A Comparative Study of Linear Receivers in Spectral Efficiency of Uplink Massive MIMO Systems with Low Resolution ADCs

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Abstract

In Massive MIMO systems, Spectral Efficiency mainly depends on the number of antennas used at the Base Station (BS). In 5G, the data requirement is very much high and the increase in Spectral Efficiency is crucial. This letter proposes the uplink Massive MIMO systems with low resolution ADCs architecture. We show that the massive number of antennas compensate the performance degradation of low resolution ADCs and increase the Spectral Efficiency which will be the major requirements in 5G Internet of Things (IOT) networks. We compare the performance of Massive MIMO systems for different receivers with low resolution ADCs

Keywords: *Massive MIMO, MRC, ZF, MMSE Receivers, low resolution ADC, uplink Spectral Efficiency.*

1. Introduction

Massive multiple Input Multiple Output (MaMIMO) is a new technology for next generation 5G communication networks [1]. MaMIMO can achieve very high Spectral Efficiency by simply increasing the number of antennas at the base station (BS) [2]. The increase in number of antennas increases the Spectral Efficiency as well as the power requirements of the system [3]. The three linear receivers: Maximal Ratio Combiner (MRC), Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) are analysed. MMSE receiver provides good sumrate compared to other linear detectors. ZF outperforms MRC and it requires less statistical knowledge than MMSE receiver. In this work, ZF receiver is considered. The quantized MaMIMO systems give solution for increase in Spectral Efficiency, where each receive antenna in the BS utilizes a relatively low-resolution analog-to-digital converter (ADC). The additive quantization noise model (AQNM) proposed in [11] is widely used to describe the effect of low-resolution ADCs. A new architecture [16] is proposed to increase the sumrate is the mixed ADC architecture. In mixed ADC architecture, some of the ADCs will have high resolution and most of the ADCs will have low resolution ADCs. In [17], the mixed ADC architecture improves the Spectral Efficiency with MRC and ZF receiver is studied. If the coherence interval is small, mixed ADC architecture is preferred. The capacity and estimation accuracy of MaMIMO systems with non-ideal transceiver hardware is analysed and said that the hardware impairments create non-zero errors irrespective of the SNR and the number of base station antenna [5]. The power consumption of the system also suffers the Spectral Efficiency [6]. The uplink Spectral Efficiency of MaMIMO systems with low resolution ADCs over Rician fading channels [7] showed that there is a fixed loss of Spectral Efficiency when the ADC quantization bit is increased. The one-bit ADCs are the least power consumption ADCs which can simplifies the RF chain in MaMIMO system. The capacity of one bit ADC in RF chains of the receiver is analysed in [4], [13] with known channel state information at the transmitter. In [8], a near-maximal likelihood detector for one-bit quantized signals and the linear detectors were proposed in [9], [10].

2. system model

We consider a single-cell multiuser MIMO system. We also assume the uplink transmission in which K single-antenna users (UE) transmit information to M Base station antennas and it

ISSN: 2005-4238 IJAST

becomes a Massive MIMO system. The flat-fading channel $h_k \in C^M$ between the antenna at k^{th} active UE and one of the antennas at the BS is assumed to be Rayleigh block fading with its fading coefficients $h_k \sim CN(0,\lambda_k I_k)$. Uniform distribution with equal datarate is assigned to all the users as in [1]. In the considered massive MIMO system, both small scale fading parameter and large scale fading parameter which is the path loss factor are taken into account. The large-scale fading is modeled [14] via $\beta_k = z_k/(d_k/r_d)^{-3.8}$, where z_k is a log-normal random variable with 8dB standard deviation, d_k is the distance between the BS and the k^{th} user and r_d is the radius of the cell. The received signal at any one of the BS antennas can be expressed as

$$\mathbf{y} = \sqrt{p_u}\mathbf{G}\mathbf{x} + \mathbf{n} \tag{1}$$

where p_u is average of each user's transmit power, $\mathbf{G} \in C^{M\times K}$ represents the channel matrix (considers both small scale and large scale fading) between all the users and all M antennas at the BS. K users' data is combined and the transmitted signal is assumed as $\mathbf{x} \in C^{K\times 1}$ and it is also assumed to be Gaussian. Symbol $\mathbf{n} \sim CN(0,\mathbf{I}_M)$ is the complex normally distributed additive white Gaussian noise vector. The variance of the noise n is assumed to be σ^2 Joule/symbol.

Consider g_{mk} be the $(m,k)^{th}$ entry of $\mathbf{G}$, denoting the channel gain between the k^{th} UE and the m^{th} BS antenna. Channel g_{mk} is modeled as $g_{mk} = \sqrt{\beta_k}h_{mk}$ where β_k represents the path loss component which is considered as the large scale fading given above. h_{mk} represents the Rayleigh flat fading between the k^{th} user's antenna and m^{th} antenna at the BS. $\mathbf{G}$ is the channel matrix which considers both the large scale and small scale fading and it can be written as

$$\mathbf{G} = \mathbf{H}\mathbf{D}^{1/2} \tag{2}$$

where $\mathbf{H}$ is the small-scale fading and $\mathbf{D}$ is the large-scale fading components of $\mathbf{G}$ as explained above as different channel affecting parameters. The received analog signal $\mathbf{y}$ is then quantized by the ADCs which has b bit resolution at the BS. We assume the appropriate automatic gain control. Consider the Additive Quantized Noise Model (AQNM) to obtain the quantized output as [3].

$$\mathbf{y}_q = \mathbf{Q}(y) \approx \Lambda\mathbf{y} + \mathbf{n}_q \tag{3}$$

where $\mathbf{Q}(\cdot)$ represents the quantization operation. The quantization is applied separately to the real and imaginary parts of the given signal and $\mathbf{n}_q \in C^M$ is the additive Gaussian quantization noise. The received signal $\mathbf{y}$ and the quantization noise $\mathbf{n_q}$ are assumed to be independent. Let ρ_m be the inverse of the signal-to-quantization-noise ratio. The matrix Λ is constructed from the value of ρ_m as $\Lambda = \mathrm{Diag}(a_1,a_2,...,a_M)$. The coefficient a_m, m = 1,2,...,M is calculated by $a_m = 1-\rho_m$.

The covariance matrix of the quantization noise n_q is

$$\begin{aligned}\mathbf{R}_{nq} &= E\{\mathbf{n}_q\mathbf{n}_q^H \mid \mathbf{G}\} \\ &= \Lambda(\mathbf{I}_M - \Lambda)diag(p_u\mathbf{G}\mathbf{G}^H + \mathbf{I}_M)\end{aligned} \tag{4}$$

and the channel $\mathbf{G}$ is a fixed channel.
On substituting the equation (1) on (3), the quantized signal becomes

$$\mathbf{y}_q = \sqrt{p_u}\Lambda\mathbf{G}\mathbf{x} + \Lambda\mathbf{n} + \mathbf{n}_q \tag{5}$$

3. Uplink Spectral Efficiency

Asymptotic Equivalent of the uplink Spectral Efficiency

In this section, we analysed the performance of low resolution ADCs. The asymptotic equivalent of the uplink SE for massive MIMO systems with low-resolution ADCs is analysed for the different linear receivers [4] using random matrix theory. Assume Channel State Information (CSI) is known at the BS. With the perfect CSI at the receiver, the received signal detected through the linear detector.

The different detectors analysed here are Maximal Ratio Combining (MRC), Zero Forcing (ZF) and MMSE receivers.

$$\mathbf{F} = \begin{cases} \mathbf{G} & for\, MRC \\ \mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1} & for\, ZF \\ \mathbf{G}^H(\mathbf{G}\mathbf{G}^H + \alpha\mathbf{I}_M)^{-1} & for\, MMSE \end{cases}$$

where $\alpha = 1/p_u$.

a. Analysis of Low resolution ADCs with MRC receiver

In this section, we analysed the achievable uplink rate of low-resolution Massive MIMO system with MRC receiver $\mathbf{F}^H = \mathbf{G}^H$. The received vector of the MRC receiver can be expressed as

$$\mathbf{r} = \mathbf{F}^H\mathbf{y}_q = \Lambda\sqrt{p_u}\mathbf{G}^H\mathbf{G}\mathbf{x} + \Lambda\mathbf{G}^H n + \mathbf{G}^H n_q \tag{6}$$

The k^th user's received data is

$$r_k = a\sqrt{p_u}\mathbf{g}_k^H\mathbf{g}_k x_k + a\sqrt{p_u}\sum_{i=1,i\neq k}^{K}\mathbf{g}_k^H\mathbf{g}_i x_i + a\mathbf{g}_k^H n + \mathbf{g}_k^H n_q \tag{7}$$

The achievable uplink rate of k^th user is

$$R_k = E\left\{\log_2\left(1 + \frac{p_u a^2\|\mathbf{g}_k\|^4}{I}\right)\right\} \tag{8}$$

where *I* is the interference noise power with zero mean and variance is given by

$$I = a^2 p_u \sum_{i=1,i\neq k}^{K}\left|\mathbf{g}_k^H\mathbf{g}_i\right|^2 + a^2\|\mathbf{g}_k\|^2 + a(1-a)\mathbf{g}_k^H diag(p_u\mathbf{G}\mathbf{G}^H + \mathbf{I}_M)\mathbf{g}_k \tag{9}$$

The exact solution for the above equation is difficult to achieve. Hence, the approximate solution is obtained by applying the MRC receiver as in Theorem 1[18],

$$\tilde{R}_k = \log_2\left(1 + \frac{ap_u\beta_k(M+1)}{I}\right) \tag{10}$$

where *I* is given by,

$$I = ap_u\sum_{i=1,i\neq k}^{K}\beta_i + (1-a)p_u\left(\sum_{i=1}^{K}\beta_i + \beta_k\right) + 1$$

b. Analysis of Low resolution ADCs with ZF receiver

ZF receiver $\mathbf{F}^H = (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H$ is applied to the quantized output vector $\mathbf{y_q}$, the output vector of ZF detector is given as

$$\mathbf{r} = \mathbf{F}^H\mathbf{y}_q = \Lambda\sqrt{p_u}\mathbf{F}^H\mathbf{G}\mathbf{x} + \Lambda\mathbf{F}^H n + \mathbf{F}^H n_q \tag{11}$$

The received signal from the k^th user can be expressed as

$$\mathbf{r}_k = a\sqrt{p_u}\mathbf{f}_k^H\mathbf{g}_k x_k + a\sqrt{p_u}\sum_{i=1,i\neq k}^{K}\mathbf{f}_k^H\mathbf{g}_i x_i + a\mathbf{f}_k^H\mathbf{n} + \mathbf{f}_k^H\mathbf{n}_q \tag{12}$$

where $\mathbf{f}_k$ is the k^th column of **F**, $\mathbf{g}_k$ is the k^th column of **G** and x_i is the i^th element of **x**. We can say, $\mathbf{f}_k^H\mathbf{g}_k = 1$ and the interference caused by other users is considered as $\mathbf{f}_k^H\mathbf{g}_i = 0$, for $i = 1\, to\, K,\ i \neq k$. Therefore the received signal of k^th user is modified as

$$\mathbf{r}_k = a\sqrt{p_u}\mathbf{f}_k^H\mathbf{g}_k x_k + a\mathbf{f}_k^H\mathbf{n} + \mathbf{f}_k^H\mathbf{n}_q \tag{13}$$

The k^th user's achievable uplink rate for the ZF receiver is expressed as

$$R_k = E\left\{\log_2\left(1 + \frac{a^2 p_u}{a^2\|\mathbf{f}_k\|^2 + a(1-a)\mathbf{f}_k^H diag(p_u\mathbf{G}\mathbf{G}^H + \mathbf{I}_M)\mathbf{f}_k}\right)\right\}$$

(14)
The solution of the above equation cannot be obtained directly. The approximate solution of the uplink rate can be obtained by substituting $\mathbf{F}^H = (\mathbf{G^H G})^{-1}\mathbf{G}^H$ in to the equation (5). The quantized output vector is then modified as

$$\begin{aligned}\mathbf{r} &= \Lambda\sqrt{p_u}(\mathbf{G^H G})^{-1}\mathbf{G}^H\mathbf{Gx} + \Lambda(\mathbf{G^H G})^{-1}\mathbf{G}^H n + (\mathbf{G^H G})^{-1}\mathbf{G}^H n_q \\ &= \Lambda\sqrt{p_u}\mathbf{x} + (\mathbf{G^H G})^{-1}\mathbf{G}^H(\Lambda n + n_q)\end{aligned} \quad (15)$$

By assuming n and n_q are uncorrelated, the received SINR of the k[th] user is obtained as

$$SINR_k^{ZF} = \frac{a^2 p_u}{[a^2 + a(1-a)[diag(p_u\mathbf{GG}^H)]_{mm}](\mathbf{G}^H\mathbf{G})_{kk}^{-1}} \quad (16)$$

By [15],

$$[diag(p_u\mathbf{GG}^H)]_{mm}] = Kp_u + 1 \quad (17)$$

Thus the k[th] user's received SINR for the ZF receiver is obtained as

$$SINR_k^{ZF} = \frac{a^2 p_u}{[a^2 + a(1-a)(Kp_u+1)(\mathbf{G}^H\mathbf{G})_{kk}^{-1}} \quad (18)$$

The Ergodic uplink rate of the k[th] user is

$$R_k^{ZF} = E\left\{\log_2\left(1 + \frac{a^2 p_u}{[a^2 + a(1-a)(Kp_u+1)(\mathbf{G}^H\mathbf{G})_{kk}^{-1}}\right)\right\} \quad (19)$$

The lower bound on the achievable uplink rate of the k[th] user is calculated using Jensen's inequality as

$$\begin{aligned}R_k^{ZF} &\geq \tilde{R}_k^{ZF} = \log_2(1 + E[SINR_k]) \\ \tilde{R}_k^{ZF} &= \log_2\left(1 + E\left\{\frac{a^2 p_u}{[a^2 + a(1-a)(Kp_u+1)(\mathbf{G}^H\mathbf{G})_{kk}^{-1}}\right\}\right) \\ &= \log_2\left(1 + \frac{a^2 p_u}{[a^2 + a(1-a)(Kp_u+1)E\{(\mathbf{G}^H\mathbf{G})_{kk}^{-1}\}}\right)\end{aligned} \quad (20)$$

When using ZF, with the number of antennas M>K, by proposition 3 in [3], $E\left\{\left[\left(G^H G\right)^{-1}\right]_{kk}\right\} \approx \frac{1}{(M-K)\beta_k}$

Thus the achievable uplink rate of the k[th] user is rewritten as

$$\tilde{R}_k^{ZF} = \log_2\left(1 + \frac{a^2 p_u (M-K)\beta_k}{[a^2 + a(1-a)(Kp_u+1)}\right) \quad (21)$$

Assume, equal power is allocated to all the users and hence all the users have same data rate. Hence, the total sumrate is expressed as the sum of datarates of all users.

$$\begin{aligned}\tilde{R} &= \sum_{k=1}^{K}\tilde{R}_k^{ZF} \\ &= K\log_2\left(1 + \frac{a^2 p_u (M-K)\beta_k}{[a^2 + a(1-a)(Kp_u+1)}\right)\end{aligned} \quad (22)$$

c. Analysis of Low resolution ADCs with MMSE receivers

MMSE receiver is designed as $\mathbf{F} = \mathbf{G}^H(\mathbf{GG}^H + \alpha\mathbf{I}_M)^{-1}$, where $\alpha = \frac{1}{p_u}$. By multiplying $\mathbf{F}$ with $\mathbf{y}_q$, the quantized output vector is proposed as

$$\mathbf{r} = \mathbf{Fy}_q = \sqrt{p_u}\mathbf{F}\Lambda\mathbf{Gx} + \mathbf{F}\Lambda\mathbf{n} + \mathbf{Fn}_q \quad (23)$$

The MMSE detected signal for the k[th] user can be expressed as

$$\mathbf{r}_k = \sqrt{p_u}\mathbf{g}_k^H\mathbf{Vg}_k x_k + \sqrt{p_u}\sum_{i\neq k}\mathbf{g}_k^H\mathbf{Vg}_i x_i + \mathbf{g}_k^H\mathbf{Vn} + \mathbf{g}_k^H\mathbf{Vn}_q \quad (24)$$

where $\mathbf{V} = (\mathbf{G}\mathbf{G}^H + \alpha \mathbf{I}_M)^{-1}$, $\mathbf{g}_i$ is the i[th] column of $\mathbf{G}$ and x_i is the i[th] element of **x**. The achievable uplink rate of the k[th] user is expressed as

$$R_k = \log_2\left(\frac{1 + p_u \left|\mathbf{f}_k^H \mathbf{V}\mathbf{g}_k\right|^2}{\mathbf{I}_k}\right) \quad (25)$$

where the variance of the interference-plus-noise term is $I_k = p_u\left|\sum_{i \neq k} \mathbf{f}_k^H \mathbf{V}\mathbf{g}_i\right|^2 + \mathbf{f}_k^H \mathbf{V}\mathbf{A}^2 \mathbf{V}\mathbf{g}_k + \mathbf{f}_k^H \mathbf{V}\mathbf{R}_{n_q} \mathbf{V}\mathbf{g}_k$

4. Numerical results

We consider a hexagonal cell with a radius of 1000 meters and the user locations are generated randomly in the cell by following uniform distribution. The radius of the central disk is set as r_d =100 meter. The distance between each user and the BS is at least 100 meters. As mentioned earlier, one of the parameter which is included in the channel matrix is the log normal shadowing. So, the large-scale fading is modeled through $\beta_n = z_n(r_n/rd)^{-3.8}$, where z_n is shadowing which is modeled as a log-normal random variable with variance 16dB (standard deviation of 8dB) and r_n is the distance between the n[th] user and the BS. Let b_m gives the resolution of ADCs i.e the number of quantization bits serving the m[th] antenna. The relation between b_m and ρ_m for $b_m \leq 5$ is listed in Table I [11]. For $b_m > 5$, values of ρ_m are calculated through $\rho_m \frac{\pi\sqrt{3}}{2} 2^{-2b_m}$, which is an approximation under high-resolutions [12].

TABLE I

ρ_m for different values of $b_m > 5$

b_m	1	2	3	4	5
ρ_m	0.3634	0.1175	0.03454	.009497	0.002499

In Fig. 1, the Spectral Efficiency of three different linear detectors is compared with number of BS antennas for quantization bit sizes of 1, 2 and infinity. Here, the power p_u is considered to be 10 dB. We also considered the number of users as K=10. It is observed that the uplink Spectral Efficiency is increased with the number of BS antennas. Also, we observe that the ZF and MMSE provide higher spectral Efficiency compared with MRC receiver.

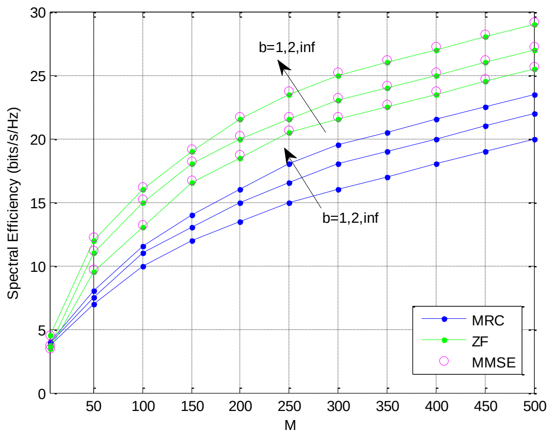

Fig.1. Spectral Efficiency versus number of BS antennas for Linear detectors with P_u=10dB and K=10.

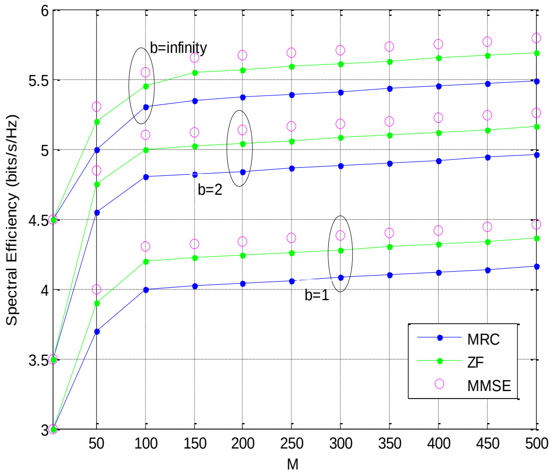

Fig. 2. Spectral Efficiency versus Transmit Power for different ADC quantization bits with M=200.

According to power scaling law [2], we assume that the transmit power is scaled down by M for each user, i.e $p_u = \frac{E_u}{M}$, where E_u is fixed. As $M \rightarrow \infty$, the uplink rate is improved. Fig. 2 shows that the Spectral Efficiency is increased with the quantization bit size. However, the gap between the SE with quantization bit size 2 and infinity is not high. This shows that by increasing the number of antennas, quantization error can be compensated.

5. Conclusion

In this paper, we have investigated the Energy Efficiency of the Massive MIMO systems with low resolution ADCs. Here, three different receivers are compared for Low resolution ADCS. We have identified that the effect of massive number of antennas at the Base station influences the bit resolution of ADCs in RF chain. Spectral Efficiency is increased with the quantization bit size and massive number of antennas. The result proves that the quantization error can be compensated by the massive number of antennas at the Base station. Also, we identified that the MMSE and ZF receivers outperforms MRC receiver.

References

1. E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, “Massive MIMO for next generation wireless systems,” IEEE Commun. Mag., vol. 52, no. 2, pp. 186–195, Feb. 2014.
2. F. Rusek et al., “Scaling Up MIMO: Opportunities and Challenges with Very Large Arrays,” IEEE Sig. Proc. Mag., vol. 30, Jan. 2013, pp. 40–60.
3. H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, “Energy and spectral efficiency of very large multiuser MIMO systems,” IEEE Trans. Commun., vol. 61, no. 4, pp. 1436–1449, Apr. 2013.
4. Yitian Dong, Ling Qiu, “Spectral Efficiency of Massive MIMO systems with low-Resolution ADCs and MMSE Receiver” IEEE Communication letters, vol. 21, No. 8, pp.1771-1774, August, 2017.
5. Emil Bjornson, Jakob Hoydis, Marios Kountouris, Merouane Debbah, “Massive MIMO systems with Non-Ideal Hardware: Energy Efficiency, Estimation and Capacity limits”, IEEE Trans. on Information Theory, vol. 60, no. 11, pp. 7112-7139, Nov 2014.
6. E. Björnson, L. Sanguinetti, J. Hoydis, and M. Debbah, “Optimal design of energy-efficient multi-user MIMO systems: Is massive MIMO the answer?” IEEE Trans. Wireless Commun., vol. 14, no. 6, pp. 3059–3075, Jun. 2015.

7. J. Zhang, L. Dai, S. Sun, and Z. Wang, "On the spectral efficiency of massive MIMO systems with low-resolution ADCs," IEEE Commun. Lett., vol. 20, no. 5, pp. 842–845, May 2016.
8. J. Choi, J. Mo, and R. W. Heath, Jr., "Near maximum-likelihood detector and channel estimator for uplink multiuser massive MIMO systems with one-bit ADCs," IEEE Trans. Commun., vol. 64, no. 5, pp. 2005–2018, Mar. 2016.
9. S. Jacobsson, G. Durisi, M. Coldrey, U. Gustavsson, and C. Studer, "Massive MIMO with low-resolution ADCs," ArXiv E-Print, Feb. 2016, arXiv:1602.01139 [cs.IT].
10. C. Risi, D. Persson, and E. G. Larsson, "Massive MIMO with 1-bit ADC," ArXiv E-Print, Apr. 2014, arXiv:1404.773
11. A. K. Fletcher, S. Rangan, V. K. Goyal, and K. Ramchandran, "Robust predictive quantization: Analysis and design via convex optimization," IEEE J. Sel. Topics Signal Process., vol. 1, no. 4, pp. 618–632, Dec. 2007.
12. J. Max, "Quantizing for minimum distortion," IRE Trans. Inf. Theory, vol. 6, no. 1, pp. 7–12, Mar. 1960.
13. J.Moand, R.W.HeathJr, "Capacity analysis of one-bit quantized MIMO systems with transmitter channel state information," IEEE Trans. Signal Process., vol. 63, no. 20, pp. 5498–5512, Oct. 2015.
14. Tianle Liu, Jun Tong, Qinghua Guo, Jiangtao Xi, Yanguang Yu, Zhitao Xiao, "Energy Efficiency of uplink Massive MIMO systems with Successive Interference Cancellation", IEEE Letters, vol. 21, no. 3, pp. 668-671, March 2017.
15. A.Azizzadeh, R.Mohammadkhani,S.V.A.D. Makki and E. Björnson, "BER performance analysis of coarsely quantized uplink Massive MIMO", Signal Processing-Elsevier, 2019
16. Ning Liang and Wenyi Zhang, "Mixed – ADC Massive MIMO, "IEEE Journal on Selected areas in Communications", vol. 34, no. 4, pp. 983-997, Apr. 2016.
17. Hessam Pirzadeh, A.Lee Swindlehurst, "Spectral Efficiency of Mixed – ADC Massive MIMO", IEEE Transactions on Signal Processing, vol. 66, no. 13, pp. 3599-3613, May 2018.
18. Li Fan,Shi Jin, Chao-Kai Wen, Haixia Zhang, "Uplink Achievable rate for Massive MIMO Systems with Low-Resolution ADC", IEEE Communication Letters, vol.19, no.12, Dec. 2015.