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Equitable Edge Coloring of Some Classes Of Graphs

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Abstract

Equitable Coloring was introduced by Meyerin 1973. Many authors discussed about the equitable coloring. Then the concept of equitable edge coloring was introducedby Hilton and De. Werra in 1994.In our work, we discuss about the equitable edge coloring of some classes of simple graphs

Keywords*: *Proper edge coloring, Equitable edge coloring, Banana Tree, Flower Graph, Jewel Graph, Sunlet Graph, Star Graph, Helm Graph.

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1. INTRODUCTION

Graph Theory is the study of relation between the discrete objects. The term Graph simply means the two non-empty sets namely Vertex set V and Edge set E .Here V means the set of all objects called vertices and E means the set of all unordered pairs of vertices. Also G is a simple finite connected graph. The concept of graph coloring came from the idea of map or region coloring [1]. Here we conclude this section by going to state the Basic and preliminary concepts necessary need for our main results. In the next section we are going to state the equitable edge colorable graphs.

2. Preliminaries

Definition 2.1.1: Proper Edge Coloring: [7]

An Edge Coloring of a graph G is an assignment of colors to the edges of G. An Edge Coloring of G is called Proper K edge Coloring if no two adjacent edges have assigned same color.

Definition 2.1.2: Equitable Edge Coloring [5]

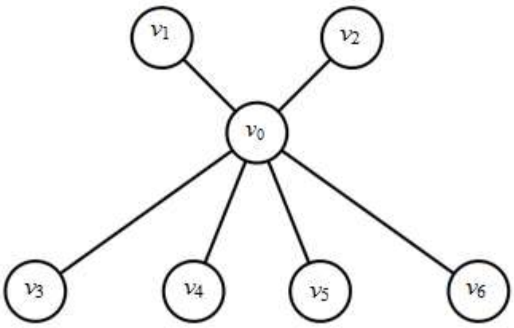
A Proper Edge Coloring of G is called Equitable if $|n(s) - n(t)| \leq 1$ .where n(s) and n (t) denote the number of edges in the color classes s and t respectively.

Definition 2.1.3: Approximately Equitable Edge Coloring [5]

A Proper Edge Coloring of G is calledapproximately Equitable if
$|n(s) - n(t)| \leq 2$wheren(s) and n (t) denote the number of edges in the color classes s and t respectively.

Definition 2.1.4: Star Graph[1]:

A Star Graph is a graph on k+1 vertices designated by $K_{1,k}$ is obtained By joining the k vertices to a single vertex called central vertex.



Qae= Diagram 1.1.1 Star Graph $K_{1,6}$

Definition 2.1.5: Banana tree: [6]

A (n, k) - banana tree is defined as a graph obtained by one leaf of each of the n-copies of an k-star graph with a single root vertex that is distinct from all the stars.
Example :

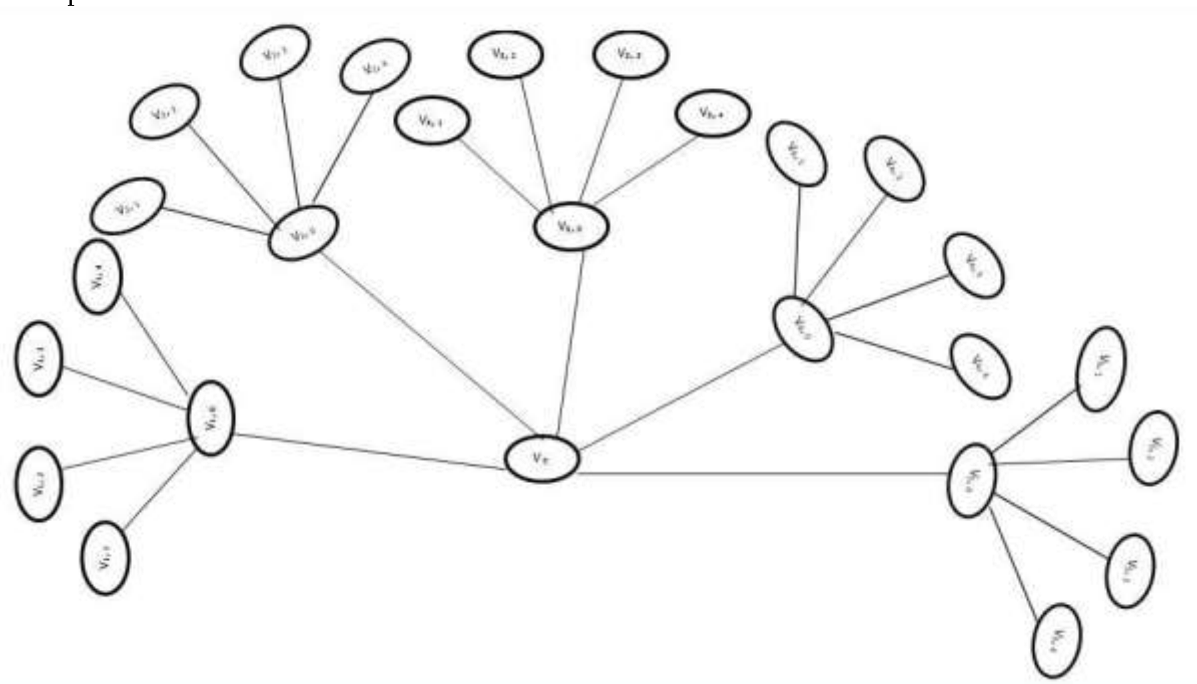


Diagram 2.1. 2 Banana Tree $B_{5,5}$

Definition 2.1.6: Flower Graph Fl_n[2]

A flower graph denoted by Fl_n is defined as a graph obtained from the Helm graph by joining each pendant vertex to the central vertex of the helm.

Example

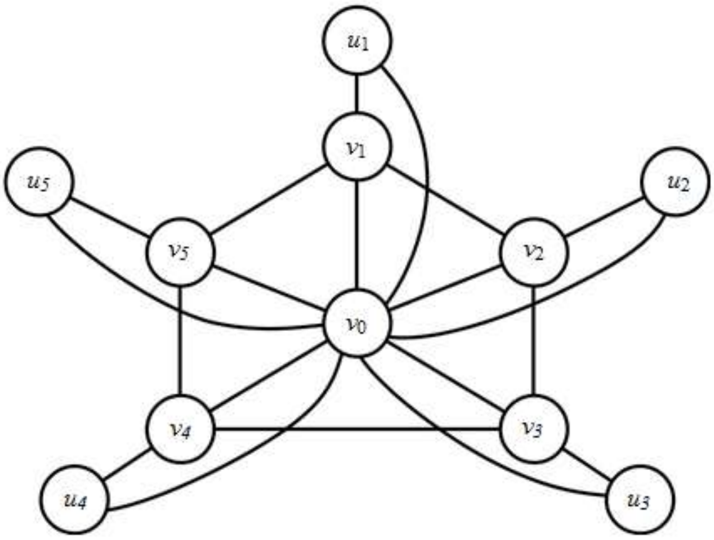


Diagram 1.1.3 Flower graph: Fl_5

Definition 1.1.7Jewel Graph:[3]

The Jewel graph designated by $J_n$is defined as a graph with vertex set V (J_n) = {u,x,v, y,v_i;$1 \leq i \leq n\}$ and the edge set E($J_n) =$ { $ux, vx, uy, vy, xy, uv_i, vv_i$; 1 ≤ i ≤ n}

Example :

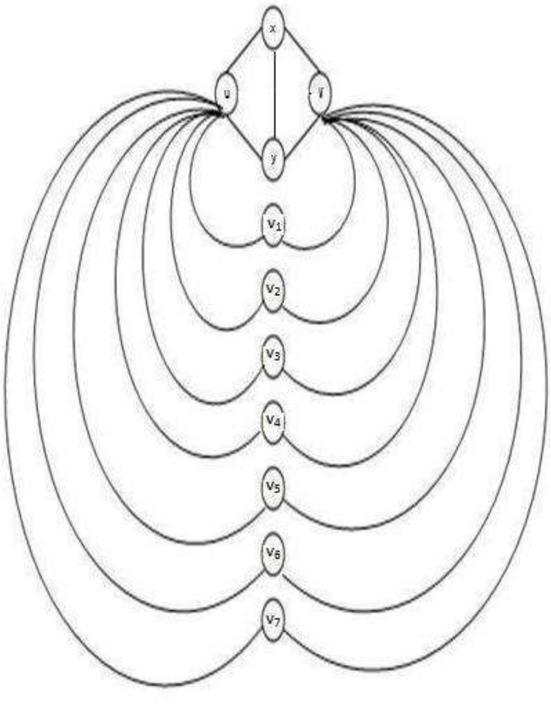


Figure : 1. Jewel Graph When n = 7

Diagram 1.1.4 : Jewel Graph J_7

Definition 1.1.8: Sunletgraph S_n[9]:

The Sunlet graph S_n is defined as the graph obtained from the cycle C_n by adjoining the pendant edge to each vertex of the cycle C_n.

Example :

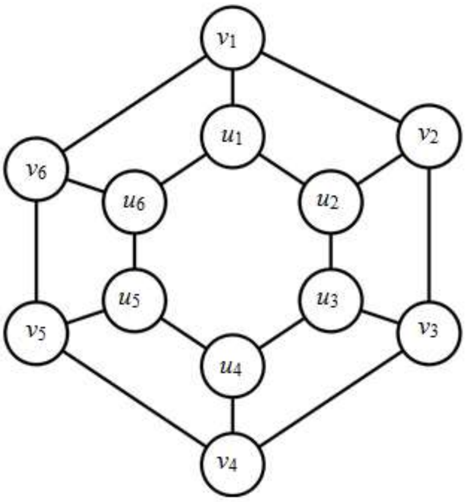


Diagram 1.1.5 Sunlet graph S_6

Definition 1.1.9 Helm Graph H_{n+1} [9]:

The Helm Graph H_{n+1} is defined as the graph obtained from the wheel graph by adjoining a pendant edge to each vertex of the n-cycle in W_{n+1} .

Example : H_7

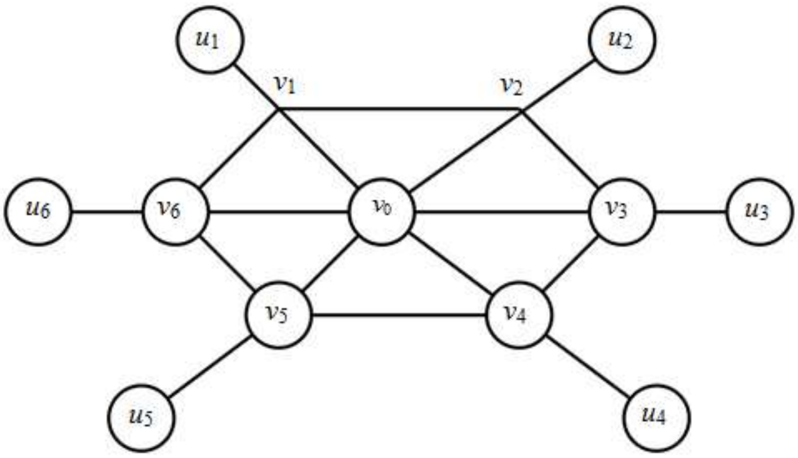


Diagram 1.1.6 Helm Graph H_7

Lemma 1.1 [VIZING'S THEOREM] [5]& [7]

Every simple undirected graph G may be Edge colored using a number of colors that is at most one larger than the highest degree Δ of the graph G.

3. MAIN RESULTS

Theorem 3.1:

The flower graph admits equitable edge coloring and its Equitable edge chromatic number is χ'_e(G) =2n.

Proof: By Lemma1.1, we require at least 2n colors for proper edge coloring of this graph

To color the edges of this graph, let us define a function f: E (G)$\rightarrow${1,2,3,…Δ} by

$$f(uv)=\begin{cases} 2i-1 \ for\ u=v_0 \ \ and\ v=v_i, i=1,2,3..n \\ 2i \ for\ u=v_0 \ \ and\ v=w_i, i-1.2,3 \dots n \\ 2i-3 \ for\ u=v_i \ and\ \ v=\ w_i, i=2,3 \dots n \\ 1\ for\ u=v_1 \ and\ \ v=\ w_1 \\ 2i\ for\ u=\ v_i\ and\ \ v=\ v_{i+1}, i=1,2,3 \dots n-1 \\ 2n\ for\ u=v_n\ and\ \ v=\ w_n \end{cases}$$

Hence from this mapping $|n(s)-n(t) \le 1|$ for all s ,t$\in$ $\{1,2,3,\dots \Delta\}$ and it has been proved that the flower graph admits the equitable edge coloring whose equitable edge chromatic number is χ'_e(G)=2n.

Theorem 3.2:

The Jewel graph admits the equitable edge coloring whose equitable edge chromatic number is n+2

Proof:

By Lemma1.1, werequire at least n+2 colors for proper edge coloring of this graph

To color the edges of this graph, let us define a function f: E(G)$\rightarrow${1,2,3,…Δ} by

$$f(UV)=\begin{cases} 1\ for\ U=u\ and\ V=x \\ 1\ for\ U=v\ \ and\ V=y \\ 2\ for\ U=u\ and\ \ V=\ y \\ 2\ for\ U=v\ and\ \ V=\ x \\ n+2\ for\ U=\ x\ and\ \ V=\ y \\ i+3\ for\ U=u\ and\ V=v_i, i=1,2,3,..n-1 \\ i+3\ mod\ n\ \ for\ U=u\ and\ V=v_n\ \ and\ i+3>n \\ i+2\ for\ U=v\ and\ V=\ v_i, i=1,2,3,\dots n \end{cases}$$

Hence from this mapping $|n(s)-n(t) \le 1|$ for all s, t$\in$ $\{1,2,3,\dots \Delta= n+2\}$ and it has been proved that the Jewel graph admits the equitable edge coloring whose equitable edge chromatic number is χ'_e(G) =n+2.

Theorem 3.3

The Banana tree $B_{n,k}$ admits the equitable edge coloring whose equitable edge chromatic number is $\chi'_e\ (B_{n,k}) = \Delta$ where $\Delta=\begin{cases} n\ if\ n \ge k \\ k\ if\ n<k \end{cases}$.

Proof:

In the Banana tree $B_{n,k}$ there are n star graphs attached to a single vertex say central vertex v_0 .In each star graph there are k leaves attached to a single vertex. Let the leaves of the each of the n star graphs attached to the n points designated by $v_{1,0}, v_{2,0}, v_{3,0}, v_{4,0} \dots v_{n,0}$Let the remaining vertices of the Banana tree by$v_{1,1}, v_{1,2}, \dots v_{1,k-1,} v_{2,1}, v_{2,2}, v_{2,3} \dots v_{2,k-1} \dots v_{n,1}, v_{n,2}, \dots v_{n,k-1}$.

By Lemma 1.1, we require at least n+2 colors for proper edge coloring of this graph

Here there are two cases for proving this theorem.

Case 1: n<k

We color the edges of the Banana tree $B_{n,k}$ by defining a function f: E(G)$\rightarrow$ {1,2,3,… Δ} as

$$f(u\,v) = \begin{cases} i \; for \; u = v_0 \;\; and \; v = v_{i,0}, i = 1,2,3, \dots n \\ i + j \; for \; u = v_{i,0} and \; v = v_{i,j}, and \; i + j \le n \; i = 1,2,3, \dots n \; and \\ j = 1,2,3..k-1 \\ (i+j) mod \; k \; for \; u = v_{i,0} and \; v = \; v_{i,j}, i = 1,2,3,4 \dots n \; and \; j = 1,2,3 \dots k-1. \end{cases}$$

Hence from this mapping we obtained $|n(s) - n(t)| \le 1$ and hence the Banana tree $B_{n,k}$ is an Equitable edge colorable graph whose equitable edge chromatic number is $\chi'_e\,(B_{n,k}) = k$.

Case 2: n≥k

We color the edges of the Banana tree $B_{n,k}$ by defining a function f: E(G)$\rightarrow$ {1,2,3,… Δ} as

$$f(u\,v) = \begin{cases} i \; for \; u = v_0 \;\; and \; v = v_{i,0}, i = 1,2,3, \dots n \\ i + j \; for \; u = v_{i,0} and \; v = v_{i,j}, and \; i + j \le n \; i = 1,2,3, \dots n \; and \\ j = 1,2,3..k-1 \\ (i+j) mod \; n \; for \; u = v_{i,0} and \; v = \; v_{i,j}, i = 1,2,3,4 \dots n \; and \; j = 1,2,3 \dots k-1. \end{cases}$$

Hence from this mapping we obtained $|n(s) - n(t)| \le 1$ and hence the Banana tree $B_{n,k}$ is an Equitable edge colorable graph whose equitable edge chromatic number is $\chi'_e\,(B_{n,k}) = n$.

3.4 CONSTRUCTION ALGORITHM FOR SUNLET-STAR GRAPH:

Step 1: Draw a cycle $C_n$with n vertices say$v_0, v_1, v_2, \dots, v_{n-1}$.

Step 2: Attach the pendant edges on the vertices of the cycle C_n.Let the other end of these pendant edges be$u_0, u_1, u_2, \dots, u_{n-1}$.

Step 3: Attach the k-star graph at each of the pendant vertices u_i, i=0, 1, 2, 3…n-1 by joining the central vertex of the star graph with the pendant edge of the Sunlet graph. Thus we get a graph called Sunlet-Star Graph denoted by$S_{n,k}$.

Observations 2.4:

In the Sunlet-Star graph $S_{n,k}$, there are n (k+2) vertices, n (k+2) edges. Sum of the degree of all the vertices of $S_{n,k}$= 2n (k+2). Degree of the vertices v_i, i=0,1,2,…n-1 are equal to 3, Degree of the vertices u_i,i=0,1,2,3...n-1 are equal to k+1 and the remaining vertices are of degree 1.

Theorem 2.4 :

The Sunlet-Star Graph with admits the Equitable edge coloring if either n=k+1 or k+ 1 is a multiple of n with its Equitable edge chromatic number be $\chi'_e(S_{n,k}) = k+1$

Proof:

Let G be a Sunlet-Star graph.

Case 1: n = k+1

Here the maximum degree of G is k+1. Hence by Theorem 1.1, we need at least k+1 colors for proper edge coloring of G. To color the edges of the graph G define a function

f: E (G) → {1, 2, 3,…,k+1} as

$$f\,(u\,v) = \begin{cases} (i+3) mod \; n + 1 \; for \; u = v_i, and \; v = v_{i+1}, i = 0,1,2, \dots, n-1 \\ i + 1 \; for \; u = \; v_i, and \; v = u_i, i = 0,1,2, \dots, n-1 \\ i + j + 1 mod \; n + 1 \; for \; u = u_i \; and \; v = \; u_{i,j}, i = 0,1,2, \dots, n-1 \\ j = 1,2,3, \dots k-1 \end{cases}$$

Hence from this mapping we find that $|n(s) - n(t)| \leq 1$ for all s ,t belongs to {1,2,3….n} and hence the Sunlet graph admits the equitable edge coloring for this case. Also its equitable edge chromatic number is $\chi'_e(S_{n,k}) = n$=k+1

Case 2: k+1 is a multiple of n

Let k+1=t(n) where t is some positive integer >1.

Here the maximum degree of G is k+1. Hence by Theorem 1.1, we need at least k+1 colors for proper edge coloring of G. To color the edges of the graph G define a function f: E (G) → {1, 2, 3 , … , k+1} as

$$f(u\,v) = \begin{cases} (n+i+1) \bmod n + 1 \; for \; u = v_i, and \; v = v_{(i+1) \bmod n}, i = 0,1,2,\ldots,n-1 \\ (i+1) \; for \; u = v_i, and \; v = u_i, i = 0,1,2,\ldots,n-1 \\ (i+j+1) \bmod k + 1 \; for \; u = u_i \; and \; v = u_{i,j}, i = 0,1,2,\ldots,n-1 \\ j = 1,2,3,\ldots k-1 \end{cases}$$

Hence from this mapping we find that $|n(s) - n(t)| \leq 1$ for all s, t belongs to {1, 2, and 3….n} and hence the Sunlet graph admits the equitable edge coloring for this case. Also its equitable edge chromatic number is $\chi'_e(S_{n,k}) = k+1$.

3.5 CONSTRUCTION ALGOTITHM FOR TRIANGLE_HELM SILICATE GRAPH

Step 1: Draw a cycle C_n with n vertices say $v_1, v_2, v_3, \ldots, v_n$.

Step 2: Insert a Central vertex V_0 and join this vertex with all the vertices of the Cycle C_n.We will get the wheel graph.

Step 3: Insert a vertex inside each triangular face of the wheel graph. Let these vertices be $y_1, y_2, y_3, \ldots, y_n$.

Step 4: Jointhe vertex y_i.with the vertices in that triangular face ,i=1,2,3,…n.

Step 5: Insert 3 sets of vertices say u_i ,y_i,w_i outside the Cycle $C_n$where i=1, 2, 3,…n.

Step 6: Join the vertex $v_i$with the vertexu_i, vertex $w_i$with the vertex $x_i$and vertex $x_i$with the vertex u_i.and the vertex $w_i$with the vertex u_i.Thus we get a graph called Triangle –Helm Silicate Graph.

Observations 2.5:

In this graph there are 5n+1 vertices, 9n edges and sum of the degree of all the vertices = 18n. Degree of the Central vertex V_0 is 2n, Degree of the vertices u_i and y_i are equal to 3, Degree of the vertices x_i and w_i are equal to 2 and degree of the vertices v_i are equal to 6,where I = 1,2,3,...,n. Here the maximum degree Δ=2n. By Theorem 1.1 we need at least 2n colors for proper edge coloring of this graph.

Theorem 3.5:

The Triangle-Helm Silicate graph admits the Equitable Edge coloring and its Equitable Edge Chromatic number is $\chi'_e(THS_n)$ = 2n.

Proof :

By Observation given above this graph needed at least 2n colors for proper edge coloring of this graph.

To color the edges of this graph equitably let us define a function f: E(G)→{1,2,3,…Δ=2n} by

$$f(U\,V)=\begin{cases} 2\,i \; for \; U = v_i \; and \; V = v_{i+1}, i = 1,2,3, \dots, n-1. \\ 2\,n \; for \; U = v_n \; and \; V = v_1 \\ 2\,i \; for \; U = V_0 \; and \; V = y_i, i = 1,2,3, \dots, n. \\ 2\,i-1 \; for \; U = V_0 \; and \; V = v_i, i = 1,2,3, \dots, n \\ 2\,i+1 \; mod \; 2n \; for \; U = y_i \; and \; V = v_{i+1}, i = 1,2,3, \dots, n-1. \\ 2\,i-1 \; for \; U = y_i \; and \; V = v_{i+1}, i = 1,2,3, \dots, n-1. \\ 2\,n-1 \; for \; U = y_n \; and \; V = v_1 \\ 2\,i+4 \; mod \; 2n \; for \; U = v_i \; and \; V = u_i, i = 1,2,3, \dots, n. \\ 2\,i+4 \; mod \; 2n \; for \; U = w_i \; and \; V = x_i, i = 1,2,3, \dots, n. \\ 2\,i-1 \; for \; U = u_i \; and \; V = w_i, i = 1,2,3, \dots, n. \\ 2\,i+1 \; mod \; 2n \; for \; U = u_i \; and \; V = x_i, i = 1,2,3, \dots, n. \end{cases}$$

Therefore from the above mapping it is found that $|n(s) - n(t)| \leq 1$ for all s, t belongs to {1, 2,3….2n} and hence the Triangle – Helm silicate graph admits the equitable edge coloring for this case. Also its equitable edge chromatic number is $\chi_e'(G) = 2n$.

4. *CONCLUSION*

Hence in this work, we have discussed the equitable edge coloring of some special classes of graphs and also their equitable edge chromatic number. Also for two classes of special graphs we have discussed their construction algorithm and their obvious observations.

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