ is an integer variable and we assume that β_k is not an integer, the partitioning of β_k into the integer and fractional components is given by

$$\beta_k = [\beta_k] + f_k, \ 0 \leq f_k \leq 1 \tag{17}$$

suppose we desire to escalate $(\boldsymbol{x_B})_k$ to its nearest integer, $([\beta]+1)$. Based on the idea of suboptimal solutions we may raise a non-basic variable, say $(\boldsymbol{x_N})_{j^*}$, above its bound of zero, given α_{kj^*}, as one of the elements of the vector α_{j^*}, is negative. Let Δ_{j^*} be amount of movement of the non-variable $(\boldsymbol{x_N})_{j^*}$, such that the numerical value of scalar $(\boldsymbol{x_B})_k$ is integer. Referring to Eqn. (16), Δ_{j^*} can then be expressed as

$$\Delta_{f^*} = \frac{1-f_k}{-\alpha_{kj^*}} \tag{18}$$

while the remaining non-basic stay at zero. After replacing (18) into (16) for $(\boldsymbol{x_N})_{j^*}$ and reflecting the partitioning of β_k given in (17), we get

$$(\boldsymbol{x_B})_k = [\beta] + 1$$

Therefore, $(\boldsymbol{x_B})_k$ is now an integer.

It is now obvious that a non-basic variable shows a significant part to integerize the corresponding basic variable.

6. The Algorithm Proposed for Solving the Problem

Now, we describe roughly about the algorithm to be used for solving the model created in Section 4.

Step 1. (Test for convergence). If $\|\boldsymbol{h}\| >$ TOLRG go to step 3.

Step 2. ("PRICE", i.e., estimate Lagrange multipliers, add one superbasic).

(a) Calculate $\boldsymbol{\lambda} = \boldsymbol{g}_N - N^T \pi$

(b) Select $\lambda_{q_1} < -$TOLDJ ($\lambda_{q_2} >$ +TOLDJ), the largest elements of λ corresponding to variables at their lower. (upper) bound. If none, STOP; the Kuhn-Tucker necessary conditions for an optimal solution are satisfied.

(c) Otherwise,

(i) Choose $q = q_1$ or $q = q_2$ corresponding to $|\lambda_q| = \max(|\lambda_{q_1}|, |\lambda_{q_2}|)$;

(ii) add a_q as a new column of S;

(iii) add λ_q as a new element of $\boldsymbol{h}$;

(iv) add a suitable new column to $\boldsymbol{R}$.

(d) Increase s by 1.

Step 3. (Compute direction of search, $\boldsymbol{p} = Z\boldsymbol{p}_s$).

(a) Solve $R^T R \boldsymbol{p}_s = -\boldsymbol{h}$.

(b) Solve $LU\boldsymbol{p}_B = -S\boldsymbol{p}_s$.

Copyright ⓒ 2020 SERSC

(c) Set $\boldsymbol{p} = \begin{bmatrix} \boldsymbol{p}_B \\ \boldsymbol{p}_S \\ 0 \end{bmatrix}$

Step 4. (Ratio test to maintain feasibility).

(a) Find $\alpha_{\max} \geq 0$, the greatest value of a for which $\boldsymbol{x} + \alpha\boldsymbol{p}$ is feasible.

(b) If $\alpha_{\max} = 0$ go to step 7.

Step 5. (Linesearch to find the step length).

(a) Find α, an approximation to α^*, where

$$F(\boldsymbol{x} + \alpha^* \boldsymbol{p}) = \min_{0 < \theta \leq \alpha_{\max}} f(\boldsymbol{x} + \theta \boldsymbol{p})$$

(b) Change $\boldsymbol{x}$ to $\boldsymbol{x} + \alpha\boldsymbol{p}$ and set f and $\boldsymbol{g}$ to their values at the new $\boldsymbol{x}$.

Step 6. (Compute reduced gradient, $\bar{\boldsymbol{h}} = Z^T\boldsymbol{g}$).

(a) Solve $U^T L^T \pi = \boldsymbol{g}_B$.

(b) Compute the new reduced gradient, $\bar{\boldsymbol{h}} = \boldsymbol{g}_s - S^T\pi$.

(c) Modify R to reflect some variable-metric recursion on $R^T R$, using □, $\boldsymbol{p}_s$ and the change in reduced gradient, $\bar{\boldsymbol{h}} - \boldsymbol{h}$.

(d) Set h = $\bar{\boldsymbol{h}}$.

(e) If □ < □$_{\max}$ go to step 1. No new constraint was encountered so we remain in the current subspace.

Step 7. (Change basis if necessary; delete one superbasic). Here □ < □$_{\max}$ and for some p ($0 < p \leq m + s$) a variable corresponding to the p-th column of [B S] has reached one of its bounds.

(a) If a *basic* variable hit its bound ($0 < p \leq m$),

(i) interchange the p-th and q-th columns of

$$\begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{x}_B^T \end{bmatrix} \text{ and } \begin{bmatrix} S \\ \boldsymbol{x}_S^T \end{bmatrix}$$

respectively, where q is chosen to keep B nonsingular (this requires a vector π_p which satisfies $U^T L^T \pi_p = e_p$);

(ii) modify L, U, R and π to reflect this change in B;

(iii) compute the new reduced gradient $\boldsymbol{h} = g_s - S^T\pi$;

(iv) go to (c).

(b) Otherwise, a *superbasic* variable hit its bound ($m < p \leq m + s$). Define $q = p - m$.

(c) Make the q-th variable in S nonbasic at the appropriate bound, thus:

(i) delete the q-th columns of

$$\begin{bmatrix} \boldsymbol{S} \\ \boldsymbol{x}_S^T \end{bmatrix} \text{ and } \begin{bmatrix} R \\ \boldsymbol{h}^T \end{bmatrix}$$

(ii) restore **R** to triangular form.

(d) Decrease s by 1 and go to step 1.

5. Conclusions

This paper offers how to solve location-routing problems in which we reflect the distance of facilities concerning the condition of route. The model developed a large-scale mixed integer program. We wolve the model using a reduced gradient based search approach.

ISSN: 2005-4238 IJAST
Copyright ⓒ 2020 SERSC

6. References

[1] Nagy G, Salhi S. 2007. Location-routing: Issues, models and methods. European Journal of Operational Research, 177, 649-672.

[2] Drexl M, Schneider M. 2014. A survey of the standard location-routing proble. Working paper, Logistic Planning and Information Systems, TU Darmstadt, Germany.

[3] Albareda Sambola M. 2015. Location-routing and location-arc-routing. In Laporte G., Nickel S., Saldanha da Gama F. (Eds.), Location Science, Springer, Berlin-heidelberg.

[4] Laporte G. 1988 *Location routing problems in golden B*, Assad A (eds) Vehicle Routing: Methods and Studies (North-Holland: Amsterdam) pp 293–318

[5] Prodhon C, Prins C. 2014. A survey of recent research on location-routing problems. European Journal of Operational Research, 238, 1-17.

[6] G Laporte 1988 *Location-routing problems. In: golden, B.L.*, Assad, A. A. (eds.), Vehicle Routing: Methods and Studies (North-Holland: Amsterdam) pp 293–318

[7] S Sitepu, H Mawengkang, I Husein "*Optimization model for capacity management and bed scheduling for hospital*"IOP Conference Series: Materials Science and Engineering 300 (1), 01,2016.

[8] G Laporte and Y Nobert 1981 An exact algorithm for minimizing routing and operating cost in depot location *European Journal of Operational Research* **6:** 224–226

[9] G Laporte, Y Nobert, and D Arpin 1986 An exact algorithm for solving a capacitated location-routing problem *Annals of Operations Research* **6:** 293–310

[10] Hemmelmayr V C, Cordeua J F, Crainic T G. 2012. An adaptive large neighborhood search heuristic for two echelon vehicle routing problems arising in city logistics. Computer & Operations Research 39, 3215-3228.

[11] Renaud Masson, Anna Trentini, Fabien Lehuédé, Nicolas Malhéné, Olivier Péton, et al.. Optimization of a city logistics transportation system with mixed passengers and goods. EURO Journal on Transportation and Logistics, Springer, 2017, 6 (1), pp.81-109. 10.1007/s13676-015-0085-5.

[12] Husein, Ismail, Herman Mawengkang, Saib Suwilo, and Mardiningsih. "Modelling Infectious Disease in Dynamic Networks Considering Vaccine." *Systematic Reviews in Pharmacy* 11.2, pp. 261-266, 2020.

[13] C. Koc¸ Bektas¸, T., Jabali, O., Laporte, G., 2016a, The fleet size and mix location-routing problem with time windows: Formulations and a heuristic algorithm. European Journal of Operational Research 248, 33–51.

[14] J Desrosiers, Y Dumas, M M Solomon, and F Soumis 1995 Time Constrained Routing and Scheduling M O Ball, T L Magnati, C L Monma, and G L Nemhauser, eds. *Handbooks in Operation Research and Management Science: Network Routing* vol 8 (North-Holland: Amsterdam, The Netherlands) pp 35-139.

[15] M Soland M W P Savelsbergh 1994 A branch-and-price algorithm for the pickup and delivery problem with time windows *Memorandum COSOR* 94-22 (Department of Mathematics and Computing Science, Eindoven University of Technology: Eindoven, The Netherlands)

[16] K L Hoffman and M W Padberg 1993 Solving airline crew scheduling problems by branch-and-cut *Management Sci* **39** 657-683

[17] Husein, Ismail H Mawengkang, S Suwilo "Modeling the Transmission of Infectious Disease in a Dynamic Network" Journal of Physics: Conference Series 1255 (1), 012052, 2019.

[18] L Guerra, T Murino, and E Romano 2007 A Heuristic Algorithm for the Constrained Location-Routing Problem *Int. J. of syit. Appl. Eng & Dev.***4** (1) 146-154

[19] H Toyoglu, O E Karasan, and B Y Kara 2012 A new formulation approach for location-routing problems *Netw. Spat. Ecom.* DOI 10.1007/511067-011-9170-y. Published

Copyright ⓒ 2020 SERSC

online: 12 January 2012.

[20] Z. Azis and H. Mawengkang. 2017. Time dependent heterogeneous vehicle routing problem for catering service delivery problem, IOP Cof. Series: Journal of Physics: Conf. Series 890(2017) 012103.

[21] Husein, Ismail, YD Prasetyo, S Suwilo "Upper generalized exponents of two-colored primitive extremal ministrong digraphs"AIP Conference Proceedings 1635 (1), 430-439, 2014

Copyright ⓒ 2020 SERSC