International Journal of Advanced Science and Technology
Vol. 29, No. 6, (2020), pp. 3590 – 3598

Convective Flow of a Ferrofluid Between Rotating Stretchable Disks in An Anisotropic Porous Medium

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Abstract

In this paper, Ferro-fluid flow between1 two stretchable 1rotating disks in an orthotropic porous medium is presented. Governing equations1 for the problem are formulated and non-dimensionalized using appropriate transformations. Results are computed using shooting method. Results are explained using figures and table values of various values of physical parameters effective magnetization parameter, volume concentration of magnetic parameter and rotation parameter
KEYWORDS: *Ferro fluid, Orthotropic porous media, Rotating Stretchable disks.*

1. Introduction

A Ferro liquid or ferromagnetic liquid is a fluid which turns out to be distinctly charged inside the sight of an attractive field. Ferro fluids are colloidal fluids and are constituted of nanoscale ferromagnetic debris suspended in a transporter liquid. Typically water or an organic dissolvable like lamp gas. Conventional Nano beverages rely upon colloidal magnetite. Ferro fluids are artifically combined inside the lab. The manner in the direction of making Ferro liquid changed into first imagined with the aid of NASA's Steve Papell in the year 1963 to make fluid rocket fuel that facilitates in drawing closer to a pump inlet in a weightless situation via making use of an magnetic field. Ferro fluids are utilized to frame fluid seals across the turning drive shafts in hard disk. Ferro fluids have contact lessening capabilities. Ferro liquids is the mix of ordinary fluid conduct with an attractive check of their circulation and effects is their peculiarity. These fluids are determined to have fantastic applications in distinctive fields. Finlayson et al. [1] considered ferromagnetic liquids convective precariousness under sight of a uniform vertical magnetic area. Nanjundappa et al. [2] tested the onset of buoyancy driven convective for an inactive ferrofluid immersed level permeable layer in nearness of a uniform vertical magnetic area. Lee et al. [3] clarified the nearby thermal non-equilibrium results for convection thermomagnetic in a ferromagnetic liquid immersed by means of flat permeable medium warmed from underneath sight of a uniform vertical magnetic field. Sekar et al. [4] clarified the direct solidness of soret-driven thermohaline convection unsteadiness of multi phase liquid in an anisotropic permeable medium is studied with the aid of Brinkman model for numerous estimations of orthotropic variable. Chand [5] brought the triple-diffusive convection direct balance of micropolar ferromagnetic liquid layer in immersed permeable medium. Ordinary mode investigation is applied to carry out convection. Various parameters, for example, solute slope, micropolar, the non-buoyancy magnetization and medium porousness parameter are examined. Seliemefendigil et al. [6] built up a numerical study on the thermal transfer circulate and stream of liquid qualities of a pivoting chamber under the impact of appealing dipole. Habibur et al.[7] considered the stability of various thermal layer of ferrofluid of oblique uniform outside magnetic field beneath zero gravity conditions. Satyajit et al. [8] dissected the affect of magnetic field on natural convection heat transfer for c-shaped cavity inside the sight of ferrofluid. The selected ferrofluid is cobalt-lamp gasoline. The method implemented to get numerical solution is Galerkin weighted residual method. Tasawar et al. [9] built a version for 2-dimensional ferrofluid chemical reactions. The effect of mixed convection and appealing dipole is analyzed. The solution given traditional differential equations is illuminated through Euler's explicit method. Akbar et al. [10] discovered the effect of ferrofluid on tube channel of cu-water nanofluid for special shaped nanoparticles. Required solutions are tackled by various generalized governing equations. Krauzina et al. [11] broadened the work on thermal convection of non-Darcy restriction layer circulation in a round hollow with ohmic dissemination over a penetrable extending sheet. The impacts of different parameters like pressure, velocity, Nusselt number and skin friction coefficient is presented. Zeeshan and Majeed [12] researched attractive liquid over a penetrable extending sheet with ohmic dissemination for regular dimensional incompressible mixed convection flow.

Industries of mechanical engineering utilize those liquids in vacuum seals, and vibration dampers. Electronic and electric designing organizations use ferrofluids in transformer coolants, cutting down inductive elements. These fluids are utilized as stepper engines in calculation groups. In biomedical to focusing on the tumor, the usage of in MRI, gout, repair arthritis complications, spondylitis magnet treatment. Mangathai et al. [13] analyzed MHD free convective circulate over a preceding a vertical plate in a permeable medium inside the sight of radiation and heat generation. Chandra Sekhar Reddy et al. [14] taken into consideration radiation effect and heat dissemination on precarious MHD unfastened-convective move past over an impulsively moving plate with sloped wall concentration and inclined wall temperature. Muhammad et al. [15] taken into consideration the influences of attractive dipole of thermal definition on a ferrofluid for certain via extending sheet. Sohail et al. [16] examined the parts of heat definition on the ferrofluid inserted in a porous medium from the affect of dipole. Optimal HAM and Bvph2 is applied to get solution for the obtained differential equations for the flow problem. Hayat et al. [17] analyzed attractive dipole in Non-Darcy1Forchheimer stream of second grade liquid soaking permeable medium. Majeed et al. [18] taken into consideration the effect of chemical reaction and thermal transfer examination on boundary layer Maxwell soaked Ferro-liquid over a stretching sheet below magnetic dipole with Soret and Suction impact. The resultant D.E's are tackled by R-K technique depending on shooting algorithm.

2. MODEL PROBLEM

Consider two dimensional viscous incompressible flow between rotating disks as appeared in fig-1. The lower disk is positioned at z=0 and upper plate is about at z=d. The lower and upper disks are rotating with different angular velocities Ω1 and Ω2 respectively. Both the disks are assumed to be stretched in radial direction for various stretching rates b_1 and b_2 respectively. The lower and upper disks are maintained with various uniform constant temperature T_0 and T_1 respectively. The total system is embedded in an orthotropic porous medium. With the above assumption the polar coordinates governing equation are:

$$\frac{\partial U}{\partial r}+\frac{U}{r}+\frac{\partial W}{\partial z}=0 \quad (1)$$

$$U\frac{\partial U}{\partial r}+W\frac{\partial U}{\partial z}-\frac{V^2}{r}=\frac{-1}{\rho}\frac{\partial p}{\partial r}+\upsilon\left(1+\frac{3}{2}\gamma g\right)\left(\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}+\frac{\partial^2 U}{\partial z^2}-\frac{U}{r^2}\right)-\frac{\upsilon}{K_r}U \quad (2)$$

$$U\frac{\partial V}{\partial r}+W\frac{\partial V}{\partial z}+\frac{UV}{r}=\upsilon\left(1+\frac{3}{2}\gamma g\right)\left(\frac{\partial^2 V}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}+\frac{\partial^2 V}{\partial z^2}-\frac{V}{r^2}\right)-\frac{\upsilon}{K_\theta}V \quad (3)$$

$$U\frac{\partial W}{\partial r}+W\frac{\partial W}{\partial z}=\frac{-1}{\rho}\frac{\partial p}{\partial z}+\upsilon\left(1+\frac{3}{2}\gamma g\right)\left(\frac{\partial^2 W}{\partial r^2}+\frac{1}{r}\frac{\partial W}{\partial r}+\frac{\partial^2 W}{\partial z^2}\right)-\frac{\upsilon}{K_z}W \quad (4)$$

$$U\frac{\partial T}{\partial r}+W\frac{\partial T}{\partial z}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right) \quad (5)$$

Associated boundary conditions are

$$\begin{aligned} &U=rb_1, V=r\Omega_1, W=0, T=T_0 \text{ at } z=0 \\ &U=rb_2, V=r\Omega_2, W=0, T=T_1 \text{ at } z=d \end{aligned} \quad (6)$$

where U,V and W are components of velocity in r,θ and z directions respectively, ρ is density, υ is the kinematic viscosity, K_r, K_θ and K_Z are permeabilities in r,θ and z directions respectively and α is the thermal conductivity of fluid.

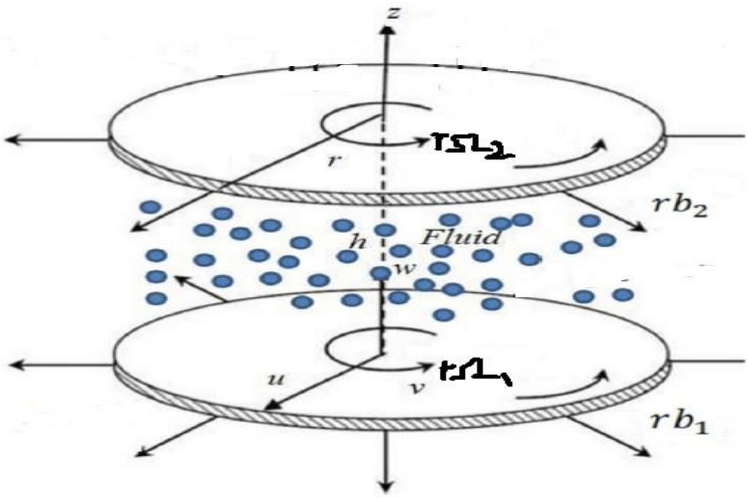

Fig.1: Geometry of the problem

In order to non-dimensionalize the eqns. (1) – (6), Introducing the following transformations

$$\begin{aligned} &\eta=\frac{z}{h}, \left[U,V,W\right]=\Omega_1\left[rF'(\eta), rG(\eta), -2hF(\eta)\right] \\ &\Theta=\frac{T-T_1}{T_0-T_1}, p=\rho\Omega_1\upsilon\left(P(\eta)+\frac{r^2}{2h^2}\varepsilon\right) \end{aligned} \quad (7)$$

Using (7), eqns. (1) – (6) becomes

$$\left(1+\frac{3}{2}\gamma g\right)F'''-\frac{1}{\mathrm{Re}}\left((F')^2-2FF''-G^2\right)-\varepsilon-\frac{1}{K_1}F'=0 \quad (8)$$

$$\left(1+\frac{3}{2}\gamma g\right)G''-\frac{2}{\mathrm{Re}}\left(F'G-FG'\right)-\frac{1}{K_2}G=0 \quad (9)$$

$$P'+\mathrm{Re}\,FF'+2\left(1+\frac{3}{2}\gamma g\right)F''-\frac{2}{K_3}F=0 \quad (10)$$

$$\Theta''+2\,\mathrm{Re}\,\mathrm{Pr}\,F\Theta'=0 \quad (11)$$

with the boundary conditions given as

$$\begin{aligned} &F(0)=0,\ F'(0)=R_1, G(0)=1, \Theta(0)=1 \\ &F(1)=0,\ F'(1)=R_2, G(1)=\Omega, \Theta(1)=0 \end{aligned} \quad (12)$$

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where $\mathrm{Re} = \frac{\Omega_1 h^2}{\upsilon}$ is the Reynold's number

$\mathrm{Pr} = \frac{\upsilon}{\alpha}$ is the Prandtl's number

$K_1 = \frac{K_r}{h^2}$ Darcy number along x-direction

$K_2 = \frac{K_\theta}{h^2}$ Darcy number along θ-direction

$K_3 = \frac{K_z}{h^2}$ Darcy number along z-direction

$\Omega = \frac{\Omega_2}{\Omega_1}$ rotation parameter

$R_1 = \frac{b_1}{\Omega_1}$, $R_2 = \frac{b_2}{\Omega_2}$ are scaled stretching parameters

The skin friction coefficients at the lower and upper disks are:

$$\tau_1 = \frac{\tau_w\big|_{z=0}}{\rho\left(r\Omega_1\right)^2} = \frac{\left(\left(f''(0)\right)^2 + \left(g'(0)\right)^2\right)^{1/2}}{\mathrm{Re}_r},$$
$$\tau_2 = \frac{\tau_w\big|_{z=h}}{\rho\left(r\Omega_2\right)^2} = \frac{\left(\left(f''(1)\right)^2 + \left(g'(1)\right)^2\right)^{1/2}}{\mathrm{Re}_r} \quad (13)$$

Rate of heat transfers (Nusselt numbers) at lower and upper disks are:

$$Nu_1 = \frac{-T_z\big|_{z=0}}{T_0 - T_1} = -\theta'(0)$$
$$Nu_2 = \frac{-T_z\big|_{z=h}}{T_0 - T_1} = -\theta'(1) \quad (14)$$

3. SOLUTION PROCEDURE

A set of equations (8) – (11) with conditions (12) are solved numerically, shooting method (Mallikarjuna et al. [3, 4 and 5] that uses Runge-Kutta method and Newton' method. To validate the present code the present results are compared with K. Stewartson [1] and Imatiaz et.al [2] in the absence of heat transfer and porous media for limiting cases as shown in table-1. The physical parameter values are assumed defaultly as Re=10, Pr =6.23, K_1=0.5, K_2=0.5, R_1=0.7, R_2=0.7, Ω = 0.5, and λ = 0.5 unless specified. Computational results are illustrated graphically for different cases of parameters on velocity (radial f' and tangential g) and temperature profiles and table values are reported for skin friction and Nusselt number along both disks.

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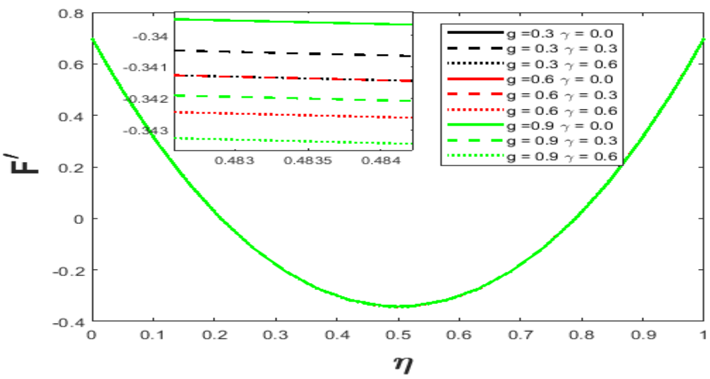

Fig. 2: Influence of radial (f') velocity profile for g and γ

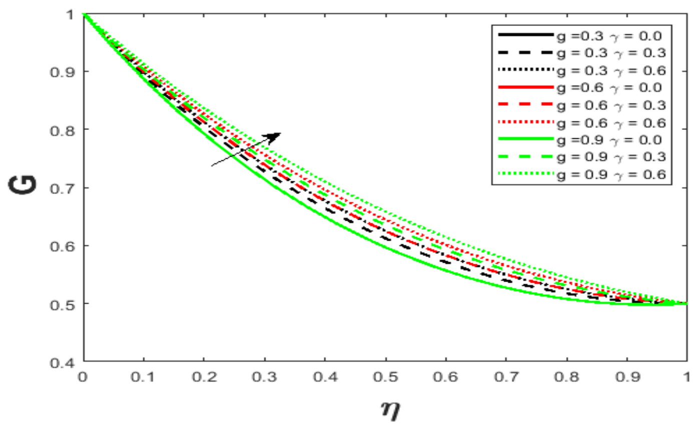

Fig. 3: Impact of tangential (G) velocity profile for g and γ

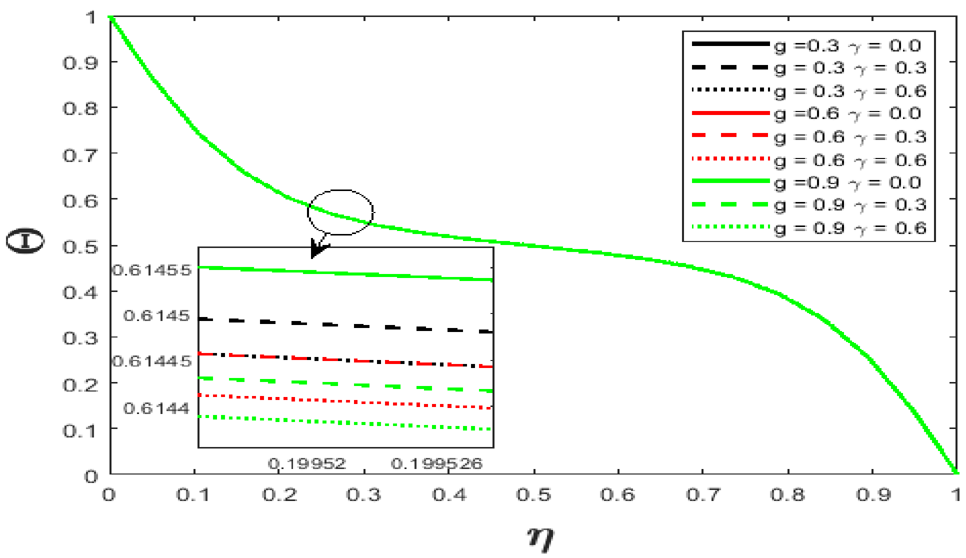

Fig. 4: Affect of temperature (Θ) profile for g and γ

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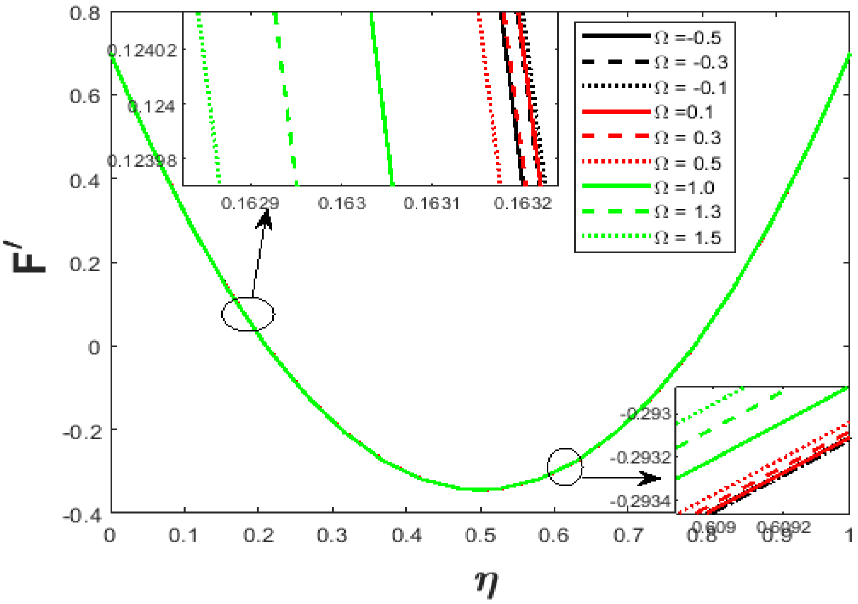

Fig. 5: Influence of radial (f') velocity profile for Ω

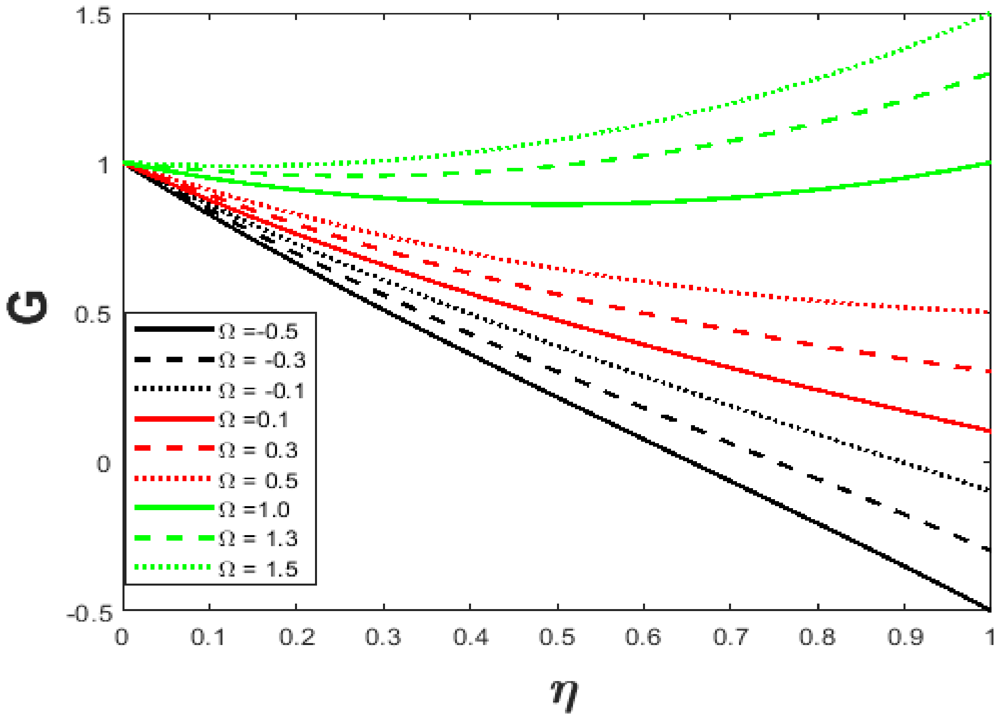

Fig. 6: Affect of tangential (G) velocity profile for Ω

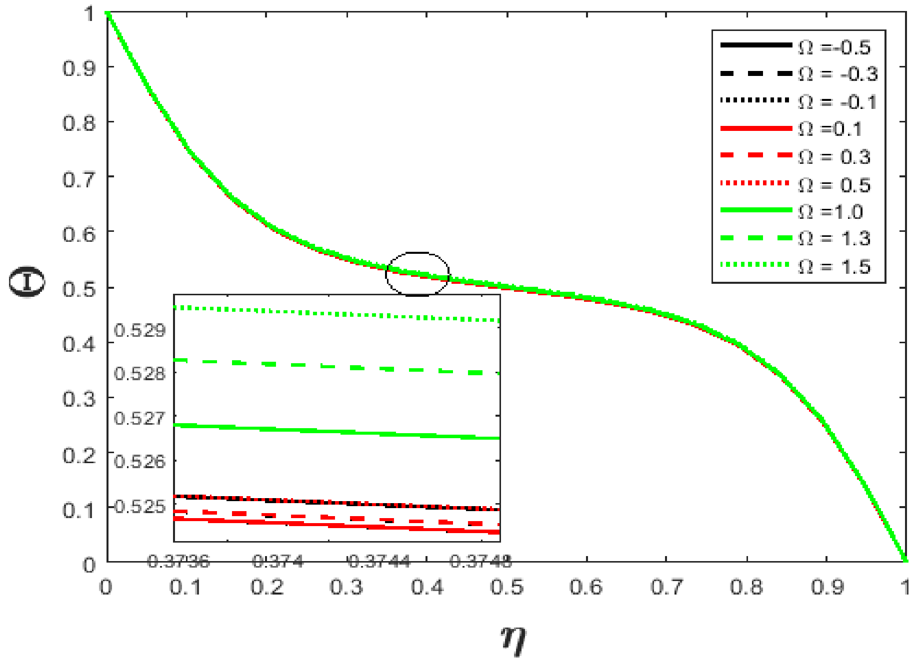

Fig. 7: influence of temperature (Θ) profile for Ω

Table 1: Comparison of $f''(0)$ and $g'(0)$ for various values of Ω when R_1=0, R_2=0 and in the absence of porous medium for Re=1

Ω	K.Stewartson [1]		M. Imtiaz et.al [2]		Present results	
	$f''(0)$	$-g'(0)$	$f''(0)$	$-g'(0)$	$f''(0)$	$-g'(0)$
-1	0.06666	2.00095	0.06666	2.00095	0.06666263	2.00095376
-0.3	0.10395	1.30442	0.10395	1.30442	0.10395043	1.30442628
0.5	0.06663	0.50261	0.06663	0.50261	0.06663394	0.50261755

Table-3: Skin friction coefficient and Nusselt number values at lower and upper disks Re=10, K_1= 0.5, K_2 = 0.5, R_1= 0.5, R_2 = 0.5 and Pr = 6.23

Ω	g	γ	τ_1	τ_2	Nu_1	Nu_2
-0.5	0.6		3.55496585	3.42097205	2.25198607	2.23566057
-0.3			3.47456678	3.30734045	2.25364004	2.23383255
-0.1			3.40033579	3.21379732	2.25444066	2.23289253
0.2			3.30145587	3.11500155	2.25404138	2.23314688
0.4			3.24443848	3.07888962	2.25270853	2.23442603
0.8			3.15348332	3.08152914	2.24748439	2.23964902
1.2			3.09546967	3.18280888	2.23885337	2.24843023
1.5			3.07474150	3.31841154	2.23014902	2.25735696
1.8			3.07401646	3.49875464	2.21953740	2.26829603
0.5	0.3		3.26140741	3.08242317	2.25086142	2.23165932

	0.6		3.21874706	3.07012617	2.25172220	2.23539858
	0.9		3.18958516	3.06256222	2.25228131	2.23808826
	0.6	0	3.32910419	3.10436565	2.24940115	2.22610062
		0.3	3.26140741	3.08242317	2.25086142	2.23165932
		0.6	3.21874706	3.07012617	2.25172220	2.23539858

4. RESULTS AND DISCUSSION

Radial (f^1) velocity for and g and ɤ is illustrated in fig 2. The graph is parabolic in nature with increased values of ɤ when g is fixed. In fig 3 impact of tangential (g) velocity profile for g and ɤ is shown. With the increased values of ɤ when g is fixed tangential (g) velocity increases. Temperature (Θ) profile for g and ɤ is observed in fig 4. With the increased values of ɤ when g is fixed temperature(Θ) profile increases with the increased values of ɤ. In fig 5 impact of radial (f^1) velocity profile for rotation parameter Ω is shown. The graph is parabolic in nature. The rotation parameter (Ω) values increases from Ω=-0.5 to Ω=1.5. Tangential (g) velocity profile for rotation parameter (Ω) is depicted in fig 6. Tangential velocity increases with increased values of Ω. Temperature (Θ) profile increases for increased values of Ω is shown in fig 7. Rotation parameter (Ω) takes values from -0.5 to 1.5.

Table 3 depicts the effect of rotation parameter on skin friction (τ_1 and τ_2) coefficients and Nusselt Number (Nu_1 and Nu_2) values at both disks. With the increased values of rotation parameter (Ω) skin friction (τ_1) coefficient at lower disk decreases and at upper disk it decreases up to Ω=0.8 and increases from Ω=1.2. With the increased values of g skin friction (τ_1 and τ_2) coefficients decreases at both disks and Nusselt Number (Nu_1 and Nu_2) increases at both disks. When Γ increases, skin friction (τ_1 and τ_2) coefficients decreases at both the disks and Nusselt Number (Nu_1 and Nu_2) increases at both disks.

CONCLUSION

Ferro-fluid flow between two stretchable rotating disks in an orthotropic porous medium is analyzed in this paper. Continuity equation, momentum and energy are formulated and non-dimensionalized using suitable transformations and then tackled using shooting technique. Results are illustrated using figures and table values for various values of physical parameters effective magnetization parameter, volume concentration of magnetic parameter and rotation parameter. The results are:

- With the increased values of rotation parameter (Ω) skin friction (τ_1) coefficient at lower disk decreases and at upper disk it decreases up to Ω=0.8 and increases from Ω=1.2.
- Increasing values of g, skin friction (τ_1 and τ_2) coefficient decreases at both disks and Nusselt Number (Nu_1 and Nu_2) increases at both disks.
- When γ increases, skin friction (τ_1 and τ_2) coefficients decreases at both the disks and Nusselt Number (Nu_1 and Nu_2) increases at both disks.

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