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On Modeling Number of District Share Transactions Using Two-Level Hierarchical Structure with Bayesian Approach

Ika Lulus Yuliatin[1], Kartika Fithriasari[2] and Nur Iriawan[3]

[1]*Department of Statistics, Faculty of Science and Data Analytics, Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia.*
[1]*lulusyuliatin@gmail.com*, [2]*kartika_f@statistika.its.ac.id*,
[3]*nur_i@statistika.its.ac.id*

Abstract

The condition of the Indonesian economy tends to fluctuate. One of the methods that can be used to observe the fluctuating development of the country's economy is looking at the development of the capital market as a leading indicator of the economy. This research intends to model the number of stock transactions in the capital market using a two-level hierarchical Judging from Social Population and Economy. The first (micro) level is the district with its characteristics of the region and the second (macro) level is the province with the distinguishing variable characteristics. A Bayesian hierarchical model approach coupled with the MCMC algorithm to the three possible data patterns of these transactions, i.e. Log-Normal 2 parameters, Log-Normal 3 parameters, and Log-Logistic distribution is proposed. The analysis shows that Bayesian hierarchy modeling is better than the Bayesian one-level, and employing the Log-Normal 3 parameter distribution is better than the others. This can be known through the smallest DIC value. The variation of the micro regression coefficients between provinces proved to be significantly influenced by the characteristics of the district and the characteristics of the province. This approach has succeeded to demonstrate that the Bayesian hierarchy model more able to illustrate the influence of socioeconomic and population factors at different levels on the growth of the number of stock transactions.

Keywords: *Bayesian hierarchy, DIC, Log-Normal, Log-Logistic, Stock transactions*

1. Introduction

At present, the economic condition of the State of Indonesia tends to fluctuate. One way that can be used to observe the fluctuating development of the country's economy is to look at capital market developments as a leading indicator of the economy. The more advanced the economy, the greater the role of the capital market. The capital market is used by the public as a medium to invest its money, with the hope of earning profits so that it can be used as business development or additional working capital. The purpose of building a capital market is to be able to move the economy of a country through the private sector. In developed countries, the capital market is the main tool in developing the economy. The development of investment in the capital market is in line with economic growth and population growth, which is highly emphasized by the government. The Indonesia Stock Exchange (IDX) has made various efforts to rectify the perceptions that are developing in the community regarding investment stocks that seem expensive, complicated, and risky. As one of the regulators and trade providers in the Indonesian Capital Market, IDX provides data in the form of trading data on shares, bonds, and derivatives. One of the data published by the IDX is demographic data on the number of share transactions in each regency in each province in Indonesia, where through this data, the value of share transactions according to the regency/city can be seen.

A person's decision to conduct a stock transaction either selling or buying shares is greatly influenced by the index and stock prices. Thus, fluctuations in the number of stock

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transactions depend on the ups and downs of the stock price index. The stock price index oscillation occurs because in the economy certain forces cause the price level to surge at the same time and take place slowly. These factors can come from economic, environmental, population, and government, and bureaucratic factors. Based on this description, this research intends to model the number of stock transactions that follow 2-parameter Log-Normal, 3-parameter Log-Normal, and 3-parameter Log-Logistic distribution using a two-level hierarchical structure with the Bayesian approach.

The reason for grouping hierarchical data is formed by the similarity of members in one group, making among members in the group have similar characteristics. While between members of one group with other group members is different or in other words, there are variants between groups [2]. The hierarchical model has several advantages. First, it can be used together to analyze at several different levels in statistical analysis. Second, the response variance can be calculated at each level [3]. This makes it possible to find out the contribution of variance at each data level to the response variance. In Bayesian hierarchical modeling, Deviance Information Criterion (DIC) is used. The smaller the DIC value, the better the model is. In the classical method, a hierarchical model that involves many variables causes the model to become complex. Therefore, it is necessary to do modeling through the Bayesian approach. The Bayesian method is very flexible and easy to estimate the parameters of a complex hierarchical model [4,5,6]. The thing that needs to be considered in modeling the Bayesian approach is the information distribution pattern of observations. Observational data in Bayesian viewpoints are stated to originate from a probability distribution that has parameters that are not certain. Therefore, the distribution of these parameters must be determined in advance called the prior distribution [7]. The accuracy in determining the prior distribution will greatly affect the estimation results. In this study, WinBUGS software will be used to model the number of share transaction values.

2. Bayesian Analysis

Conceptually the Bayesian method was developed based on the Bayes theorem, which combines the prior distribution and data information (likelihood function) into a posterior distribution. the joint distribution of and *y* can be written in the equation.

$$f(y,\theta)=f(y\mid\theta)\,f(\theta)=f(\theta\mid y)\,f(y), \tag{1}$$

where, given data *y*, the conditional probability density function of θ when $f(y) \neq 0$ is:

$$f(\theta\mid y)=\frac{f(y\mid\theta)\,f(\theta)}{f(y)}. \tag{2}$$

By assuming that $f(y)=E\left[f(y\mid\theta)\right]=c^{-1}$, then (2) can be alternatively written as

$$f(\theta\mid y)=cf(y\mid\theta)\,f(\theta). \tag{3}$$

The Bayes' formula in (2), therefore, can be written as $f(\theta\mid y)\propto f(y\mid\theta)\,f(\theta)$ which denotes that the posterior $f(\theta\mid y)$ is proportional to $f(y\mid\theta)\,f(\theta)$ [8]

3. Linear Hierarchical Model

In hierarchy modeling, each level is represented by a sub-model that describes the relationship between variables in one level and explains the effect of relationships with

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variables at other levels. This model is also known as the micro model and macro model [3]. The model equation at first level for each group is

$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + \beta_{2j} X_{2ij} + ... + \beta_{kj} X_{kij} e_{ij}, \quad i = 1, 2, ..., n_j \text{ , and } j = 1, 2, ..., m \tag{4}$$

where

$$\mathbf{Y}_j = \begin{pmatrix} Y_{1j} & Y_{2j} & \dots & Y_{nj} \end{pmatrix}^T$$

$$\mathbf{X}_{ij} = \begin{pmatrix} 1 & X_{11j} & X_{21j} & \cdots & X_{k1j} \\ 1 & X_{12j} & X_{22j} & \cdots & X_{k2j} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & X_{1n_j j} & X_{2n_j j} & \cdots & X_{kn_j j} \end{pmatrix}$$

$$\boldsymbol{\beta}_j = \begin{pmatrix} \beta_{0j} & \beta_{1j} & \cdots & \beta_{kj} \end{pmatrix}^T$$

$$\mathbf{e}_j = \begin{pmatrix} e_{1j} & e_{2j} & \cdots & e_{n_j j} \end{pmatrix}^T$$

As many m regression models will be generated from the micro model, the parameter values of the regression models are $\beta_{rj}, r = 0,1,...,k$. The value of these variations can be explained by regressing each coefficient β_{rj} with all the predictors in the second level model. The relationship of this regression model is known as the macro model whose equation is [9]

$$\beta_{rj} = \gamma_{0r} + \gamma_{1r} W_{1j} + \gamma_{2r} W_{2j} + \cdots + \gamma_{lr} W_{lj} \quad r = 0,1,...,k \text{ and } j = 0,1,...,m. \tag{5}$$

where

$$\mathbf{W}_j = \begin{pmatrix} 1 & w_{11} & w_{21} & \cdots & w_{lk} \\ 1 & w_{12} & w_{22} & \cdots & w_{l2} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & w_{1m} & w_{2m} & \cdots & w_{lm} \end{pmatrix},$$

$$\boldsymbol{\beta}_r = \left(\beta_{r1} \quad \beta_{r2} \quad \dots \quad \beta_{rm} \right)^T$$

$$\boldsymbol{\gamma}_j = \left(\gamma_{0r} \quad \gamma_{1r} \quad \cdots \quad \gamma_{lr} \right)^T$$

4. The 2-parameter Log-Normal, 3-parameter Log-Normal, and 3-parameter Log-Logistic distribution

As the nature of the value of stock transactions that will always have a positive value, the pattern of the data value of these transactions will be able to have patterns such as 2-parameter Log-Normal, 3-parameter Log-Normal, and 3-parameter Log-Logistic. The probability density function (p.d.f) of these three distributions is as follows. The p.d.f of 2-parameter Log-Normal is

$$f\left(y \mid \mu, \sigma^2\right) = \frac{1}{y\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}\left(\ln(y) - \mu\right)^2\right], y > 0, \mu > 0 \text{ and } \sigma > 0. \tag{6}$$

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This p.d.f would state that the value of the stock transaction, Y, will lean to the right centrally at μ and spread as wide as σ. Furthermore, if the 2-parameter Log-Normal distribution is expanded by adding one parameter λ such that the probability value of the stock transaction, Y, which less than λ is equal to zero, then the 3-parameter Log-Normal p.d.f would happen and the p.d.f as the following [1]

$$f\left(y\mid\mu,\sigma^2,\lambda\right)=\frac{1}{\left(y-\lambda\right)\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2\sigma^2}\left(\ln\left(y-\lambda\right)-\mu\right)^2\right], \tag{7}$$

$y>\lambda$, $\lambda>0$, $\mu>0$ and $\sigma>0$, λ is a threshold parameter. This threshold parameter would be used for stating the minimum value of stock transactions in each province, which can also tell the liquidity difference of stocks among provinces. The last form used to represent the value of the stock transaction pattern is the 3-parameter Log-Logistic distribution which also involves λ as a threshold value with p.d.f as follows:

$$f(y\mid\mu,\sigma,\lambda)=\frac{\exp\left(\dfrac{\ln(y-\lambda)-\mu}{\sigma}\right)}{(y-\lambda)\sigma\left[1+\exp\left(\dfrac{\ln(y-\lambda)-\mu}{\sigma}\right)\right]^2} \tag{8}$$

5. Research Variables

This research will be carried out modeling the recapitulation value of stock transaction variables as the dependent variable (Y) published by the financial services authority and there are two types of independent variables, namely micro variable (X) and macro variable (W) published by the Indonesian Statistics Agency. The micro variables are population density (X_1), population growth rate (X_2), human development index (X_3), labor force participation rate (X_4), open unemployment rate (X_5). the number of large and medium industrial companies (X_6), the number of large and medium industrial workers (X_7). The macro variables, on the other hand, are inflation (W_1), rupiah and foreign currency loan positions provided by commercial banks and rural banks according to the province project location (W_2), and the position of Rupiah public deposits commercial banks and rural banks by province (W_3). In this study, the observation unit was divided into two levels, namely the regency level as the first level and the provincial level as the second level. In the observation unit at the first level, there are 90 regencies, while in the second level observation unit there are four provinces, namely the provinces of Bali, West Java, Central Java, and East Java.

6. Methodology

The three distributions approach as the pattern of the value of the stock transaction would be set as responses in the regression model at the first level. Their link functions are unity. The regression model applied in the first level, therefore, can be arranged naturally, namely as a form of a linear model. The steps of analysis are described as follows:

1. Form a response vector for each province, $\mathbf{Y}_j$ and its population distribution, i.e. $\left(\mu_{[y]},\sigma^2_{[y]}\right)$.

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2. Create a Directed Acyclic Graph (DAG) two-level hierarchy model to declare the relationship between the data used, the parameters and the prior and hyperprior distributions in the model
3. Establish the Likelihood function.

$$f_L\left(\mathbf{y} \mid \boldsymbol{\mu}_{[y]}, \boldsymbol{\sigma}^2_{[y]}, \boldsymbol{\lambda}\right) = \prod_{j=1}^{m}\prod_{i=1}^{n_j} f\left(\mathrm{y}_{ij} \mid \boldsymbol{\beta}_{[y]}, \tau^2_{[y]j}, \lambda_j\right) \tag{9}$$

4. Determine the prior β_{rj} distribution $p\left(\beta_{rj}\right)$ and $\tau_{[y]j}$, namely $p\left(\tau_{[y]j}\right)$, and hyperprior γ_{qr}, namely $p\left(\gamma_{qr}\right)$ of the parameters and hyperparameter to be estimated. The independent form of priors can be drawn as Figure 1.

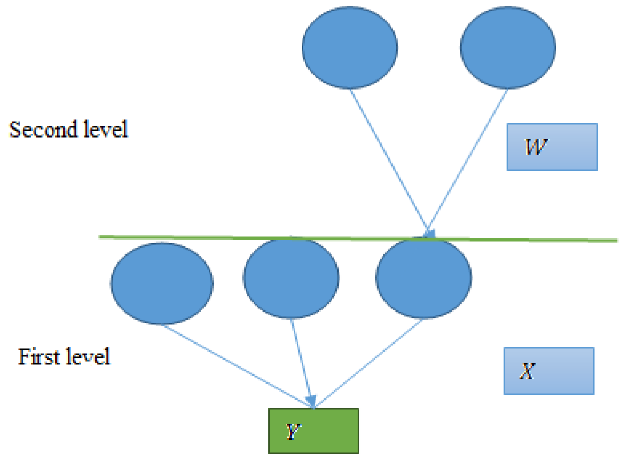

Figure 1. Prior and Hyperprior Structures

5. Form a proportional joint posterior distribution by multiplying the likelihood function and the prior distribution function with its related hyperprior distribution function [10]

$$\begin{aligned} p\left(\boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\tau}_{[y]}, \boldsymbol{\tau}_{[\beta]} \mid y\right) &\propto f_L\left(\mathrm{y} \mid \boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\tau}_{[y]}\right) p\left(\boldsymbol{\beta} \mid \boldsymbol{\gamma}, \boldsymbol{\tau}_{[\beta]}\right) p\left(\boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\tau}_{[y]}, \boldsymbol{\tau}_{[\beta]}\right) \\ &\propto f_L\left(\mathrm{y} \mid \boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\tau}_{[y]}\right) p\left(\boldsymbol{\beta} \mid \boldsymbol{\gamma}, \boldsymbol{\tau}_{[\beta]}\right) p(\boldsymbol{\gamma}) p(\boldsymbol{\lambda}) p\left(\boldsymbol{\tau}_{[y]}\right) p\left(\boldsymbol{\tau}_{[\beta]}\right) \end{aligned} \tag{10}$$

after the following normalized constant

$$h(\mathbf{y}) = \int \cdots \int p\left(\boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\tau}_{[y]}, \boldsymbol{\tau}_{[\beta]} \mid \mathrm{y}\right) d\beta_{01} \cdots d\beta_{km} d\gamma_{00} \cdots d\gamma_{1k} d\lambda_1 \cdots d\lambda_m d\tau_{[y]1} \cdots d\tau_{[y]m} d\tau_{[\beta]0} \cdots d\tau_{[\beta]k}$$

is not included in the equation, where $f_L\left(\mathrm{y} \mid \boldsymbol{\beta}, \lambda, \tau_{[y]}\right)$ is a likelihood function, $p\left(\boldsymbol{\beta} \mid \gamma, \tau_{[\beta]}\right)$ is a prior distribution function at the first level, $p\left(\gamma, \tau_{[y]}, \tau_{[\beta]}\right)$ is a second stage prior distribution function. Among those priors, β_{rj} and λ_j is prior for parameters where $\beta_{rj} \sim N\left(\mu_{[\beta]r}, \sigma^2_{[\beta]r}\right)$ and $\lambda_j \sim N\left(\mu_{[\lambda]j}, \sigma^2_{[\lambda]j}\right)$, while $\tau_{[y]j}$, γ_{qr}, and

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$\tau_{[\beta]\mathrm{r}}$ are hyperprior for hyperparameter, where $\tau_{[\mathrm{y}]\mathrm{j}} \sim \mathrm{Gamma}\left(a_{[\tau_{[y]}]j}, \mathrm{b}_{[\tau_{[y]}]j}\right)$, $\gamma_{qr} \sim N\left(\mu_{\gamma_{qr}}, \sigma^2_{\gamma_{qr}}\right)$, and $\tau_{[\beta]\mathrm{r}} \sim \mathrm{Gamma}\left(a_{[\tau_{[\beta]r}]}, \mathrm{b}_{[\tau_{[y]r}]}\right)$,

6. Apply the full conditional posterior for each parameter to be estimated based on equation (8) by giving values to all of the conditioning parameters with the following steps:
   a) generate $\hat{\beta}_{rj}$ from $p\left(\beta_{\mathrm{rj}} \mid y, \boldsymbol{\beta}_{\backslash rj}, \gamma, \lambda, \tau_{[y]}, \tau_{[\beta]}\right)$ where $\beta_{\backslash rj}$ is a vector $\boldsymbol{\beta}$ on equation (10) without elements β_{rj}
   b) generate $\hat{\gamma}_{qr}$ from $p\left(\gamma_{qr} \mid y, \boldsymbol{\beta}, \lambda_j, \boldsymbol{\gamma}_{\backslash qr}, \tau_{[y]}, \tau_{[\beta]}\right)$ where $\gamma_{\backslash qr}$ is a vector $\boldsymbol{\gamma}$ on equation (10) without elements γ_{qr}
   c) generate $\hat{\tau}_{[\beta]\mathrm{r}}$ from $p\left(\tau_{[\beta]\mathrm{r}} \mid y, \boldsymbol{\beta}, \lambda_j, \boldsymbol{\gamma}_{\backslash qr}, \tau_{[y]}, \tau_{[\beta]\backslash\mathrm{r}}\right)$ where $\tau_{[\beta]\backslash\mathrm{r}}$ is a vector $\boldsymbol{\tau}_{[\beta]}$ on equation (10) without elements $\tau_{[\beta]\mathrm{r}}$
7. Estimate two-level hierarchy model parameters iteratively using an MCMC and Gibbs Sampling [11,15] by determining the initial value for each parameter $\left(\boldsymbol{\beta}^{(0)}, \boldsymbol{\gamma}^{(0)}, \boldsymbol{\lambda}^{(0)}, \boldsymbol{\tau}_{[y]}^{(0)} \text{ and } \boldsymbol{\tau}_{[\beta]}^{(0)}\right)$ for running the 6-th step above.
8. Catch a number of the B samples of all generated estimated posterior parameters when the process reached the equilibrium distribution to get the parameter estimator characteristics.
9. Evaluate the model using their credible intervals. If there are insignificant predictors, then an alternative model is built by issuing these predictors.
10. Choose the best model based on DIC value.
    DIC is one measure that can be used to compare two or more models that can be identified through the following equation [12]:

$$\begin{aligned} DIC(m) &= 2\overline{D(\theta_m, m)} - D\left(\overline{\theta_m}, m\right) \\ &= D\left(\overline{\theta_m}, m\right) + 2p_m \end{aligned} \tag{11}$$

7. Experiment Result

In this section, there are four sub-discussions, namely how the characteristics of the value of stock transactions in the four provinces, how economic growth in Indonesia is seen from the modeling of the value of stock transactions using the One-Level Bayesian and Hierarchical Bayesian methods and the best modeling of all methods used.

7.1. Characteristics of stock transactions in each region

One indicator to see fluctuations and economic growth rates in each province is the activity of stock transactions in the area. Table 1 shows the highest and the lowest share transaction value is in East Java province. These conditions indicate that the East Java province varies greatly in the value of its stock transaction, it can also be proven from the highest standard deviation value. However, the highest average value in the stock transaction data is in West Java province.

Table 1. Descriptive statistics

Statistics	Bali	West Java	Central Java	East Java
Mean	1,741.74	6,989.24	2,816.01	6,255.65

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Standard Deviation	4,117.57	12,709.90	6,656.57	21,401.36
Minimum	133.00	141.09	134.06	31.62
Maximum	12,689.12	48,005.16	30,017.23	119,130.36

7.2. Growth Modeling using One-Level Bayesian Methods

The parameter estimation of a one-level Bayesian model involves 7 predictor variables, where the estimation results show that it is compatible with the nature of MCMC, namely irreducible, aperiodic, and recurrent. In the one-level Bayesian modeling, the modeling comparison is done by following 3 distribution patterns, namely 2-parameter Log-Normal, 3-parameter Log-Normal, and Log-Logistic distributions. The following is a comparison coefficient value of the three distribution models where the priors used are the same using pseudo and conjugate priors [13].

Table 2. Summary of Regression Parameters Coefficient of one-level Bayesian model

Coefficient	Distribution		
	2-parameter Log-Normal	3-parameter Log-Normal	3-parameter Log-Logistic
β_0	-56,730.00	-56,730.00	-56,730.00
β_1	1.27*	1.28*	-0.11*
β_2	-80.01	-80.00	-80.00
β_3	801.90	801.90	802.00
β_4	-58.01	-58.01	-57.96
β_5	269.00	269.00	269.00
β_6	0.10*	0.10*	0.05*
β_7	-0.01*	-0.012*	0.03*

Note : * the parameter estimates were not significant at $\alpha = 5\%$

Table 2 is the coefficient of modeling stock transactions with economic and population variables using one-level Bayesian. β_0 is the intercept value while β_1 through β_7 is a regression coefficient which states the change in the value of the micro variable to the value of the stock transaction. This table demonstrates that using one-level Bayesian modeling, the population density (X_1), the number of large and medium industrial companies (X_6), and the number of large and medium industrial workers (X_7) are not significant to the three models approaches. The other four variables i.e. the population growth rate (X_2), human development index (X_3), labor force participation rate (X_4), open unemployment rate (X_5), on the other hand, have significant influence to the transaction value of shares in the four provinces. Comparison of the three models shows that the 3-parameter Log-Logistic distribution is the best model on one-level Bayesian modeling of the value of the stock transactions with the smallest of the DIC value of 3,266.20. The 3-parameter Log-Normal distribution is the second with the DIC value is 3,487.77, and the last rank is the 2-parameter Log-Normal distribution response approach with the DIC value is 3,488.86. The smallest DIC value shows the modeling level of the best modeling so that in Bayesian modeling one-level of 3-parameter log-logistic is chosen as modeling with the following model.

$$Y = -56730 - 80X_2 + 802X_3 - 57.96X_4 + 269X_5$$

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7.3. Economic Growth Modeling with Hierarchical Bayesian Methods

The hierarchical model is built from two sub-models, namely first level (micro-model) and second level (macro-model). By using 7 predictor variables as district characteristics, 32 coefficients of micro-level regression estimation will be obtained. While at level two there are 3 variables at the provincial level so that we will get 21 macro-level regression coefficients. In Bayesian hierarchical modeling, a combination of pseudo prior, informative prior, and conjugate prior will be employed [13, 8]. The following are the results of estimating the hierarchical modeling with the same prior used for three distributions approaches in the one-level Bayesian regression modeling.

Table 3. Coefficient of the Hierarchical Model with 2-parameter Log-Normal Distribution

Province	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
Bali	6.96*	1.11*	0.49	-0.17	-0.11	0.06	-0.21	0.65
West Java	6.68*	0.06	0.56*	0.41	-0.54	-0.22	0.13	0.72
Central Java	6.97*	0.79*	0.01	0.18	-0.32	-0.26	-0.25	0.87*
East Java	6.73*	0.85	-0.13	0.33	-0.36	0.80	-0.21	0.77

Table 4. Coefficient of the Hierarchical Model with 3-parameter Log-Normal Distribution

Province	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
Bali	6.96*	1.12*	0.48	-0.17	-0.12	0.06	-0.22	0.66
West Java	6.67*	0.06	0.56*	0.41	-0.54	-0.23	0.13	0.72
Central Java	6.98*	0.79*	0.01	0.18	-0.32	-0.26	-0.24	0.87*
East Java	6.73*	0.85	-0.13	0.34	-0.36	0.79	-0.20	0.76

Table 5. Coefficient of the Hierarchical Model with 3-Parameter Log-Logistic Distribution

Province	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
Bali	6.90*	1.13*	0.50	-0.20	-0.11	0.05	-0.24	0.63
West Java	6.64*	0.06	0.55*	0.40	-0.48	-0.15	0.13	0.72
Central Java	6.87*	0.76*	-0.01	0.27	-0.38	-0.25	-0.25	0.87*
East Java	6.59*	0.84	-0.10	0.43	-0.33	0.67	-0.20	0.77

* the parameter estimates were significant at $\alpha = 5\%$

Table 3 shows the first level hierarchy modeling of the 2-parameter Log-Normal distribution, Table 4 for the 3-parameter Log-Normal distribution, and Table 5 for the 3-parameter Log-Logistic distribution response approaches. By employing the credible interval built using the highest posterior distribution (HPD), the significant parameter can be determined by testing if the zero value is not in the credible interval [16]. All three tables provide similar modeling results, where significant model parameters are in the same position. These parameters are all of β_0, β_1 in the provinces of Bali and Central Java, and β_7 in central java. The equation from the modeling can be seen in Table 6, 7, and 8.

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Table 6. Equation Modeling using 2-parameter Log-Normal

Province	2-parameter Log-Normal
Bali	$6.96 + 1.11X_1$
West Java	$6.68 + 0.56X_2$
Central Java	$6.97 + 0.79X_1 + 0.87X_7$

Table 7. Equation Modeling using 3-parameter Log-Normal

Province	3-parameter Log-Normal
Bali	$6.96 + 1.12X_1$
West Java	$6.67 + 0.56X_2$
Central Java	$6.98 + 0.79X_1 + 0.87X_7$

Table 8. Equation Modeling using 3-parameter Log-Logistic

Province	3-parameter Log-Logistic
Bali	$6.90 + 1.13X_1$
West Java	$6.64 + 0.55X_2$
Central Java	$6.87 + 0.76X_1 + 0.87X_7$

The significance of all intercept coefficients means that there is a difference in the influence of the provincial level, even small, with the level of the share transaction value. Based on the significant parameter values of Bali Province, the value of share transactions is influenced by variable X_1, West Java Province is influenced by X_2, Central Java Province is influenced by X_2 and X_7. Whereas in East Java province is not influenced by these seven variables.

Because there are differences in share transactions at the provincial level, the hierarchical modeling can be done at level two to determine the effect of these differences. Table 9 shows the regression coefficient of the macro-level model. These estimated coefficients are significant on $\alpha = 5\%$. They explain that the Inflation (W_1) represented by γ_1, Rupiah and foreign currency loan positions given by Commercial Banks and BPRs according to the Provincial project location (W_2) shown by γ_2, and Rupiah and foreign currency community deposit positions of commercial banks and BPRs by Province (W_3) given as γ_3, are affecting the coefficient value of the micro-level regression model or district characteristics.

Table 9. The regression coefficient of the macro-level model

Parameter	γ_1	γ_2	γ_3
β_1	64	1.819	-1.722
β_2	64	1.820	-1.722
β_3	64	1.819	-1.722
β_4	64	1.819	-1.722
β_5	64	1.819	-1.722
β_6	64	1.819	-1.722
β_7	64	1.819	-1.721

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7.4. Methods Model Selection

The next step is to compare all the modeling that has been done using one-level Bayesian regression and Bayesian Hierarchy in all distributions, namely 2-parameter Log-Normal, 3-parameter Log-Normal, and 3-parameter Log-Logistic distribution. The comparison of the model is seen from the DIC value. The smaller the DIC value indicates that the model is getting better. Table 10 summarizes the DIC values in the comparison of all methods that have been used. The table shows that in Hierarchical Bayesian modeling have smaller DIC values than those using One-level Bayesian modeling.

Table 10. DIC of the three compared models

Model	2-parameter Log-Normal	3-parameter Log-Normal	3-parameter Log-Logistic
One-level Bayesian	3,488.86	3,487.77	3,266.20
Hierarchical Bayesian	1,524.83	1,524.80	1,526.77

In Hierarchical Bayesian modeling using the 2-parameter Log-Normal distribution the DIC value is 1,524.83 while using the 3-parameter Log-Normal the DIC value is 1,524.8 and using the 3-parameter Log- Logistic distribution, the DIC value is 1,526.77. Modeling with the 3-parameter Log-Normal has the smallest DIC value, it shows that the modeling with the distribution pattern is the best model in Hierarchical Bayesian methods. In One-level Bayesian modeling using the 2-parameter Log-Normal distribution the DIC value is 3,488.86 while using the 3-parameter Log-Normal the DIC value is 3,487.77and using the Log-Logistic distribution the DIC value is 3,266.20. Modeling with the 3-parameter Log-Logistic has the smallest DIC value, it shows that the modeling with the distribution pattern is the best model in One-level Bayesian methods.

8. Conclusion

From the results of the analysis conducted, it can be concluded that; in the one-level modeling the best model is obtained with 3-parameter Log-Logistic distribution pattern where the significant variables are the population growth rate (X_2), the human development index (X_3), the labor force participation rate (X_4), the level of openness (X_5). Whereas in Bayesian hierarchical modeling gets the best model by following 3-parameter Log-Normal in the province of Bali, the significant variable is (X_1), Central Java province (X_2) variable, and in West Java province are (X_1) and (X_7) variables. When done, the best model for each method is modeling the Bayesian Hierarchy is the best.

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