CONTEMPORARY ISSUES IN TEACHING AND LEARNING TECHNIQUES OF DIFFERENTIAL EQUATIONS: A REVIEW AMONG ENGINEERING STUDENTS

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Abstract

The Differential Equations are becoming trending now-a-days in the development of modern technology based findings, which plays a vital role in our routine life day-to-day. We always have a query that students learn Engineering and Mathematicians as their subject forever. But in the current scenario, students learn Mathematics as just a subject only, but without Mathematics intuition, students cannot build an empire around the world. In this regard, here in this paper we will review the relation between the Engineering students and Mathematics to show that they are co-related. That too how Mathematics is applied to different branches of Engineering like Civil, Mechanical, Electrical and Electronics, Computer Science in its own efficient way. This study will pave a new way to teaching and learning methods of Differential Equations and also indicate where the further research is essential to improve the development

Keywords: *Differential Equations, Analytic methods, Numerical methods, Graphical methods, Technology based improvements.*

1. Introduction

Differential equations are playing an effective role in this world of hustle and bustle of Engineering. All countries are developing rapidly having Engineering to be their pulse of life. There is necessity of Engineering in minute may be nano problems of solving compared with the late 17th century. For centuries, Differential equations have developed rapidly in different scenoaric problems. Differential equations is not only Mathematics, it is a branch of physical, astrological, economical and biological sciences. It is well said that Mathematics is called as the Queen of Science and in it Differential equation has become an all-rounder right now.

Though the differential equation has become a king, it is not easy to become an Emperor i.e., we mean the teaching and learning techniques of differential equations (Mallet D.G & Mccue,S.W, 2009). Differential equations can be teached and learned analytically, numerically and graphically. Students feel it difficult in studying differential equations because it is a topic entirely new to them in their higher secondary education. They enter new to the topics of differential calculus, integral calculus and its applications.

The process of conveying differential equations into the minds of students is little bit a tough task as they have no previous history of study in this topic (Cobb, P, 1995). Some special inference of attention is required for the students to start their new subject of differential equations or else

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they will face it as an alien paper of study and they will miss the fruitness of the paper (Kwon,O.N., Rasmussen,C., & Allen.K, 2005)and their further understanding and development will be stopped. When this stops, solutions to simple day-to-day real life problems cannot be arrived.

This generation students are not willing to learn the theoretical concepts deeper. They are very much interested video based interactive learning. That is the new trend of the students is they give much more interest when the concept is explained to them with videos and colourful images (Gollwitzer, H, 1991) and Powerpoint presentations in the screen or even with the objects. The students get into it and aspire, when things are explained to them video based, audio based, experiment based in an active learning method. The students of this generation tell that they get grasped the concepts quickly when it is a visual treat education to them through their eyes and ears. Our teachers in the previous generation also had taught the same concepts without leaving a little effect in the concept putting their atmost energy. The generation during that time had deep listening skill and learned the concepts. But to this generation also the same concepts are conveyed to the students by their teachers, but they are more interesting in developing new ideologies as most of the students are engineers. Engineers, the name itself denotes developing new technological firms.

The approaches of teaching community to the students must also change according to the need of the hour. Students are not showing much attention in differential calculus teaching and learning process when the teachers approach them analytically in depth. But when the concept is understood only deeper the innovations can be made. They are not completely avoiding the study techniques of differential equations, but they are showing much interest when differential equations are approached numerically and graphically to them. The Engineering students feel that analytic approach is somewhat tedious to them, but of course for the teaching faculty also it is sometimes a difficult task for the query of the students where we use it in their own engineering application. But when it becomes graphical or numerical approach students actively involve in their course of study of engineering and they are ready to take it to the next level of development.

When the teaching and learning process of differential equations to the engineering students is by analytic approach, they feel that they have to remember too much analytic methods of solving, formulae and theoretical steps where as when it is graphical or numerical approach, they enter into the concepts of mathematical modeling and just only to the physical interpretation and relation of the equation.

Literature study of this paper reveals that many researchers have done their own research work in the teaching learning process of differential equations in a very betterment way of their own. But no one has reviewed recently the better way of approach of teaching learning process by the teachers who teaches differential equations to the present 2020 student community of engineering and also the difficulty faced by both the teachers and the student who studies differential equations in their engineering degree. Therefore, in this article, the problems faced by both these communities have been peerly reviewed.

Research Objectives:

The main objective of this research article is to relate to the issues of the teaching and learning process between the mathematics teachers and the engineers in the peak topic of differential equations. It is exponentially calculated that this study would enhance the students, teachers, researchers with enlightning ideas in making models, physical solutions to problems using the conceptual area of differential equations.

The questions that arise during this study are as stipulated below:

1) What is the best method to implement for the teaching learning process of differential equations?
2) What are the difficulties faced in the teaching learning process of differential equations?

3) Why should an engineering student be teached with an engineering mathematics especially differential equations for semesters for his under graduation degree?
4) How will the development lead the society by the teaching and learning process of the differential equations?
5) Where does the further improvement in the field of differential equations required in its teaching and learning process?

The forthcoming topics have given concentration in the above said regards and finally it gives us conclusions, problems faced and where the leakage had to be exponentially tightened.

2. Methodology:

The study of literature has been keenly reviewed in the several researchers and ongoing research projects which have been applied on teaching and learning differential equations. In the late 90's, teaching differential equations was like a sculpture of art in a wall with all traditional ideas (Arslan, S, 2010b), that ideas used all analytic approaches and differential and integral calculus operations. Now at the present year 2020, students feel the concept to be little easier by utilizing simpler techniques of solving differential equations in a modern way of teaching learning process. They prefer analytical, graphical and numerical approaches.

Ideas pertaining to differential equations in its process of Teaching and Learning:

Figure - 1 demonstrates the overall framework for the current work. Details of each section is furnished below:

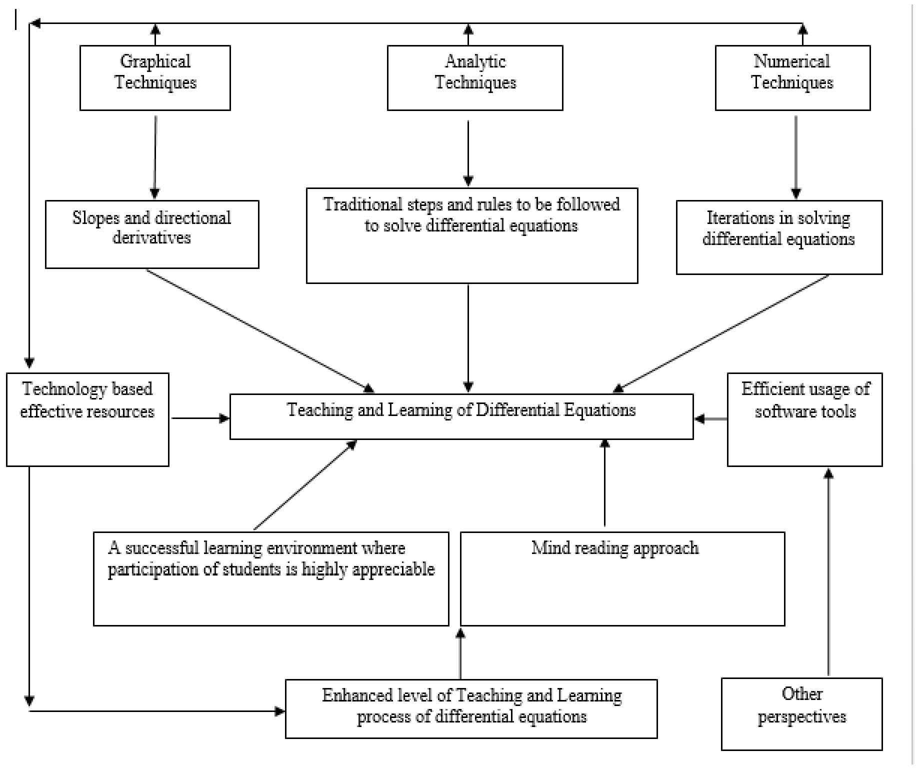

Figure -1

3. *Analytic Techniques*

Analytic method, the name itself procures among the student community that this procedure is going to be a tougher task of understanding and solving. Right from the younger age of schooling, the students are afraid of analyticity as it comprise of too many identities and sometimes notations and symbols too. Analytically when could we arrive a solution to a differential equations? We, we must arrive the value of an unknown and must satisfy the ordinary differential equations and also its nature may be exact, linear, separable, homogeneous and other types.

When the students were given a problem in solving differential equations, they used all the possible methods of differentiation and even they used integration. The senior second year under-graduate students solved partial differential equations using all such techniques. Their effort were appreciable but the students made mistakes in understanding the type of an equation, utilization of symbols and also of course their methodology went wrong without proper knowledge of applications.

Compared with the students, teaching teachers an differential equation concept was easier with limited efforts as they were somehow aware of the procedure to be followed. When in equation of differential calculus was given to the engineering student community they went with the steps of analytic methods claiming that they would get the exact answer and would arrive the solutions very quickly though it is tedious. They applied integration in this equation solving. But many students were not aware where to apply what step and started solving the differential equation in a blind approach.

First of all some students tried to solve it mathematically and only one student from electrical background who was interested in circuits and its designs coined that it is we have to find the current I and so it is necessary to find the charge dq/dt and cleared that we have to find out an equation for charge (Dawkins, P.C., & Epperson, J.A.M, 2014) instead of finding current. Rowland (2006) reported that this student was able to coin out this concept as he was taught in term of rate of change. But all times this concept can't work and because rate of change itself denotes it changes and it can't be a constant and so we can't arrive at a final solution (Czocher,J.A., Tague,J., & Baker.G, 2013). This mentality among the students was high because they were following that to find a quantity may be a vector its rate of change of a particular quantity finding is enough and it has been taught in all soughts of teaching resources including their on-board classroom assignments, tutorials, notes so on and so forth.

Though the students studying engineering come forward readily to solve the differential equations, they finally come out with some technical issues like where to solve the problem i.e., at what step it is necessary to arrive at a solution and where must we integrate the concept wisely, what notations we must use, where must we apply integration and also mostly they use wrong synchronizing of symbols in solving ordinary differential equations. First of all the engineering students are not clear with the differentiation concept symbols, first derivative, second derivative, third derivative…, functions, variables, constants, equation type whether it is quadratic, first order(linear), second order or so on.

Nevertheless students possess a mismatch for differentiation, integration, representation of functional derivative, but they are able to use it when they are subject to visualized teaching. The focal point to the students should be how they are elegantly utilizing the mathematical resources instantly involving differentiation and integration in electrical problems so that it as contribution towards both fields of electrical and mathematics (Pollak, H.O, 2015).

Example for Mathematical Modeling

We call Differential Equations are of great importance in engineering and science because many physical laws and relations appear mathematically in the form of differential equations, for reasons that will soon become apparent (Erwin Kreyzig, 2011). Here, let us consider a basic physical application that will illustrate the typical steps of modeling (Moore,T.J., Miller, R.L., Lesh, R.A., Stohlmann, M.S., &Kim,Y.R, 2013), that is, the steps that lead from physical situation (physical system) to a mathematical formulation (mathematical model) and solution (Fadlelmula, F. K., Cakiroglu. E., & Sungur.S,2015), and to the physical interpretation of the result. This may be the easiest way of obtaining a first idea of the nature and purpose of differential equations and their applications (Hu,D., & Rebello,N.S,2013). Though this is an easy and traditional way it is very much essential for the student to have a brief knowledge about the concepts like dependent and independent variables, initial conditions, proportionality constant. Only when a student is aware about these concepts, the mathematical modeling can be done. This is the effective way of teaching the concepts clearly and getting into the concepts.

Considering an experiment, it shows that a radioactive substance decomposes at a rate proportional to the amount present. Starting with a given amount of substance, say, 2 grams, at a certain rate, say, t=0, what can be said about the amount available at a later time?

Setting up a Mathematical Model (a differential equation) of the Physical Process

We denote y(t) the amount of substance still present at time t. The rate of change is dy/dt. According to the physical law governing the process of radiation, dy/dt is proportional to y:

$$\frac{dy}{dt} \alpha y$$

$$\frac{dy}{dt} = ky \qquad (1)$$

Hence y is the unknown function, depending on t, so the equation we get will be an ordinary differential equation as there is only one dependent variable, or else it will be a partial differential equation. This knowledge of understanding is very important for the student to frame a differential equation and then construct the mathematical model. The constant k is a definite physical constant whose numerical value is known for various radioactive substances. Clearly, since the amount of substance is positive and decreases with time dy/dt is negative, and so is k. We see that the physical process under consideration is described mathematically by an ordinary differential equation of the first order. Hence this equation is the mathematical model of that physical process. Whenever a physical law involves a rate of change of a function, such as velocity, acceleration, etc., it will lead to a differential equation. For this reason, differential equation occur frequently in physics and engineering.

Solving the Differential Equation

We do not yet know methods of solution, but calculus will help us here. Equation (1) tells us that if there is a solution y(t), its derivative must be proportional to y. Now we remember from calculus that exponential functions have this property. Indeed, by differentiation and substitution we see that a solution for all t is $y(t) = e^{kt}$ because $y'(t) = (e^{kt})' = ke \wedge kt = ky(t)$. More generally, a solution for all t is

$$y(t) = ce^{kt}$$

(2)

with any constant c because $y'(t) = cke^{kt} = ky(t)$. Since c is arbitrary, (2) is the general solution of (1), by definition.

Determination of a particular solution from an initial condition:

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Clearly, our physical process behaves uniquely. Hence we should be able to get from (2) a unique particular solution. Now the amount of substance at some time t will depend on the initial amount y = 2 grams at time t = 0, or, written as a formula,

$$y(0) = 2 \tag{3}$$

This is called an initial condition. We use it to find c in (2):

$y(t) = ce^{0} = 2,$ thus $c = 2.$

With this c, equation (2) gives as the answer the particular solution

$$y(t) = 2e^{kt} \tag{4}$$

Thus the amount of radioactive substance shows exponential decay (exponential decrease with time). This agrees with physical experiments.

Checking of the solution:

From (4) we have

$\frac{dy}{dt} = 2ke^{kt}$ = k y and y(0) = $2e^{0}$ = 2

We see that the function (4) satisfies the equation (1) as well as the initial condition (3). The student should never forget to carry out this important final step, which shows whether is (or is not) the solution of the problem.

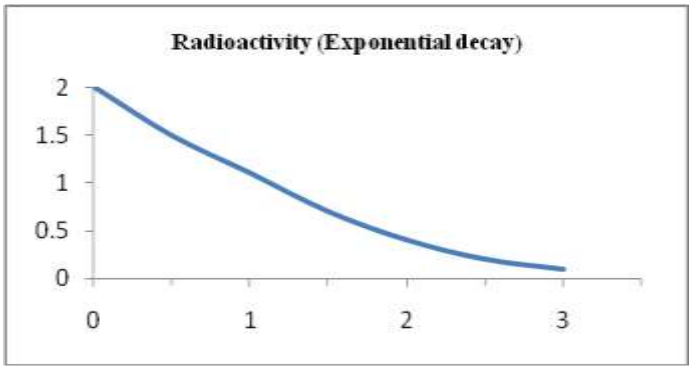

Figure - 2

A differential equation together with an initial condition, as in our example, is called an initial value problem. With x as the independent variable (instead of t) it is of the form

$$y' = f(x, y), y(x_0) = y_0 \qquad (5)$$

where x_0 and y_0 are given values. (In our example, $x_0 = t_0$ =0 and $y_0 =$ y(0)=2. The initial condition $y(x_0) = y_0$ is used to determine a value of c in the general solution.

Once when the student is thorough with the mathematical knowledge modeling, the next step simulation process can occur and then it can be experimented at a wider range. This is what the concept of application of a differential equation applies to the needs of the development of the society.

Different approaches of a Differential equation which is solved Analytically, Graphically and Numerically:

Example:

- Let us consider $y' = x + y$

$dy/dx = x + y$

Auxiliary equation is $m - 1 = 0, m = 1$

Complementary function = Ce^{x}

Particular Integral $= -x - 1$

$y(x) = Ce - x - 1$ is the the required solution (i)

Also a differential equation is said to be linear when the dependent variable and its derivatives occur only in the first degree. The above said differential equation can also be solved by the following method also:

- Let us consider $y' = x + y$

$\frac{dy}{dx} = x + y$

ISSN: 2005-4238 IJAST

$\frac{dy}{dx} - y = x$ and we've , the linear equation to be the form

$$\frac{dy}{dx} + Py = Q$$

Here, P = -1 and Q = x

We've, $ye^{\int Pdx} = \int Qe^{\int Pdx}dx + C$

$e^{\int -dx} = \int xe^{\int -dx}dx + C$

$y(x) = Ce - x - 1$

is the required solution. (ii)

From (i) and (ii) we can see that the same solution has arrived, though the analytic approaches are different. This implies the lot more ways of solving a differential equation analytically where the student must be able to equip themselves by mastering in this field.

4. Graphical Techniques

Engineering students feel that when the solution of a differential equation is arrived when it is given pictorically i.e., when graphically or interpreted geometrically, they feel it very easy and understanding also made them quite better. But some engineering students also show their interest in analytic approaches but they are not able to arrive the exact solution. Engineering students show a wide range of interest in graphical approach (Lomen, D., & Lovclock.D, 1996) because it is easy for them to differentiate between the solutions when decreasing, increasing or maintaining an equivalent or an equilibrium state (Zandieh,M., & Donald M, 1999). The understanding is far better for them when it is pictorial or geometrical so that they get an excellent idea when applying it to the practical real life problems for arriving a better excellent solution. Visualization plays a vital role in graphical approach amidst the students. The analytic approach students did not show much interest in graphical approach as they were striking to their own methodology of school teaching, though they get struck at certain steps and efforts were quietly appreciable.

Example:

- Let us consider the following initial value problem $y' = x + y$, y(0) = 1

In solving most of the differential equation, it is impossible to solve the differential equation in the sense of obtaining an explicit formula for the solution. But despite the absence of an explicit

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solution, we can still learn a lot about the solution through a graphical approach (direction fields) or a numerical approach (Euler's method).

We don't have a formula for the solution, but the possible graph can be sketched.

Let's think about what the differential equation means.

The differential equation $y' = x + y$ tells us that the slope at any point (x,y) on the graph (called the solution curve) is equal to the sum of the x and y co-ordinates of the point. In particular, because the curve passes through the point (0,1) i.e., our initial consition needs to be met, its slope must be 0+1=1. So a small portion of the solution curve near the point (0,1) looks like a short line segment through (0,1) with slope 1. Let's draw short line segment at a number of points (x,y) with slope x+y. The result is called a direction field and is shown in the figure below.

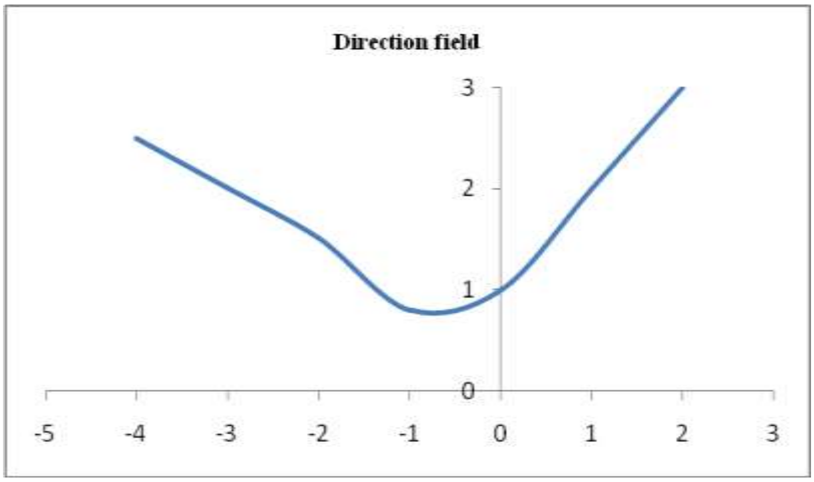

Figure - 3

For example, the line segment at the point (1,2) has slope 1+2=3.

The direction field allows us to visualize the general shape of the solution curves by indicating the direction in which the curves proceed at each point. Now, we can sketch the solution curve through the point (0,1) by following the direction field. We have to draw the curve so that it is parallel to nearby line segments. In general, suppose we have a first order differential equation of the form: y' = F(x,y), where F(x,y) is some expression in x and y. The differential equation says that the slope of a solution curve at a point (x,y) on the curve is F(x,y). If we draw short line segments with slope F(x,y) at several points (x,y), the result is called a direction field(or slope field). These line segments indicate the direction in which a solution curve is heading, so the direction fields helps us to visualize the general shape of these curves (even if we don't know the explicit formula for the solution).

ISSN: 2005-4238 IJAST

5. *Numerical Approaches*

Engineering students while approaching differential equation solving also switch for numerical approaches. They engage in this approach also as they feel easy because, most of the students are bored in solving a differential equation elaborately, instead they store the values of the unknowns in the differential equation using a scientific calculator and they get the values quickly within short period of time and solutions are arrived at first attempt at once. They enjoy the active participation because it is fun for them in the scientific calculator by following the steps instructed by their teacher. They ask the students to follow each step and sometimes the students even tell the teaching community more shortcuts and active involvement. Some cases, the students show a mindset of jumping to graphical method as it is pictorial grasping as no visualization is possible in numerical approach as it easily fixes into the minds and also but most of the students to numerical approach as it is fun with numbers in mathematics like school kids though it is an errorable approach. When numerically solved, definitely we will get the answers with little accuracy of errors, but the studens actively involves in this approach only (Alpaslan, M. M., Yalvae. B.. Loving, C, C., & Willson, V, 2016).

Example:

When a differential equation is to be solved numerically, a student must be able to decide which method is applicable for solving an ordinary differential equation. First this preliminary knowledge is important for a student to solve it. So, let's see how to opt the method for solving differential method numerically. Let us consider the same differential equation and solve it by both Euler's method and Runge - Kutta method to know the accuracy error.

Euler's method

Euler's method is a numerical method that is used to approximate the solution to an initial value problem with a differential equation that can't be solved using a more traditional method, like the methods we use to solve separable, exact, or linear differential equations. Euler's method give inaccurate results when compared to other methods since the step size is larger here. So, student must be able to choose a larger step size to get more accuracy in the answer. Though Euler method is neither efficient nor accurate as other methods, it is useful for the better understanding for the students of numerical methods in differential equations.

Let us consider the following initial value problem, choosing h = 0.2 and computing $y_1, .., y_5$ to

$$y' = x + y, y(0) = 0 \quad (6)$$

Here $f(x, y) = x + y$, and we have, $y_{n+1} = y_n + 0.2(x_n + y_n)$

Table-1 shows the computation, the values of the exact solution, $y(x) = e \wedge x - x - 1$ obtained from (6) and the error. In practice the exact solution is unknown, but an indication of the accuracy of the values can be obtained by applying Euler's method once more with step with step 2h = 0.4 and comparing corresponding approximations. This computation is:

x_n	y_n	$0.4(x_n + y_n)$	y_n in Table-1	Difference
0.0	0.000	0.000	0.000	0.000
0.4	0.000	0.160	0.040	0.040
0.8	0.160		0.274	0.114

Since the error is of order h^2, in a switch from h to 2h it is multiplied by $2^2 = 4$, but since we then need only half as many steps as before, it will only be multiplied by 4/2 = 2. Hence the difference $2\varepsilon_2 - \varepsilon_2$= 0.040 indicates the error ε_2 of y_2 in Table-1 (which actually is 0.052), and 0.114 that of y_4 (actual: 0.152)

Table-1 – Euler Method Applied to (6) and Error

n	x_n	y_n	$0.2(x_n + y_n)$	Exact values	Error
0	0.0	0.000	0.000	0.000	0.000
1	0.2	0.000	0.040	0.021	0.021
2	0.4	0.040	0.088	0.092	0.052
3	0.6	0.128	0.146	0.222	0.094
4	0.8	0.274	0.215	0.426	0.152
5	1.0	0.489		0.718	0.229

Runge - Kutta method

Runge - Kutta method of fourth order is a numerical technique to solve ordinary differential equation. Usually error in Euler method is higher than higher order RK method because truncation error in higher order methods is less compared to Euler method. If the exact solution to the differential equation is a polynomial of order n, it will be solved exactly by an n-th Runge-Kutta method.

Let us consider the following initial value problem, choosing h = 0.2 and computing $y_1,.., y_5$ to

$$y' = x + y, y(0) = 0 \tag{6}$$

Here f(x,y) = x + y. Hence

$$k_1 = 0.2(x_n + y_n)$$
$$k_2 = 0.2(x_n + 0.1 + y_n + 0.5k_1)$$
$$k_3 = 0.2(x_n + 0.1 + y_n + 0.5k_2)$$
$$k_4 = 0.2(x_n + 0.2 + y_n + k_3)$$

Since these expressions are so simple, we find it convenient to insert k_1 into k_2, obtaining the auxiliary quantity

k_2 = 0.22(x_n + y_n)+0.02 insert this into k_3, finding k_3 = 0.222(x_n + y_n)+0.022 and finally insert this into k_4, finding

k_4 = 0.2444(x_n + y_n)+0.0444. If we use these expressions, the formula for y_{n+1}, we get

$$y_{n+1} = y_n + 0.2214(x_n + y_n) + 0.0214 \tag{7}$$

Our present inserting process is not typical of the Runge-kutta method and should not be tried in general. Table-2 shows the computations.

Table – 2 – Runge – Kutta method applied to (6), Computations by the use of (7)

n	x_n	y_n	$y_{n+1} = y_n + 0.2214(x_n + y_n) + 0.0214$	Exact values $y(x) = e^x - x - 1$	10^6 * Error of y_n
0	0.0	0	0.021400	0.000000	0
1	0.2	0.021400	0.070418	0.021403	3
2	0.4	0.091818	0.130289	0.091825	7
3	0.6	0.222106	0.203414	0.222119	11
4	0.8	0.425521	0.292730	0.425541	20

ISSN: 2005-4238 IJAST

5	1.0	0.718251		0.718282	31

Table – 3: Comparison of the Accuracy of the Two methods under consideration in the case of the Initial Value Problem (6) with h = 0.2

χ	$y(x)=e^{x}-x-1$	Euler method (Table -1)	Runge-kutta (Table -2)
0.2	0.021403	0.021	0.000003
0.4	0.091825	0.052	0.000007
0.6	0.222119	0.094	0.000011
0.8	0.425541	0.152	0.000020
1.0	0.718282	0.229	0.000031

From Table -3 we see that the values are much more accurate in Runge-kutta method than in Euler method. Students must have a very good knowledge in adapting the apt methods to solve a differential equation.

6. *Mind Reading Approach*

Here is a little like solution in solving differential equations to the engineering students. It is that the writing practice. Both the community of teachers and engineering students strongly agree with this as it helps them to last in their mind forever (Braten,I., & Stromso,H.I,2005) once or twice when it is handwritten. This might look like a simple solution in the teaching and learning process of differential equation to engineering students since the ancient period that writing practice has a vast impact. Though it is an old idea that writing a piece of anything can't be replaced by any technology however recent the technology it is (Lave,J, 1988). Many researchers in the past and the recent days tell that whatever the notes of lessons and concepts a teacher teaches a student in whatever the interesting way it is, the student community must take its own steps which is the need of the hour.

In this present scene among the students, there are too much deviations for them. They have large number of groups among them, usage of mobile phones, neighbouring disturbances, learning environment where he/she must have the same mindset of travelling in their studies, surrounding in the classroom and less student teacher interactions are going on. Leaving all these beyond, every student must attend a class attentively (Muis, K.R.,&franco, G.M, 2009) and write their notes in their own style explained by their teacher to them. Our Brain is always conscious and grasp concepts easily only when it is written by our own handwriting and it is better for future reference also. Whenever mind starts to approach reading, a student must have the first and the best practice of writing and grasping a concept (Raychaudhuri, D, 2008)which will never fade away as it sticks strong into their reading mind where a base becomes stronger.

7. Teaching and Learning of Differential Equations

Teaching and learning of differential equation can be well-versedly done by the utilization of technology advancements (Meier,J.,Rishel .T, 1998). Many effective tools have been introduced. Right from the year 1996 to 2020, many software tools have been introduced for the benefits of students and teachers. Many software tools like Odetool, which is a Java program that helps in the differential equations to calculate, visualize and explore solutions for the users who may be a researcher, student or a teacher. Some more tools loke Matlab, Maple, vee diagrams, concept maps (Afamasaga-Fuata i. K, 2001) play a vital role in research and problem solving differential equation. An effective way is highly appreciated by the engineering students as in the present situation they are much interested in learning software.

The new trend now in schools followed widely is teaching visually. It is incorporated by the smart classroom teaching. All classrooms even government is taking initiative to incorporate in the school classroom. This gives them a visualized learning (Borrelli, R.L.,& Coleman,C.S,1999) and when tools are used, they are much interested in using computers, students take it a special case to learn and of course interesting too.

This takes a while to start from schools and then get transferred to colleges, universities, where we have an another software ActivInspire which is now enabling smart board culture classrooms in a most efficient way. The software almost like an advanced paint application in Microsoft word, is where we can import the videos, teaching methodologies, interesting simple exercises to the students to twist their brains in differential equations. First a simple differentiation problem and then increasing the grade. Through this ActivInspire, we can import presentation, draw figures, tables, graphs in a colourful way in the board and the students grasp the concepts easily in their own way.

Traditionally, we have chalk and talk method by the teachers which had a great impact by the students. But this modern generation students are interested in colourful active based learning with practical applications and assignments. When differential equation is taught by such tools they easily recognize the ups, downs or the constant values of the solution of the given differentiation equation (Habre,S,2000).

Apart from all the schemes of teaching and learning differential equations, the foremost important thing a student must do his he/she must know the course plan of the paper which he/she is studying. Because once he knows the topics to be taught the next day by his/her teacher, he can have a key idea about that topic by surfing. When this system has been carried out, the next day what the teacher comes to explain briefly can be understood quickly by correlating the concepts what he/she has grasped already (Keene,K.A., Glass, M., & Kim, J, H, 2011). It is not quite simple to motivate a student (Wolters, C.A., Shirley ,L.Y .,& Pintech, P.R, 1996) newly into a topic of study. Instead the student himself/herself must have a simple motivational interest towards the subject (Velayuthum ,S., Aldridge, J.M.,&Fraser.B, 2012).

The main field where the teaching community faces problems in teaching differential equations is that engineering students with less rate of interest towards differential equations(Camacho-Machin.M. & Guerreo-Ortiz,C,2015). This bridge must be breaked down and only it can be done by the teachers (Camcho-Machin. M., Perdomo-Diaz,J.,& Santos-Trigo,M, 2012a). Some concepts in classrooms must be revamped in this generation 2020 to make the teaching and learning of differential equation much more accessible (Hofer, B.K,1999). Almost all the developing countries face this problems in teaching process. Some of them are more number of student strength in a class which leads to not able to provide attention to the whole class, then an hour of class is not enough for effective teaching (Camacho-Machin,M., Perdomo-Diaz.J., & Satos-Trigo,M, 2012b). A class of a subject must be two hours so that a teacher may not rush up with the concepts, clear all the doubts of the students (Porter,M.K & Masingila ,J.O., 1995)and once a concept is started it can be done on the same day without taking the balance portion to the next day, so that it gives a chain like continuity in the concepts.

8. Other Perspectives

Getting into a solution of the real life problem using differential equation is not that much easy on the other hand both by the communities of teachers and engineering students. Teachers must be well-versed in the concepts of softwares and theories, where they can inculcate in the minds of the engineering students. This is not such an easy task. Everyone can't be superior in all the concepts. The students to be getting developed in the field of differential equations and to invent new technologies by applying these concepts must be very thorough from the basic point of differentiation. To test these skills, multiple choice questions of this domain must be frequently conducted as a test to make them superior from the base (Habre,S, 2002).

The concept what we are telling to enhance the teaching and learning process of differential equation is active learning. Objective type questions from simple concepts of differentiation can be frequently conducted as a test to the engineering students. But there are many practical difficulties were among more works other than academics to a teacher in an instituition, they don't have time to value the answer scripts of the students. To overcome this difficulty, softwares like KAHOOT can be used where a teacher can set a certain set of questions for that particular day and display it (Kingir,S., Tas, Y., Gok, G., & Vural, S.S, 2013). The engineering students can actively involve in it using their login through mobile phone (YurtsevenAvci ,Z., Vasu E,S Oliver,K.,keene ,K A .,&Fusarelli,B, 2014). When the question is displayed by the teacher with or without the options, they can point the option through their mobile phones and immediately the teacher will be able to know the answering capacity of the students from the topper to the last one. Using such type of softwares is not only for effective teaching but psychologically it is relief and stress buster to both the student and a teacher. Both the communities learn concepts through game mode. The students play as well as learn. Though it seems to be something simple, it takes a deeper place in the teaching and learning of differential equations (Machin M.C.,Diaz,J.P.,&Trigo,L.M.S, 2009). Continuous practices in differential equations to the engineering students (Arslan, S, 2010a) is the only way to master in the field which paves the new way for inventions.

In any mathematical domain, students may be some higher count will have the capacity of solving linear, second order, third order, homogeneous or non-homogeneous differential equations, But when questioned in a basic concept of differentiation, engineering students struggle at most of the times. They must be cornered from the basics (Perels, F.,Gurtler ,T.,& Schmitz,B, 2005), that is the only way for effective teaching and learning (Afamasaga-Fuata i. K, 2003a). Students who do problems for pages by memorizing feel really difficult when asked questions in the beginner level (Rasmussen,C.L, 1998).

9. Conclusions

The study that we wish to conclude from this discussion it is a belting relationship between the students and teachers in studying differential equations. Inspite of using all softwares and technical learning perspectives to the engineering students by the teachers/professors, the students must actively participate in the learning strategy. The articles from 1996 to 2020 have been critically reviewed and analyzed in the teaching and learning techniques in differential equations. Some students solve the problems without the clear cut concept of the domain and applications. Though the teaching and learning is traditional based or new technological based it is an ever-lasting relation between both the communities.

Compared to the analytic traditional approach, students give high importance to the numerical, graphical and technological based learning of differential equations. It is a remarkable note in the change of teaching and learning process of differential equations. The most effective way of learning differential equations by the engineering students is their regular self disciplined writing practice approach in an elegant manner. The inspiring words and methodology of teaching handled by the teachers and the self disciplined strategy together found effective in the development of differential equations in the mathematical domain to the core. This pedagogy decides whether a teacher can introduce a researcher or an engineer to this innovation needy world. This combination highly enhanced the novelity of differential equations both at the school and college levels of study.

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