

Modeling Indonesian Crude Price Using Markov Switching Generalized Autoregressive Conditional Heteroscedasticity

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Abstract

*Indonesian Crude Price (ICP) is one of the important factors in determining the state budget. If ICP prediction is far from its realization value, it will give a risk to state revenue. The risks given by ICP predictions can be minimized by learning the characteristics and patterns of ICP data. Like other financial time series data, the crude oil price has a nonlinear pattern and fairly high volatility so that it has a nonconstant variance (heteroscedasticity). Moreover, ICP data also have a structural change. Previous studies showed that Generalized Autoregressive Conditional Heteroscedasticity (GARCH) can handle the heteroscedasticity property in the data. However, GARCH can not capture the structural change. Markov Switching is an alternative of time series data modeling having structural change. In Markov Switching, structural change is considered as a random event. In this study, the combination of GARCH and Markov Switching called Markov Switching GARCH (MS-GARCH) was applied to model Indonesian Crude Oil Price and compare it to the single regime GARCH model. Both of volatility models are obtained under normal and Student's *t* distribution. The best model is chosen based on the smallest Akaike Information Criterion (AIC). According to AIC values, Markov Switching GARCH gives better performance than single regime GARCH and Student's *t* distribution beat the normal distribution.*

Keywords: AIC, GARCH, Indonesian Crude Price, Markov Switching, Time Series Data.

1. Introduction

Oil prices are one of the macroeconomic indicators. The main cause of macroeconomic fluctuations is oil shocks. Hence, oil price volatility prediction is considered a critical research problem [1,2,3]. In Indonesia, oil prices are one source of uncertainty in the preparation of the State Budget (APBN). The reference used for the assumption of oil prices is the price of Indonesian Crude Price (ICP). ICP is one of the macroeconomic indicators, so the amount of ICP is very influential on several important posts in the APBN such as oil and gas revenue, energy subsidies, and Oil and Gas Revenue Sharing Funds. Calculation of the correct ICP assumptions which is close to its realization value is important in the preparation of the APBN. A big difference between ICP predictions and their realization can pose a big risk. If the ICP prediction exceeds the actual value, then it can pose a risk of decreasing state revenues, whereas if the ICP prediction is lower than the actual value then it will be at risk of an increase in state spending. As a result, the government's planned programs that were previously planned were not achieved and affected the development of the national economy.

Like the other financial data, Indonesian Crude Price (ICP) has a nonlinear pattern and fairly high volatility so that it has nonconstant variance (heteroscedasticity). Generalized Autoregressive Conditional Heteroscedasticity (GARCH) can be applied to modeling data that have a nonstationary process, especially in variance or namely heteroscedasticity [4]. But, the GARCH model can not capture the structural change in the data. Parameter

estimates of the GARCH model can be biased by structural breaks in the volatility dynamics that imply poor prediction [5]. To capture the structural changes in the data, the regime-switching process introduced the switching GARCH model [6]. In the Markov switching process, structural changes are considered a random event and assume that the state or regime occurred in the past can be repeated in the future. Between states are connected by a variable called a transition matrix. This matrix shows the probability of a state, whether a state will stay in the same state or transition to another state. In the previous research, Markov switching was applied for modeling the volatility of precious metals [7] and for modeling the West Texas Intermediate (WTI) crude oil market [8].

This paper uses monthly average ICP which are obtained from Directorate General of Oil and Gas, Ministry of Energy and Mineral Resources (ESDM), Republic of Indonesia's website from January 1989 to December 2019 which consists of 372 observations. The first step is identifying the data characteristics and does several statistical tests. The next step is fitting the model. Then, a combination of GARCH and Markov Switching called Markov Switching GARCH (MS-GARCH) is proposed for modeling ICP. The proposed model is expected to capture the change of high volatility and low volatility regime. MS-GARCH will be compared to a single regime GARCH model. In a single regime GARCH, there is no regime change or assume that there is only one regime. All of the models are formed under normal and Student's t distribution. The goodness of fit criteria for choosing the best model is the Akaike Information Criterion (AIC). The model with the smallest AIC is the best.

2. Methodology

This section contains theories related to the proposed method and contains two sub-sections. The first sub-section discusses Markov switching Generalized Autoregressive Conditional Heteroscedasticity. While the second subsection contains the estimation method used in this study.

2.1. Markov Switching Generalized Autoregressive Conditional Heteroscedasticity

Let $y_t \in R$ is the crude oil price at time t . It assumes that y_t has zero mean and not serially correlated. In this study just allow regime-switching in the conditional variance process. Markov Switching GARCH can be specified as follows [5]:

$$y_t | (s_t = k, I_{t-1}) \sim D(0, h_{k,t}) \quad (1)$$

where $D(0, h_{k,t})$ is a continuous distribution with zero mean and time-varying conditional variance $h_{k,t}$ in regime k . The state variable s_t is defined on discrete space $1, 2, \dots, K$ evolve according to a first-order ergodic homogenous Markov Chain of K finite number of states with transition probability matrix $\mathbf{P} = \{p_{i,j}\}_{i,j=1}^k$ where $p_{i,j} = P[s_t = j | s_{t-1} = i]$. Moreover I_{t-1} represents the information set available up to $t-1$. This study assumes two regimes, $K = 2$.

Conditional variance y_t is assumed to follow a GARCH type model [9]. Hence, conditionally on the regime $s_t = k$, $h_{k,t}$ is available as a function of the past observation (y_{t-1}) and past variance $h_{k,t-1}$ as follows:

$$h_{k,t} = h(y_{t-1}, h_{k,t-1}) \quad (2)$$

Two types of GARCH used in this study, namely ARCH(1) or GARCH(1,0) and GARCH(1,1). The ARCH model was introduced by [10].

$$h_{k,t} = \alpha_{0,k} + \alpha_{1,k} y_{t-1}^2 \quad (3)$$

for $k=1,2$. In this case $\theta_k = (\alpha_{0,k}, \alpha_{1,k})^T$. To ensure the positivity it constrained by $\alpha_{0,k} > 0$ and $\alpha_{1,k} \geq 0$. Covariance stationarity in each regime is obtained by requiring that $\alpha_{1,k} < 1$.

GARCH model was introduced by [4] is given by:

$$h_{k,t} = \alpha_{0,k} + \alpha_{1,k} y_{t-1}^2 + \beta_{1,k} h_{k,t-1} \quad (4)$$

for $k=1,2$. Now θ_k changes to $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \beta_{1,k})^T$. To ensure the positivity it constrained by $\alpha_{0,k} > 0$, $\alpha_{1,k} \geq 0$, and $\beta_{1,k} \geq 0$. Covariance stationarity in each regime is obtained by requiring that $\alpha_{1,k} + \beta_{1,k} < 1$.

2.2. Model Estimation

The conditional distribution in this study is the normal distribution and Student's t distribution. Parameter estimation can be done by Maximum Likelihood Estimation (MLE) [5]. Let $\Theta = (\theta_1, \theta_2, \mathbf{P})^T$ is the vector model parameters. Thus, the likelihood function can be written as:

$$L(\Theta | I_T) = \prod_{t=1}^T f(y_t | \Theta, I_{t-1}) \quad (5)$$

where $f(y_t | \Theta, I_{t-1})$ is the density of y_t given past observation I_{t-1} and model parameters ψ . For MS-GARCH, the conditional density of y_t is:

$$f(y_t | \Theta, I_{t-1}) = \sum_{i=1}^2 \sum_{j=1}^2 p_{i,j} \eta_{i,t-1} f_N(y_t | s_t = j, \Theta, I_{t-1}) \quad (6)$$

where $\eta_{i,t-1} = P(s_{t-1} = i | \Theta, I_{t-1})$ represent the filtered probability of state i at time $t-1$ obtained via Hamilton's filter [11]. The estimator $\hat{\Theta}$ is obtained by maximizing the logarithm of equation (5).

3. Result and Discussion

The objective of this study is to get the best model of Indonesian Crude Oil Price (ICP) volatility which has non-stationary property (heteroscedasticity) and structural changes. In this section, there is two part of the discussions. First, we do descriptive statistics to identify the characteristics of the data. The second part is a fitting model for the data.

3.1. Descriptive Statistics

The pattern of movement in Indonesian crude oil prices fluctuates considerably as shown in Figure 1. In the period of January 1989 to December 1998 ICP was still stable with an average of around 17.29 US\$ per barrel, except during the Gulf War from August 1990 to February 1991. The Gulf War caused a decline in oil production in a short time so that oil prices experienced a sharp increase. In September 1990 ICP rose to 27.63 US\$ per barrel and reached a peak of 34.88 US\$ per barrel in October 1990 until then slowly fell and stabilized again after the Gulf War was declared over.

ICP also tends to increase in certain periods, namely in the period of January 1999 to July 2008, the period of January 2009 to June 2014, and the period of February 2015 to May 2018. In addition, there were certain periods where the ICP experienced a significant decrease from August 2008 to December 2008. This decline occurred due to the financial crisis in the US. A sharp decline also occurred in the period from July 2014 to January 2015 caused by riots in the Middle East. Thus, it can be seen that ICP shows high uncertainty over time based on the historical review of oil prices previously described.



Figure 1. Time Series Plot of ICP

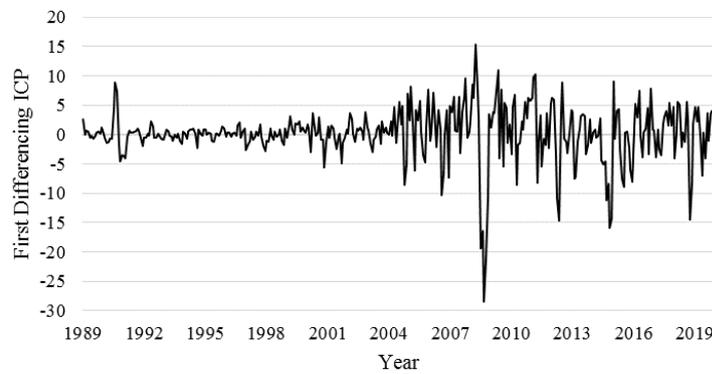


Figure 2. Time Series Plot of First Differencing ICP

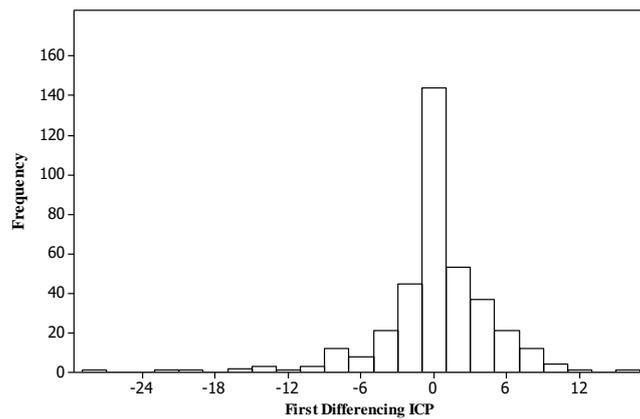


Figure 3. Histogram of First Differencing ICP

ICP data in Figure 1 shows the non-stationary pattern and the Dickey-Fuller test said not to reject the null hypothesis. It means that ICP data is not stationary in a mean. Differencing on lag 1 is needed to fulfill the stationarity assumption. So, the differencing ICP is used as the new series on the next step of the analysis. The pattern of oil price

fluctuation (ICP) can be seen through the ICP first differencing plot in Figure 2 and the distribution pattern can be seen through the histogram in Figure 3.

The histogram of the first differencing ICP shows the leptokurtic and fat tail. It has the same result with the descriptive analysis in Table 1 that the ICP series has significant kurtosis and longer left tails and fatter tails than the normal distribution, also Jarque-Bera test rejects the normality. The Dickey-Fuller test is significant, indicating that the new series is stationary. Ljung-Box test in the 10th and 20th order is rejecting the null hypothesis of no autocorrelation. ARCH(1) test indicates that there is an ARCH effect or heteroscedasticity in the ICP series. It is shown that the GARCH type model is suitable for modeling the ICP data.

Table 1. Descriptive Statistics of ICP

Statistics	Value	Statistics	Value
Mean	0.14	Jarque-Bera	87.89
Median	0.32	Dickey-Fuller	-7.81
Min	-28.40	Ljung-Box (10)	99.61
Max	15.36	Ljung-Box (20)	111.21
Std	4.58	ARCH-LM(1)	133.33
Skewness	-1.51		
Kurtosis	6.99		

3.2. ICP Volatility Modelling

This dynamic of data volatility is more interesting to be studied. To analyze the dynamics of volatility, then the autoregressive effect in the data is removed by using AR(1) filter, and the model is obtained on the residuals. In the modeling of ICP, first, we apply a single regime GARCH with normal (-N) and Student's t (-t) distribution. The results of the GARCH model are presented in Table 2. The value of α_1 in ARCH model and the sum of α_1 and β_1 in the GARCH model is less than one. That finding means that all the models fulfilled the variance stationarity. Almost all the parameters for conditional variance are significant at 5% significant level, except α_0 in the GARCH-t model. The parameter of a fat tail (ν_1) in Student's t distribution is significant. It means that the fatness of the tail can not capture under a normal distribution.

Table 2. Parameter Estimation and AIC value of ICP by using GARCH

Parameter	ARCH-N	GARCH-N	ARCH-t	GARCH-t
α_0	5.4384*	0.2637*	5.8816*	0.0637*
α_1	0.9992*	0.3129*	0.9998*	0.1633*
β_1	-	0.6640*	-	0.8215*
ν_1	-	-	3.3245*	6.3960*

Note: * indicate significance at 5%.

The second model used in this study is the MS-GARCH model. Based on Table 3, almost all of the ARCH and GARCH parameters are significant. All constant values in the MS-ARCH model are significant, but all constant values in the MS-GARCH model are not significant except MS-GARCH-N for Regime 2. From Under Student's t distribution, parameters of fat-tailness are significant in both Regime 1 and Regime 2. The value of

volatility parameter (α_{0k}) in Regime 1 and Regime 2 show that there are different behaviors. Regime 1 has a lower value than Regime 2. This condition indicates that Regime 1 shows low volatility while Regime 2 shows high volatility.

The transition probabilities of both regimes are significant. Besides, the value of transition probabilities is closed to one meaning that a high probability a state will stay in the same state or infrequently transition to another state.

Table 3. Parameter Estimation and AIC value of ICP by using MS-GARCH

Parameter	MS-ARCH-N	MS-GARCH-N	MS-ARCH-t	MS-GARCH-t
α_{01}	0.9278*	0.0000*	1.5561*	0.0000*
α_{11}	0.3344*	0.1276*	0.5705*	0.0070*
β_{11}	-	0.8723*	-	0.9929*
ν_{11}	-	-	3.4371*	7.4614*
α_{02}	19.1749*	9.4812*	27.4667*	5.7036*
α_{12}	0.4253*	0.4074*	0.1276*	0.2055*
β_{12}	-	0.3477*	9.2221*	0.6302*
ν_{12}	-	-	-	20.1834*
p_{11}	0.9751*	0.9878*	0.9971*	0.9824*
p_{22}	0.9804*	0.9864*	0.9971*	0.9740*

Note: * indicate significance at 5%.

After fitting all of the alternative models and obtained parameters, the next step is choosing the best model using the AIC value presented in Table 4. As can be seen from the rank of AIC in Table 4, we know that the MS-GARCH model gives better performance than a single regime GARCH model. On the other hand, the Student's t distribution outperforms normal distribution. According to the highest AIC, the best model is MS-GARCH-t.

Table 4. Comparison of Alternative Models

Model	AIC	Rank
ARCH-N	2000.0484	8
GARCH-N	1875.1746	6
ARCH-t	1955.1557	7
GARCH-t	1838.4753	3
MS-ARCH-N	1857.6776	4
MS-GARCH-N	1859.1361	5
MS-ARCH-t	1828.4833	2
MS-GARCH-t	1826.0948	1

From the parameter estimate of MS-GARCH-t in Table 2, it can be seen that Regime 1 shows the low volatility and Regime 2 is high volatility. The transition from one regime to another is connected by a transition probability matrix in equation (7).

$$\mathbf{P} = \begin{bmatrix} 0.9824 & 0.0158 \\ 0.0260 & 0.9740 \end{bmatrix} \tag{7}$$

The transition probability exhibited that how precious crude price dynamics move across the regimes. The probabilities of transiting from one state (regime) to others are

very low, but the probabilities of remaining in the same state are very high. The probability of Regime 1 remaining in the same state is 0.9824 and the transition to Regime 2 is 0.0158. For Regime 2, the probability to stay in the same state is 0.9740 and changes to other regime is 0.0260. This condition means that after changing to a regime, the process would stay longer in that regime rather than changing to another state. The probability of a state stays in a certain state can be seen through smoothed probability in Figure 4.

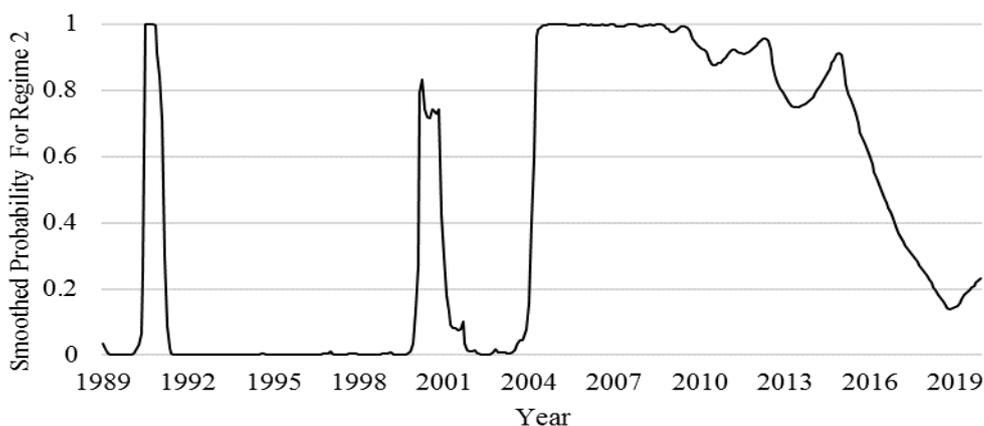


Figure 4. Smoothed Probability for High Volatility (Regime 2)

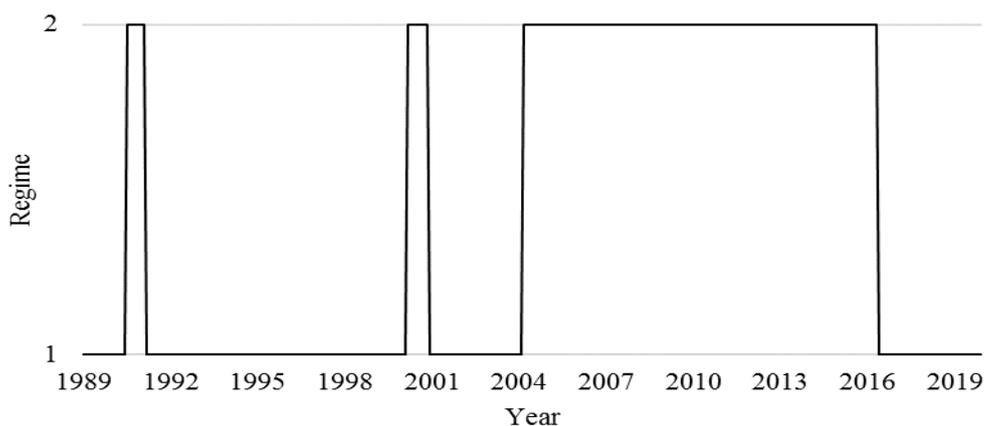


Figure 5. State for ICP

Figure 4 presents the smoothed probability for regime two means probabilities of a state stay in Regime 2 at the time t . The high probabilities occurred throughout 1991, from 2001 until 2003, from 2005 until 2017, and become decreased from 2017 until 2019. After descend from 2017 to 2019, in the middle of 2019 the graph looks ascend. This condition indicates that in the next period, the state may be changing from low volatility to high volatility. This might happen as mentioned before, the crude oil price has high uncertainties and are greatly influenced by economic and geopolitical conditions. Smoothed probabilities in Figure 3 also can be realized as a state variable in Figure 5. Thus, we can see clearly when the high volatility and low volatility occurred.

4. Conclusion

The characteristics of ICP volatility data indicate a non-stationary process in variance or heteroscedasticity. Besides, there are structural changes in the data pattern that indicates two types of volatility in ICP data, which are high and low volatility. MS-GARCH demonstrates a better model for representing the ICP volatility data by giving smaller AIC than the single regime GARCH. Besides, Student's t distribution also suitable for fat-tailness than a normal distribution. Among alternative models, Markov Switching GARCH under Student's t distribution (MS-GARCH-t) is the best model for modeling ICP volatility. Thus, from the results of this research, it was found that the appropriate distribution has a significant effect on the results. Other important contributions, such as the type of return distribution, the regime-switching properties from volatility are also important features to consider. These facts can be considered as useful knowledge in determining accurate volatility models in the financial problem or risk management.

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