International Journal of Advanced Science and Technology
Vol. 29, No. 7s, (2020), pp. 3494-3500

Modification of Regression Rating to Estimate Population Average Using Quartiles and Population Variations Coefficient

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Abstract

In conducting the assessment, in general only take a few samples of the many possible samples, and the estimated value for population parameters is only calculated based on the sample taken. So that errors in assessing will be very likely to occur. The estimator discussed in this study is to estimate the population average. The method used is the quartile method and the coefficient of population variation using a regression estimator. Quartile methods and coefficients of population variation are performed to estimate the average population in simple random sampling. The regression estimator is a biased estimator. In this study aims to compare the two estimators with the ratio estimator through MSE and the variance of the estimator. From the results of the comparative analysis it is found that the estimator $\hat{\bar{Y}}_{RR_1}$ is more efficient than the estimator $\hat{\bar{Y}}_{RR_2}$.

Keywords: *Coefficient Skewness, Mean Square Error, Quartile, Ratio Estimator, Simple Random Sampling, Quartile.*

1. Introduction

The assessment process generally only takes a few samples from a large number of possible samples, and the estimated value for population parameters is solely based on the sample taken [1], [2]. So that errors in assessing will be very likely to occur [3], [4], [5]. Therefore, an estimated value is not expected to estimate the population parameters accurately, but the expected value is not expected to deviate too far from the expected value [6], [7]. In other words, the estimation we want is a statistic in which the sampling distribution has the same average as the population parameter value [8], [17]. One estimator that can be used in estimating the population average is the regression estimator. A regression estimator is a biased estimator, whereas an efficient estimator for a biased estimator is an estimator that has the smallest Mean Square Error (MSE).

Cochran (1977) [9], explains that a good estimator comes from a sample that is repressive, i.e. the sample can represent population parameters. One of the parameters estimated through sampling is population variance. Whereas the study of Solanki et al (2012) [10] explained that if the correlation between the Y research variable and the additional variable X was positive (high), the ratio estimator method was quite effective. On the other hand, the correlation between the research variable Y and the X variable is negative and high. When the parameters are different from the variable supporting ratio modification, regression and product can be done by determining the ratio estimator class that is modified using deciles of the auxiliary variables [11]. In a study conducted. Abid et al (2016) [8] also suggested several new ratio estimators that were modified in simple random sampling. In addition, the research conducted by Subzar et al (2017) [12] also provides a modification of the new ratio estimator that uses decile, quartile, median mean with slope coefficient, correlation coefficient, and coefficient of variation as known parameters for additional variables.

Based on the description above, this research is interested in modifying the regression estimator. Where the regression estimator $\hat{\hat{\bar{Y}}}_{RR1}$ using quartiles and coefficient of variation C_x. And the regression estimator $\hat{\hat{\bar{Y}}}_{RR2}$ using quartiles and coefficient of variation C_x. Furthermore, from the two modifications proposed regression estimator, in addition to using quartiles and the regression coefficients are also modified using the ratio estimator which is a biased estimator. It aims to obtain the best average population estimator. So that in the process of taking a sample the errors that occur are smaller.

2. Simple Random Sampling

Simple random sampling is the taking of *n* sample units from *N* population units, where each element has the same opportunity to be taken as a sample member. Sampling can be done with returns or without returns, but to produce more representative samples, it is recommended to use simple random sampling without replacement. For taking samples of size n from a population of character *Y* of size *N*, the value of each population unit is expressed as y_i for $i = 1, 2, \dots, N$ and the value of each sample unit is expressed as y_i for $i = 1, 2, \dots, n$. The average population is $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ and the average sample is $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$.

2.1. Quartile, Decile mean and Variation Coefficient

One measure used to describe the characteristics of a distribution is the size of the distribution. There are several types of distribution including quartile, decile and percentile. Weiss (1999) [13], suggested that the quartile divides the data set into four equal parts with the data sequentially increasing from the smallest to the largest. The first quartile φ_1 is the number that divides the bottom 25% of the data from the top 75%, the second quartile φ_2 is the number that divides the bottom 50% of the data from the top 50% and the third quartile φ_3 is the number that divides the bottom 75% of the data from the top 25%. The following is given a formula for a single data as follows [12]:

$$Q_i \text{ lies in the data to } - \frac{i(n+1)}{4}, \tag{1}$$

Where $i = 1, 2, 3$

The interquartile distance or quartile deviation helps estimate 50% of the total observations. Quartile deviations are formulated as follows [14]:

$$QD = \frac{Q_3 - Q_1}{2}. \tag{2}$$

Quartile deviation can be used as a disperse measure in the middle of the distribution. Next, decile divides the data set into ten equal parts. The data set has nine deciles $d_1, d_2, \dots, d_9$ Thus, 10% of the data is located below d_1 , 20% of the data is located below d_2 and so on if the data is arranged in ascending order. Here is given a formula for a single data,

$$d_i \text{ lies in the data to } - \frac{i(n+1)}{10}, \tag{3}$$

Where $i = 1, 2, \dots, 9$

The decile mean denoted by (DM) is formulated as follows:

$$DM = \sum_{K=1}^{9} \frac{D_k}{9}. \tag{4}$$

The coefficient of variation is a measure that shows how far the distance of the deviation of the data values with the average value. The coefficient of variation is always positive. The following is given the definition of the coefficient of variation.

Definition 1 [14]. The coefficient of variation in a sample consisting of observations $x_1, x_2, \dots, x_n$ is the ratio of the standard deviation and the value of the average represented by

$C_x = \frac{\sigma}{\mu}$,

Definition 2 [15]. Let $\hat{\theta}_1$ be an unbiased estimator for $\boldsymbol{\theta}, \forall \theta \in \Omega$ if

$$E(\hat{\theta}_1) = \theta.$$

Definition 3 [15]. Estimator $\hat{\theta}_2$ is said to be a biased estimator for θ, the magnitude of the bias is defined by

$$B(\hat{\theta}_2) = E(\hat{\theta}_2) - \theta.$$

Montgomery and Runger [16] state the expected value of an error square called Mean Square Error or abbreviated as (MSE). The MSE definition is presented in Definition 4 as follows.

Definition 4 [16]. The Mean Square Error of the estimator $\hat{\theta}^*$ of parameter θ is defined by:

$$MSE(\hat{\theta}^*) = E(\hat{\theta}^* - \theta)^2.$$

2.2. Regression Estimator and ratio Estimator for mean population

In the ratio method an additional variable X that corresponds to the population unit Y is obtained for each unit in the sample. It is assumed that the parameters attached to population X are known. The population ratio is expressed as $R = \bar{Y}/\bar{X}$ and the sample ratio is expressed as $\hat{R} = \bar{y}/\bar{x}$. The estimator ratio for the mean population $\bar{Y}$ is expressed as $\bar{y}_R = \hat{R}\bar{X}$.

The bias of the ratio estimator for the population mean are

$$B(\bar{y}_R) \approx \frac{1-f}{n}\bar{Y}(C_x^2 - {}_{\rho}C_xC_y),$$

and MSE of the ratio estimator for the population mean are

$$MSE(\bar{y}_R) \approx \frac{1-f}{n}\left(R^2S_x^2 - 2RS_{xy}S_y^2\right) \tag{5}$$

The linear regression model is an equation that states the relationship between the independent variable Y and one of the independent variables X in the form of a linear equation. The form of a simple linear regression model is

$$Y = A + BX + e,$$

where Y is the independent variable, while X is the independent variable, A and B is the regression coefficient and e is the sampling error.

Furthermore, based on sampling with simple random sampling, a regression estimator is obtained for the population mean that is notated with and formulated with,

$$\hat{\bar{Y}}_{Reg} = \bar{y} + B(\bar{X} - \bar{x}). \tag{6}$$

3. Results and Discussion

Explained in the introduction, using information (additional variable parameters can increase the estimator's precision. In this case, utilizing these additional variables is usually referred to as an estimator modification. The Regulator Estimator Modification to estimate the average population using quartiles and the coefficient of variation is expressed by submitted as follows [12]

$$\hat{\bar{Y}}_{RR_1} = \frac{\bar{y}_n + B(\bar{X}_N - \bar{x}_n)}{\bar{x}_n C_{x_n} + \varphi_1}\left(\bar{X}_N C_{x_n} + \varphi_1\right), \tag{7}$$

and

$$\hat{\bar{Y}}_{RR_2} = \frac{\bar{y}_n + B(\bar{X}_N - \bar{x}_n)}{\bar{x}_n C_{x_n} + \varphi_2}\left(\bar{X}_N C_{x_n} + \varphi_2\right), \tag{8}$$

where $\varphi_1 = DM \times Q_2$ and $\varphi_2 = DM \times QD$

3.1 Bias and MSE Ratio $\hat{\hat{\bar{Y}}}_{RR_1}$

Will be examined which is the result of modification whether biased or unbiased, by making expectations of the estimator,

$$E\left(\hat{\hat{\bar{Y}}}_{RR_1} - \bar{Y}_N\right) = E\left(\left(\bar{y}_n + b(\bar{X}_N - \bar{x}_n)\right) - \frac{R_{R_1}}{C_{x_n}}\left(\bar{x}_n C_{x_n} + \varphi_1\right)\right)$$

$$-\frac{1}{(\bar{X}_n C_{x_n} + \varphi_1)} E\left(\left(\bar{y}_n + B(\bar{X}_N - \bar{x}_n)\right)\left(\bar{x}_n C_{x_n} + \varphi_1\right) - \left(\bar{X}_n C_{x_n} + \varphi_1\right)\right)$$

$$+\frac{R_{R_1}}{C_{x_n}(\bar{X}_n C_{x_n} + \varphi_1)} E\left(\left(\bar{x}_n C_{x_n} + \varphi_1\right)\left(\left(\bar{x}_n C_{x_n} + \varphi_1\right) - \left(\bar{X}_n C_{x_n} + \varphi_1\right)\right)\right)$$

with a pretty tough translation, finally obtained,

$$E\left(\hat{\hat{\bar{Y}}}_{RR_1}\right) \approx \bar{Y}_N + \frac{1-f}{n} R_{R_1}^2 \frac{S_x^2}{\bar{Y}_N} \tag{9}$$

because based on the definition of 3 the second term of equation (12) is not equal to zero, then based on the definition 2 of the estimator then the estimator $\hat{\hat{\bar{Y}}}_{RR_1}$ is a biased estimator with the magnitude of bias is

$$B\left(\hat{\hat{\bar{Y}}}_{RR_1}\right) \approx \frac{1-f}{n} R_{R_1}^2 \frac{S_x^2}{\bar{Y}_N}. \tag{10}$$

Next, to determine the MSE from the ratio estimator is described in the form of Taylor series of two variables in equation (5) regardless of the degree two or more, i.e.

$$MSE\left(f(\bar{x}, \bar{y})\right) \approx \left(\frac{\partial f(\bar{x}, \bar{y})}{\partial \bar{x}}\bigg|_{\bar{X},\bar{Y}}\right)^2 E(\bar{x} - \bar{X})^2$$

$$+2\left(\frac{\partial f(\bar{x}, \bar{y})}{\partial \bar{x}}\bigg|_{\bar{X},\bar{Y}}\right)\left(\frac{\partial f(\bar{x}, \bar{y})}{\partial \bar{y}}\bigg|_{\bar{X},\bar{Y}}\right) E(\bar{x} - \bar{X})(\bar{y} - \bar{Y}) + \left(\frac{\partial f(\bar{x}, \bar{y})}{\partial \bar{y}}\bigg|_{\bar{X},\bar{Y}}\right)^2,$$

$$MSE\left(\hat{\hat{\bar{Y}}}_{RR_1}\right) \approx \frac{1-f}{n}\left(R_{R_1}^2 S_x^2 + 2R_{R_1} B S_x^2 + B^2 S_x^2 - 2R_{R_1} S_{xy} - 2B S_{xy} + S_y^2\right),$$

Finally,

$$MSE\left(\hat{\hat{\bar{Y}}}_{RR_1}\right) \approx \frac{1-f}{n}\left(R_{R_1}^2 S_x^2 + S_y^2\left(1 - \rho_{xy}^2\right)\right). \tag{11}$$

3.2 Bias and MSE ratio $\hat{\hat{\bar{Y}}}_{RR_2}$

As was the case in section 3.1.expectations of the estimator $\hat{\hat{\bar{Y}}}_{RR_2}$ are carried out,

$$E\left(\hat{\hat{\bar{Y}}}_{RR_2} - \bar{Y}_N\right) = E\left(\left(\bar{y}_n + B(\bar{X}_N - \bar{x}_n)\right) - \frac{R_{R_2}}{C_{x_n}}\left(\bar{x}_n C_{x_n} + \varphi_2\right)\right)$$

$$-\frac{1}{(\bar{X}_n C_{x_n} + \varphi_2)} E\left(\left(\bar{y}_n + B(\bar{X}_N - \bar{x}_n)\right)\left(\bar{x}_n C_{x_n} + \varphi_2\right) - \left(\bar{X}_n C_{x_n} + \varphi_2\right)\right)$$

$$+\frac{R_{R_2}}{C_{x_n}(\bar{X}_n C_{x_n} + \varphi_2)} E\left(\left(\bar{x}_n C_{x_n} + \varphi_2\right)\left(\left(\bar{x}_n C_{x_n} + \varphi_2\right) - \left(\bar{X}_n C_{x_n} + \varphi_2\right)\right)\right)$$

so the expectations of the estimator $\hat{\hat{\bar{Y}}}_{RR_2}$

$$E\left(\hat{\hat{\bar{Y}}}_{RR_2}\right) \approx \bar{Y}_N + \frac{1-f}{n} R_{R_1}^2 \frac{S_x^2}{\bar{Y}_N}$$

because based on the definition of 3 the second term of equation (12) is not equal to zero, then based on the definition 2 of the estimator $\hat{\hat{\bar{Y}}}_{RR_2}$ is a biased estimator with the magnitude of bias is

$$B\left(\bar{Y}_{RR_2}\right) \approx \frac{1-f}{n} R_{R_1}^2 \frac{S_x^2}{\bar{Y}_N} \quad .$$

(12)

Next, to determine the MSE from the ratio estimator is described in the form of Taylor series of two variables in equation (5) regardless of the degree two or more, i.e.

$$MSE(f(\bar{x},\bar{y})) \approx \left(\frac{\partial f(\bar{x},\bar{y})}{\partial \bar{x}}\bigg|_{\bar{X},\bar{Y}}\right)^2 E(\bar{x}-\bar{X})^2 + 2\left(\frac{\partial f(\bar{x},\bar{y})}{\partial \bar{x}}\bigg|_{\bar{X},\bar{Y}}\right)\left(\frac{\partial f(\bar{x},\bar{y})}{\partial \bar{y}}\bigg|_{\bar{X},\bar{Y}}\right)E(\bar{x}-\bar{X})(\bar{y}-\bar{Y}) + \left(\frac{\partial f(\bar{x},\bar{y})}{\partial \bar{y}}\bigg|_{\bar{X},\bar{Y}}\right)^2 E(\bar{y}-\bar{Y})^2.$$

$$MSE\left(\hat{\bar{Y}}_{RR_2}\right) \approx \frac{1-f}{n}\left(R_{R_2}S_x^2 + 2R_{R_2}BS_x^2 + B^2S_x^2 - 2R_{R_2}S_{xy} - 2BS_{xy} + S_y^2\right)$$

Finally,

$$MSE\left(\hat{\bar{Y}}_{RR_2}\right) \approx \frac{1-f}{n}\left(R_{R_2}^2 S_x^2\left(1-\rho_{xy}^2\right)\right). \quad (13)$$

3.3 Efficient Regression Ratio Estimator For Population Mean

To determine a regression estimator that is more efficient than an ordinary estimator, it can be determined by comparing the MSE of each estimator. In comparing the MSE of each regression estimator, it can be done by looking at the relative efficiency of the following definition 5.

Definition 5 [16].]. Let $\hat{\theta}_1^*$ and $\hat{\theta}_1^*$ *be* the estimators for θ, and suppose that $MSE\left(\hat{\theta}_1^*\right)$, and $MSE\left(\hat{\theta}_2^*\right)$, are MSE of $\hat{\theta}_1^*$ and $\hat{\theta}_2^*$,then the Relative Efficiency $\hat{\theta}_1^*$ and $\hat{\theta}_2^*$ is denoted by $RE\,(\hat{\theta}_1^*,\ \hat{\theta}_2^*)$defined as follows:

$$RE(\hat{\theta}_1^*,\hat{\theta}_2^*) = \frac{MSE(\hat{\theta}_1^*)}{MSE(\hat{\theta}_2^*)}$$

If $RE(\hat{\theta}_1^*,\hat{\theta}_2^*) < 1$, it means that MSE $\left(\hat{\theta}_1^*\right)$ is smaller than MSE $\left(\hat{\theta}_2^*\right)$ so it can be concluded that the estimator $\hat{\theta}_1^*$ is more efficient than the estimator $\hat{\theta}_1^*$. Thus, to determine the relative efficiency of each estimator, it can also be done by determining the difference between MSE $\left(\hat{\theta}_1^*\right)$ and MSE $\left(\hat{\theta}_2^*\right)$ Following is the MSE comparison of each estimator with the distinguishing factors φ_1, φ_2 and B.

3.4 Comparison between $MSE\left(\hat{\bar{Y}}_{RR_1}\right)$ and $MSE\left(\hat{\bar{Y}}_R\right)$

Because the distinguishing factor of the estimator $\hat{\bar{Y}}_{RR_1}$ and the estimator $\hat{\bar{Y}}_R$ is the parameter φ_1 and parameter B, the exploratory conditions for efficiency between the estimator $\hat{\bar{Y}}_{RR_1}$ and the estimator $\hat{\bar{Y}}_R$ are one of the distinguishing factors.

The MSE difference in equations (11) and equation (6) with the difference factor parameter φ_1 is that the $\hat{\bar{Y}}_{RR1_1}$ estimator is more efficient than the $\hat{\bar{Y}}_R$ estimator if

$$MSE\left(\hat{\bar{Y}}_{RR1}\right) - MSE\left(\hat{\bar{Y}}_R\right) = \frac{1-f}{n}\left(R_{R_1}^2 S_x^2 + S_y^2\left(1-\rho_{xy}^2\right)\right) - \frac{1-f}{n}\left(R^2 s_x^2 - 2Rs_{xy} + s_y^2\right) < 0$$

with the difference factor parameter φ_1 with the difference factor found that The estimator $\hat{\bar{Y}}_{RR_1}$ is more efficient than the estimator $\hat{\bar{Y}}_R$ if,

$$\varphi_1 < -\frac{S_x\bar{Y}_N C_{x_n}}{\rho_{x_n}S_y - RS_x}\bar{X}_N C_{x_n} \text{ or } \varphi_1 > -\frac{S_x\bar{Y}_N C_{x_n}}{\rho_{x_n}S_y - RS_x}\bar{X}_N C_{x_n} \quad (14)$$

When based on the parameter differentiating factor B, it is found that the estimator $\hat{\bar{Y}}_{RR_1}$ is more efficient than the estimator $\hat{\bar{Y}}_R$, if $B > B_1$ or $B < B_2$

3.5 Comparison between $MSE\left(\hat{\bar{\bar{Y}}}_{RR_2}\right)$ *and* $MSE\left(\hat{\bar{\bar{Y}}}_R\right)$

Because the distinguishing factor of the estimator $\hat{\bar{\bar{Y}}}_{RR2_2}$ and the estimator $\bar{Y}_R$ is the parameter φ_2 and parameter B, the exploratory conditions for efficiency between the estimator $\hat{\bar{\bar{Y}}}_{RR2_1}$ and the estimator $\hat{\bar{\bar{Y}}}_R$ are one of the distinguishing factors.
The MSE difference in equations (11) and equation (6) with the difference factor parameter φ_1 is that the $\hat{\bar{\bar{Y}}}_{RR2_1}$ estimator is more efficient than the $\hat{\bar{\bar{Y}}}_R$ estimator if

$$MSE\left(\hat{\bar{Y}}_{RR2}\right) - MSE\left(\hat{\bar{Y}}_R\right) = \frac{1-f}{n}\left(R_{R_2}^2 S_x^2 + S_y^2(1-\rho_{xy}^2)\right) - \frac{1-f}{n}\left(R_{R_1}^2 S_x^2 + S_y^2(1-\rho_{xy}^2)\right) < 0$$

with the difference factor parameter φ_1 and B with the difference factor found that The estimator $\hat{\bar{\bar{Y}}}_{RR2_1}$ is more efficient than the estimator $\hat{\bar{\bar{Y}}}_R$ if,

$$\varphi_2 < -\frac{S_x\bar{Y}_N C_{x_n}}{\rho_{x_n}S_y - RS_x}\bar{X}_N C_{x_n} \text{ or } \varphi_2 > -\frac{S_x\bar{Y}_N C_{x_n}}{\rho_{x_n}S_y - RS_x}\bar{X}_N C_{x_n} \qquad (15)$$

When based on the parameter differentiating factor B, it is found that the estimator $\hat{\bar{\bar{Y}}}_{RR_2}$ is more efficient than the estimator $\hat{\bar{\bar{Y}}}_R$, if

$$B > B_1 \text{ or } B < B_2 \qquad (16)$$

3.6 Comparison between $MSE\left(\hat{\bar{\bar{Y}}}_{RR_2}\right)$ *and* $MSE\left(\hat{\bar{\bar{Y}}}_{RR1}\right)$

Because the distinguishing factor of the estimator $\hat{\bar{\bar{Y}}}_{RR1}$ and the estimator $\hat{\bar{\bar{Y}}}_{RR2}$ is the parameter φ_1 and parameter φ_2, the exploratory conditions for efficiency between the estimator $\hat{\bar{\bar{Y}}}_{RR1}$ and the estimator $\hat{\bar{\bar{Y}}}_{RR2}$ are one of the distinguishing factors.
The MSE difference in equations (11) and equation (6) with the difference factor parameter φ_1 is that the $\hat{\bar{\bar{Y}}}_{RR1}$ estimator is more efficient than the $\hat{\bar{\bar{Y}}}_{RR2}$ estimator if

$$MSE\left(\hat{\bar{\bar{Y}}}_{RR2}\right) - MSE\left(\hat{\bar{\bar{Y}}}_{RR1}\right) = \frac{1-f}{n}\left(R_{R_2}^2 S_x^2 + S_y^2(1-\rho_{xy}^2)\right) - \frac{1-f}{n}\left(R_{R_1}^2 S_x^2(1-\rho_{xy}^2)\right)$$

with the difference factor parameter φ_1 with the difference factor found that The estimator $\hat{\bar{\bar{Y}}}_{RR2}$ is more efficient than the estimator $\hat{\bar{\bar{Y}}}_{RR1}$ if,

$$\varphi_1 > \varphi_2 \text{ or } \varphi_1 < -2\bar{X}_N C_{x_n} - \varphi_2 \,. \qquad (17)$$

4. Conclusion

Based on the results of the analysis that has been done, it can be concluded that the estimator $\hat{\bar{\bar{Y}}}_{RR_{11}}$ is more efficient than a simple estimator $\hat{\bar{Y}}_R$ if the efficiency requirements on the differentiating factor φ_1 and the efficiency requirements on the differentiator factor *B* are met. Furthermore, the estimator $\hat{\bar{\bar{Y}}}_{RR2}$ is more efficient than the estimator $\hat{\bar{Y}}_R$ if the efficiency requirements on the distinguishing factor φ_1 and the efficiency requirements on the differentiating factor B variable are met. Estimator $\hat{\bar{\bar{Y}}}_{RR_{11}}$ is more efficient than the estimator, if the efficiency requirements are met, so that of the three estimators who are the best estimators are the estimators $\hat{\bar{\bar{Y}}}_{RR_{11}}$.

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