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An Analytical Approximation for unsteady MHD natural convective flow of Casson fluid over an oscillating vertical plate

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Abstract

In this article, Laplace transform and new homotopy perturbation methods are adopted to study the problem of forced convection over a horizontal flat plate analytically. In this paper, an unsteady magnetohydrodynamic natural convection, heat transfer, electrically conductive non-Newtonian Casson fluid over an oscillating vertical porous plate taken in to the account with an influence of viscous dissipation. An approximate analytical to solve non-dimensional boundary layer partial differential equations for velocity and temperature distribution with the influence of emerging dimensionless parameters. Also, the local Skin-friction and local Nusselt number coefficients are obtained and analyzed
Keywords: *Mathematical modeling; Boundary layer; Laplace transform; New homotopy perturbation method; Casson fluid; MHD flows; Viscous dissipation*

1. Introduction

The departure of many engineering problems is vulnerable to MHD analysis. In recent years, many Researchers and Engineers to apply MHD principles in designing of heat exchangers, pumps and flow meters, and power generating systems and in developing confinement schemes for controlled fusion. MHD convection problems, especially missile aerodynamics, are important in astronomy because the temperature at which those flying speeds occur is sufficient to separate or significantly ionize the air. In such a situation, MHD is very interesting from the physical point of view of the study of convection problems. Problems with mass diffusion and thermal diffusion, especially in chemical engineering, are being studied by numerous engineering disciplines for their importance

Takhar et al. [1] investigated the effects of radiation on MHD free convection flow past a semi-infinite vertical plate. The effect of radiation on free convection from a porous vertical plate was discussed by Hossain et al. [2]. Muthucumaraswamy and Kumar [3] investigated the thermal radiation effects on moving infinite vertical plate in the presence of variable temperature and mass diffusion. The joint effect of free convection and thermal radiation on MHD unsteady flow of a viscous incompressible fluid past an impulsively started vertical plate with uniform heat and mass flux was analyzed by Prasad et al. [4]. Samad and Rahman [5] obtained the similarity solution to the problem of unsteady MHD free convective flow through a porous vertical flat plate immersed in a porous medium in the presence of magnetic field with radiation. Due tothe compound behavior of the fluids, many researchers have proposed non-Newtonian fluid problems or models such as Jefrrey [6], Walter-B [7], Viscoplastic [8], Maxwell [9], Oldroyd-B [10] and Brinkman type [11] models.

There is an emerging non-Newtonian model, which is called as Casson model. Casson fluid was first introduced by Casson [12] for the prognostication of flow behavior of the pigment-oil suspension. Recently many researchers studied theCasson models; Khalid et al. [13] found the analytical solutions unsteady MHD natural convective flow of a Casson fluid on an oscillatory vertical plate with the presence of a porous medium and magnetic field with the absence of viscous dissipation.

Khalid et al. [14] found the exact solutions of an unsteady Magneto hydrodynamic natural convective flow of Casson fluid on an oscillatoryvertical plate, with the absence of porous

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medium, magnetic field and viscous dissipation. Hussanan et al. ([15,16]) studied an influence of Newtonian heating on Casson fluid past an oscillatory vertical plate by Laplace transform. Das et al. [17] studied an influence of Newtonian heating on MHD natural convective flow of heat and mass transfer over an oscillating vertical plate. Mukhopadhyay et al. [18] analyzed the heat transfer on Casson fluid flow on a symmetricwedge.

Most scientific problems such as mass transfer are nonlinear and do not have analytical solution. One of the well-known equations arising in fluid mechanics and boundary layer approach is Blasius equation [19]. The homotopy perturbation method (HPM) is established by He [20-24] in 1998 to obtain series solutions of nonlinear differential equations. Khan, et al. [25] solved the long porous slider problem by homotopy perturbation method which is coupled nonlinear ordinary differential equations resulting from the momentum equation. Esmaeilpour, D.D. Ganji [26] have used homotopy perturbation method to solve boundary layer flow and convection heat transfer over a flat plate.

Kumar [27] investigated a new approximate method, namely homotopy perturbation transform method (HPTM) which is a combination of homotopy perturbation method (HPM) and Laplace transform method (LTM) to provide an analytical approximate solution to time-fractional Cauchy-reaction diffusion equation. In this study, analytical approximation to the solution of problem of forced convection over ahorizontal flat plate using combination of Homotopy perturbation method and Laplace transform is presented.

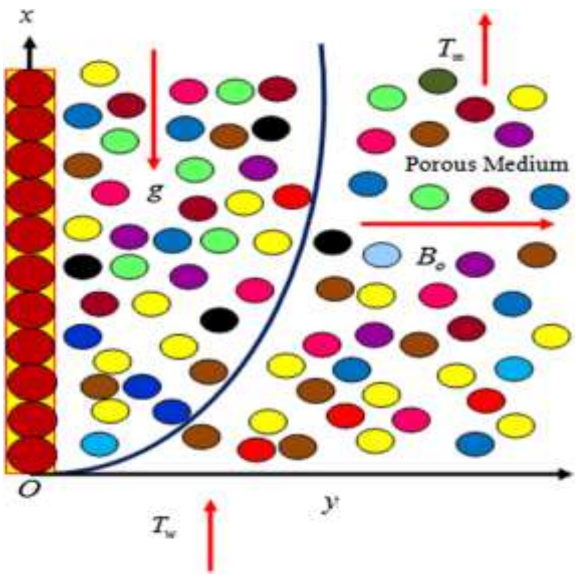

Fig. 1. Physical configuration and coordinates system[28].

2. Mathematical formulation

We consider Casson fluid over a vertical flat plate, the length of plate is infinite and embedded in a saturated porous medium. The rheological equation of state for the Cauchy stress tensor of Casson fluid [7] is written as follows:

$$\tau' = \tau_0 + \mu\gamma \quad or \quad \tau_{ij} = \begin{cases} 2\left(\mu_B + \dfrac{\Pr}{\sqrt{2\pi}}\right)e_{ij}, & \pi < \pi_c \\ 2\left(\mu_B + \dfrac{\Pr}{\sqrt{2\pi_c}}\right)e_{ij}, & \pi > \pi_c \end{cases} \quad (1)$$

where p = $e_{ij}e_{ij}$ and $e_{ij}$is the (i, j)th component of rate of the deformation.

Before we derive the governing equations, the following assumptions are made, rigid plate, incompressible flow, unsteady flow, unidirectional flow, one dimensional flow, non-Newtonian flow, free convection, oscillating vertical plate and a viscous dissipation term is considered in the energy equation. Under these conditions, we get the set of partial differential equations[8-12]. The basic equation for influence of viscous dissipation on unsteady MHD natural convective flow of Casson fluid over an oscillating vertical plate is given in ref [28]. The nonlinear coupled partial differential equations in oscillatory boundary layer flow are given below:

$$\rho\frac{\partial u'}{\partial t'}=\mu_b\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u'}{\partial y^2}-\sigma B_0^2 u'-\frac{\mu\varphi}{k_1}u'+\rho g\beta(T-T_\infty) \quad (2)$$

$$\rho C_p\frac{\partial T}{\partial t'}=K\frac{\partial^2 T}{\partial y^2}+v\left(\frac{\partial u'}{\partial y}\right)^2 \quad (3)$$

Together with the following conditions

$$t'<0:\ u'=0,\quad \text{At}\quad T=T_\infty\quad \text{for all}\ y>0$$

$$t'\geq 0:\begin{cases}u'=U\,H(t')\cos(\omega' t')\ \text{or}\ u'=U\sin(\omega' t'),\ T=T_w\ \text{at}\ y=0\\ u'\to 0,\ T\to T_\infty\ \text{as}\ y\to\infty\end{cases} \quad (4)$$

The dimensionless variables are introduced as follows:

$$u=\frac{u'}{U},\ \eta=\frac{yU}{v},\ t=\frac{U^2}{v}t',\ \theta=\frac{T-T_\infty}{T_w-T_\infty},\ \omega=\frac{\omega' v}{U^2},\ \tau=\frac{\tau'}{\rho u^2},$$

$$M^2=\frac{\sigma B_0^2 v}{\rho U^2},\ K=\frac{k_1U^2}{\varphi v^2},\ Gr=\frac{v\,g\,\beta(T_w-T_\infty)}{U^3},\ \Pr=\frac{v\rho C_p}{k},$$

$$Ec=\frac{U^2}{\rho C_p(T_w-T_\infty)},\ \gamma=\frac{\mu_B\sqrt{2\pi_c}}{P_y} \quad (5)$$

The above dimensionless variables substitute in the above Eqs.(1-3), we obtain

$$\frac{\partial u}{\partial t}=\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial\eta^2}-M^2u-\frac{1}{K}u+Gr\,\theta \quad (6)$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial\eta^2}+Ec\left(\frac{\partial u}{\partial\eta}\right)2 \quad (7)$$

where, u and θ represents, the velocity profile and temperature distributionwith the influence of emerging casson fluid flow over an oscillating vertical plate respectively.

The boundary conditions are,

$$\mathrm{t}<0:\ \mathrm{u}=0,\ \theta=0\ \text{for all}\ \eta>0 \quad (8)$$

$$\mathrm{t}\geq 0:\begin{cases}u=H(t)\cos(\omega t)\quad\text{or}\ u=\sin(\omega t)\qquad \theta=1\ \text{at}\ \eta=0\\ u\to 0,\qquad\qquad \theta\to 0\ \text{as}\ \eta\to\infty\end{cases} \quad (9)$$

Here, velocity and temperature distribution with the influence of emerging Casson fluid flow parameter (γ), Grashof number (Gr), porous medium (K), Magnetic parameter (M^2), Phase angle (ωt), Eckert number (Ec), and Prandtl number (Pr) parameter, respectively. The local

Skin-friction and Nusselt number coefficients are very important material parameters to analyze the rate of fluid velocity and temperature near to the plate. The dimensionless local Skin friction at the plate is given by

$$C_f = -\left(\frac{\partial u}{\partial \eta}\right)_{\eta=0} \tag{10}$$

The dimensionless rate of heat transfer coefficient in terms of the Nusselt number is given by

$$\text{Reduced Nusselt number} = Nu\,\mathrm{Re}_x^{-1} = -\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0} \tag{11}$$

3.Method of Solution

: Homotopy Perturbation Method (HPM)

Consider the function

$$A(u) - f(r) = 0 \tag{12}$$

with the boundary condition of

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0 \tag{13}$$

where $A(u)$ is defined as

$$A(u) = L(u) - N(u) \tag{14}$$

Homotopy Perturbation procedure is shown as:

$$H(v, p) = L(v) - L(u_0) + pL(u_0) + p\left[N(v) - f(r)\right] = 0 \tag{15}$$

or

$$H(v, p) = (1-p)\left[L(v) - L(u_0)\right] + p\left[A(v) - f(r)\right] = 0 \tag{16}$$

The solution is represented by

$$\bar{u} = \bar{u}_0 + P\bar{u}_1 + P^2\bar{u}_2 + P^3\bar{u}_3 + ... \tag{17}$$

$$\bar{\theta} = \bar{\theta}_0 + P\bar{\theta}_1 + P^2\bar{\theta}_2 + P^3\bar{\theta}_3 + ... \tag{18}$$

A homotopy perturbation method is constructed as follows

$$H(\bar{u}, p) = (1-P)\left[\frac{d^2\bar{u}}{d\eta^2} - \frac{\gamma}{1+\gamma}s\bar{u}\right] + P\left[\frac{d^2\bar{u}}{d\eta^2} - \frac{\gamma}{1+\gamma}s\bar{u} - \left(\frac{\gamma}{1+\gamma}\right)\left(M^2 + \frac{1}{k}\right)\bar{u} + \frac{\gamma}{1+\gamma}Gr\bar{\theta}\right] = 0 \tag{19}$$

$$H(\bar{\theta}, p) = (1-P)\left[\frac{d^2\bar{\theta}}{d\eta^2} - \mathrm{Pr}\,s\bar{\theta}\right] + P\left[\frac{d^2\bar{\theta}}{d\eta^2} - \mathrm{Pr}\,s\bar{\theta} + Ec\,\mathrm{Pr}\left(\frac{d\bar{u}}{d\eta}\right)^2\right] = 0 \tag{20}$$

A solution of Eqs. (13) and (14) can then be obtained in the form

$$\bar{u}(\eta) = \bar{u}_0(\eta) + p\bar{u}_1(\eta) + p^2\bar{u}_2(\eta) + \dots$$
(21)

$$\bar{\theta}(\eta) = \bar{\theta}_0(\eta) + p\bar{\theta}_1(\eta) + p^2\bar{\theta}_2(\eta) + \dots \quad (22)$$

Substituting Eqs. (11) and (12) into (13) and (14), yields

$$(1-P)\left[\frac{d^2(\bar{u}_0 + P\bar{u}_1 + \dots)}{d\eta^2} - \frac{\gamma}{1+\gamma}s(\bar{u}_0 + P\bar{u}_1 + \dots)\right] + P\left[\begin{matrix}\frac{d^2(\bar{u}_0 + P\bar{u}_1 + \dots)}{d\eta^2} - \frac{\gamma}{1+\gamma}s(\bar{u}_0 + P\bar{u}_1 + P^2\bar{u} + \dots) - \\ \left(\frac{\gamma}{1+\gamma}\right)\left(M^2 + \frac{1}{k}\right)(\bar{u}_0 + P\bar{u}_1 + \dots) + \frac{\gamma}{1+\gamma}Gr(\bar{\theta}_0 + P\bar{\theta}_1 + \dots)\end{matrix}\right] = 0$$
(23)

$$(1-P)\left[\frac{d^2(\bar{\theta}_0 + P\bar{\theta}_1 + \dots)}{d\eta^2} - \Pr s(\bar{\theta}_0 + P\bar{\theta}_1 + \dots)\right] + P\left[\frac{d^2(\bar{\theta}_0 + P\bar{\theta}_1 + \dots)}{d\eta^2} - \Pr s(\bar{\theta}_0 + P\bar{\theta}_1 + \dots) + Ec\Pr\left(\frac{d(\bar{u}_0 + P\bar{u}_1 + \dots)}{d\eta}\right)^2\right] = 0$$
(24)

Comparing the coefficients of p, p^2 and solving $\bar{u}_0, \bar{u}_1$ and $\bar{\theta}_0, \bar{\theta}_1$

Solving the coefficients of p^0 in Eqs. (23) and (24) using boundary condition

$$\bar{u}_0 = \frac{\omega}{s^2+\omega^2}e^{-\eta\sqrt{\frac{\gamma s}{1+\gamma}}}, \quad \bar{\theta}_0 = \frac{e^{-\mu\sqrt{\Pr s}}}{s}$$
(25)

Using the Eqs. (23) and (24) and boundary condition solving for the coefficients of p^1,

$$\bar{u}_1 = \frac{Gr}{s^2}e^{-\eta\sqrt{\frac{\gamma s}{1+\gamma}}} - \frac{Gr}{s^2}e^{-\eta\sqrt{\Pr s}}, \quad \bar{\theta}_1 = \frac{\gamma\omega^2 e^{-2\eta\sqrt{\frac{\gamma s}{1+\gamma}}}}{s(s^2+\omega^2)^2(\gamma - \Pr(1+\gamma))} - \frac{\gamma\omega^2 e^{-\eta\sqrt{\Pr s}}}{s(s^2+\omega^2)^2(\gamma - \Pr(1+\gamma))}$$
(26)

We obtain the solution the Eqs.(25) and (26),

$$\bar{u}(\eta) = \bar{u}_0 + \bar{u}_1 = \frac{\omega}{s^2+\omega^2}e^{-\eta\sqrt{\frac{\gamma s}{1+\gamma}}} + \frac{Gr}{s^2}e^{-\eta\sqrt{\frac{\gamma s}{1+\gamma}}} - \frac{Gr}{s^2}e^{-\eta\sqrt{\Pr s}}$$
(27)

$$\bar{\theta}()\eta = \bar{\theta}_0 + \bar{\theta}_1 = \frac{e^{-\mu\sqrt{\Pr s}}}{s} + \frac{\gamma\omega^2 e^{-2\eta\sqrt{\frac{\gamma s}{1+\gamma}}}}{s(s^2+\omega^2)^2(\gamma - \Pr(1+\gamma))} - \frac{\gamma\omega^2 e^{-\eta\sqrt{\Pr s}}}{s(s^2+\omega^2)^2(\gamma - \Pr(1+\gamma))}$$
(28)

Finally we take inverse Laplace transform for solutions (27) and (28) we get

$$u(\eta) = \sin(\omega t) erfc\left(\frac{\eta\sqrt{\gamma}}{2\sqrt{1+\gamma}\sqrt{t}}\right) + \frac{\eta\gamma\left(M^2 + \frac{1}{k}\right)}{2\sqrt{\gamma(1+\gamma)}}\left[2\sqrt{\frac{t}{\pi}}e^{-\frac{\eta^2\gamma}{4t(1+\gamma)}} - \eta\sqrt{\frac{\gamma}{1+\gamma}}\left\{erfc\frac{\eta\sqrt{\gamma}}{2\sqrt{1+\gamma}\sqrt{t}}\right\}\right]$$

$$\frac{\gamma Gr}{(1+\gamma)\Pr-\gamma}\left[\left(\frac{\eta^2\gamma}{2(1+\gamma)} + t\right) erfc\left(\frac{\eta\sqrt{\gamma}}{2\sqrt{1+\gamma}\sqrt{t}}\right) - \frac{\eta\sqrt{\gamma}\sqrt{t}e^{-\frac{\eta^2\gamma}{4t(1+\gamma)}}}{\sqrt{\pi}\sqrt{1+\gamma}}\right] -$$

$$\frac{\gamma Gr}{(1+\gamma)\Pr-\gamma}\left[\left(\frac{\eta^2 \Pr}{2} + t\right) erfc\left(\frac{\eta\sqrt{\Pr}}{2\sqrt{t}}\right) - \frac{\eta\sqrt{\Pr}\sqrt{t}e^{-\frac{\eta^2\gamma}{4t(1+\gamma)}}}{\sqrt{\pi}}\right]$$

(29)

$$\theta(\eta) = erfc\left(\frac{\eta\sqrt{\Pr}}{2\sqrt{t}}\right) + \frac{4\gamma\, Ec \Pr}{(4\gamma - (1+\gamma)\Pr)}\left[erfc\left(\frac{\eta\sqrt{\Pr}}{2\sqrt{t}}\right) - erfc\left(\frac{2\eta\sqrt{\gamma}}{2\sqrt{1+\gamma}\sqrt{t}}\right)\right]$$

(30)

Dimensionless local Skin friction at the plate becomes,

$$C_f = -\left(\frac{\partial u}{\partial \eta}\right)_{\eta=0} = \frac{-\sin\omega t\sqrt{\gamma}}{\sqrt{1+\gamma}\sqrt{\pi}\sqrt{t}} + \frac{\sqrt{\gamma}(M^2 + \frac{1}{k})\sqrt{t}}{\sqrt{1+\gamma}\sqrt{\pi}} - \frac{2\gamma Gr\sqrt{t}}{(1+\gamma)\left(\Pr - \frac{\gamma}{1+\gamma}\right)\sqrt{\pi}}\left[\sqrt{\Pr} - \sqrt{\frac{\gamma}{1+\gamma}}\right]$$

(31)

The dimensionless rate of heat transfer coefficient in terms of the Nusselt number is given by

$$\text{Reduced Nusselt number } = Nu \operatorname{Re}_x^{-1} = -\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0} = \frac{-\sqrt{\Pr}}{\sqrt{\pi}\sqrt{t}} + \frac{4\gamma Ec \Pr\left(\sqrt{\Pr} - 2\sqrt{\frac{\gamma}{1+\gamma}}\right)}{[\Pr(1+\gamma) - 4\gamma]\sqrt{\pi}\sqrt{t}}$$

Results and Discussions

In this study, MHD natural convective flow of Casson fluid over an oscillating vertical plate has been investigated for velocity profile and temperature distribution. The governing equation which is a pair partial differential equations were solved analytically by using the homotopy perturbation method (HPM) and Laplace transform.

Fig.2 represents the velocity profile u for all values of parameters for Pr, Gr, K, M^2 and γ .From the figure it is inferred that velocity profile decreases as influence of casson parameter (γ) and thermal conductivity of the fluid (K). The velocity distribution for various values of phase angle $\omega t = 0$ is illustrated in Fig. 2. It is also noted that velocity profile has no significant effect due to Prandtl number (Pr), Grashof number (Gr) and magnetic parameter M^2.

The effect of the parameters Pr, Ec and γ on the temperature distribution is shown in Fig.3. From the Figure it is observed that temperature decreases as Prandtl number Prand Eckert number Ec increases. It is also noted that temperature distribution has no significant effect due to γ influence of casson parameter.

4. Conclusions

In the study, an unsteady MHD natural convective flow of Casson fluid past an oscillator verticalplate for all values of parameters. The approximate analytical expression of velocity profile and temperature distribution using the Homotopy perturbation method and Laplace transform. The present results are in excellent for Newtonian and non Newtonian fluid flowproblems. Therefore the present model is useful to analyzethe Newtonian and non Newtonian fluid flow models.

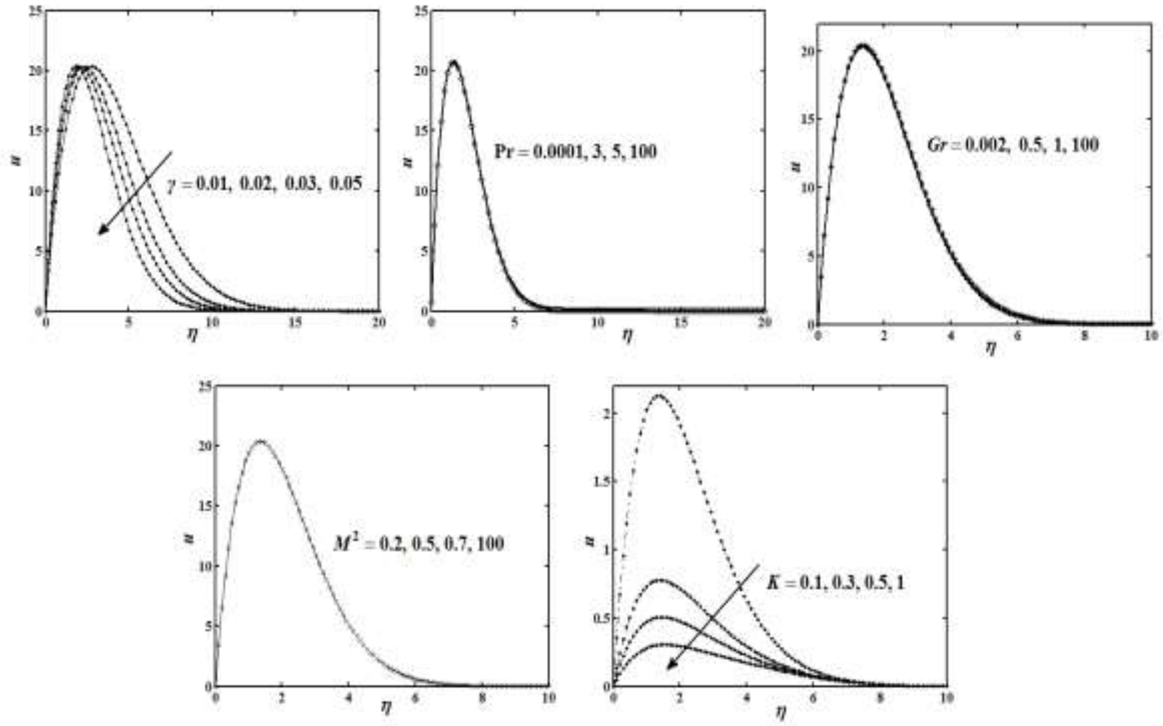

Fig.2. Velocity profile (u) for against η for $\Pr = 0.3,\ \gamma = 0.2,\ M^2 = 0.2,\ \omega t = 0,\ K = 0.01,\ Gr = 0.2,$ and $t = 1$.

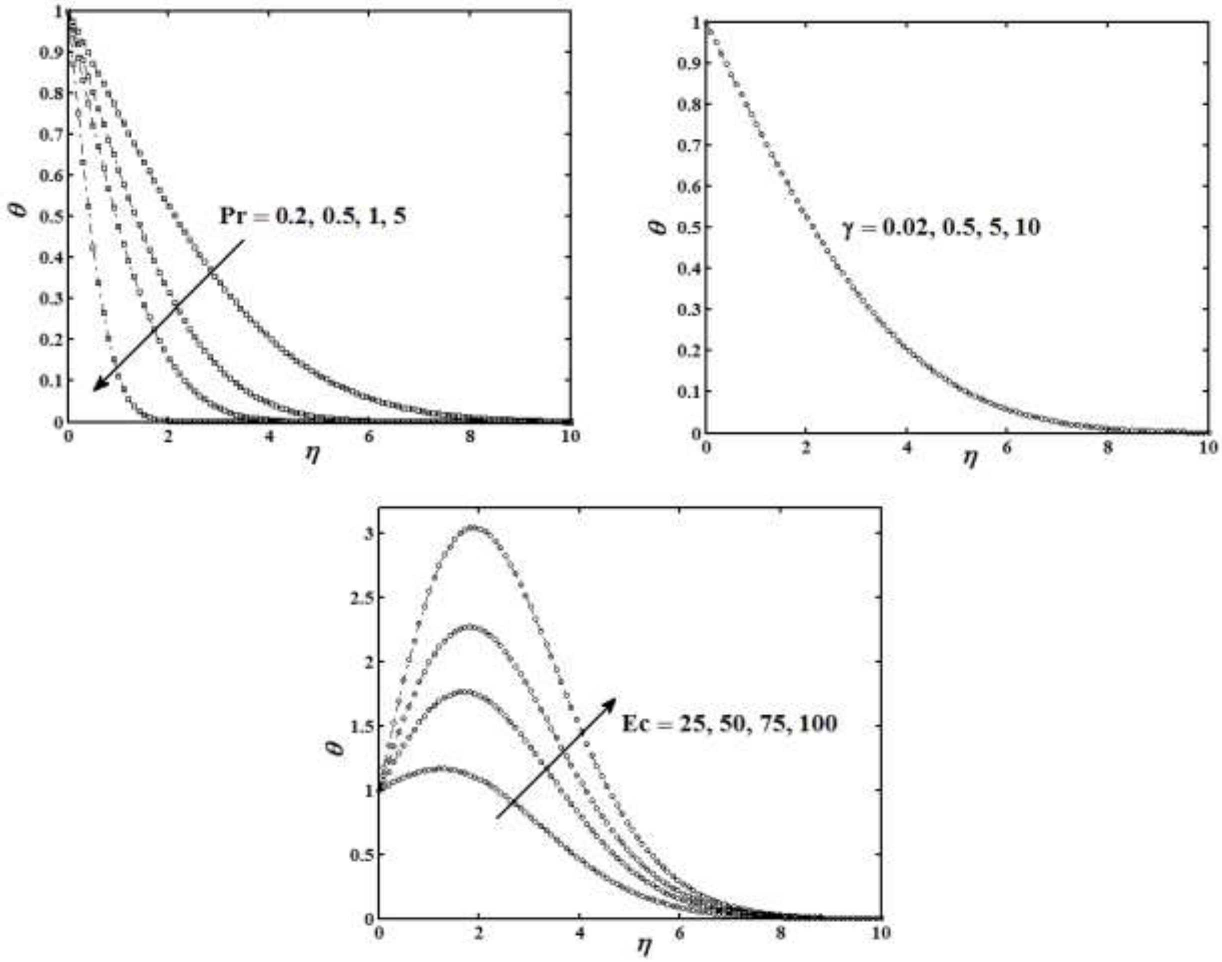

Fig.3. temperature distribution (θ) for against η for $\Pr = 0.3,\ Ec = 0.01,\ \gamma = 0.2$ and $t = 1$.

Symbols

Nomenclature	Meaning	Units
u	dimensionless velocity	None
θ	dimensionless temperature	None
C_f	the local skin friction	$N\ m^{-2}$
Ec	Eckert number	None
Pr	Prandtl number	None
M^2	magnetic parameter	None
y	dimensionless coordinate axis normal to the plate	None
t	Time	s

Greek Symbols

Nomenclature	Meaning	Units
κ	Thermal conductivity	$W \cdot m^{-1} \cdot K^{-1}$
η	dimensionless displacement	(m)
μ_b	plastic dynamic viscosity	None
γ	Casson parameter	None
ρ	the constant density	(kg m^{-3})

β	volumetric coefficient of thermal expansion	(K^{-1})
σ	electric conductivity of the fluid	$(s\ m^{-1})$
φ	porosity	None
ω	frequency of oscillation of the plate	$(m\ s^{-1})$
θ	fluid temperature	(K)
τ	stress tensor of Casson fluid	$(N\ m^{-2})$

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