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Exponential meanness of some graphs and its line graph

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Abstract

In this article, it is try to analyze that the line graph of exponential meanness of some standard graphs are also exponential mean graphs .

Keywords: *— Labeling, line graph, exponential mean labeling, exponential mean graph.*

1. Introduction

Follow [1] for the graph theory concepts and [2] for labeling survey.

The line graph $L(G)$ of a graph G is defined to have as its vertices the edges of G, with two being adjacent if the corresponding edges share a vertex in G. Arockiaraj et al. introduced the F-root square mean labeling of the graphs in [3] and Rajesh Kannan et al. defined the exponential mean graphs in [4]. A function χ is called an exponential mean labeling of a graph $G(V,E)$ with p vertices and q edges if $\chi : V(G) \rightarrow \{1,2,3,\cdots,q+1\}$ is injective and the induced function $\chi^* : E(G) \rightarrow \{1,2,3,\cdots,q\}$ defined as

$$\chi^*(uv) = \left\lfloor \frac{1}{e}\left(\frac{\chi(v)^{\chi(v)}}{\chi(u)^{\chi(u)}}\right)^{\frac{1}{\chi(v)-\chi(u)}} \right\rfloor,$$

where $\chi(u) < \chi(v)$, for all $uv \in E(G)$, is bijective. A graph that admits an exponential mean labeling is called an exponential mean graph. Motivated by the works of so many authors in the area of graph labeling, here try to analyze that the line graph of exponential meanness of some standard graphs are also exponential mean graphs.

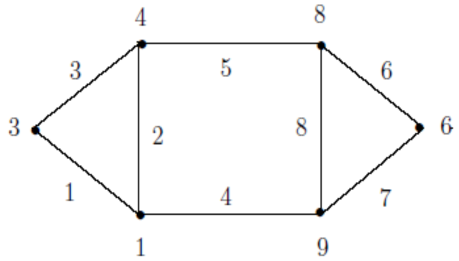

Figure 1. An exponential mean labeling of the graph

Theorem 1.1. [4]. Every path P_n is an exponential mean graph, for $n \geq 1$.

2. MAIN RESULTS

Observation 2.1 The line graph of the path P_n is an exponential mean graph, for $n \geq 2$.
Proof. Since $L(P_n)$ is again a path, by theorem 1.1, the result follows.
Theorem 2.2 Every cycle C_n is an exponential mean graph, for $n \geq 3$.
Proof. Let $u_1, u_2, u_3, \cdots, u_n$ be the vertices of the cycle C_n.
Define $\chi : V(C_n) \rightarrow \{1,2,3,\cdots,n+1\}$ by

$$\chi(v_\alpha)=\begin{cases} 2\alpha-1, & 1\le\alpha\le\left\lfloor\frac{n}{2}\right\rfloor+1 \\ 2n-2\alpha+4, & \left\lfloor\frac{n}{2}\right\rfloor+2\le\alpha\le n. \end{cases}$$

Then the induced edge labeling is as follows.

$$\chi^*(u_\alpha u_{\alpha+1})=\begin{cases} 2\alpha-1, & 1\le\alpha\le\left\lfloor\frac{n}{2}\right\rfloor+1 \\ 2n-2\alpha+2, & \left\lfloor\frac{n}{2}\right\rfloor+2\le\alpha\le n-1. \end{cases}$$

Hence, χ is an exponential mean labeling of the cycle C_n. Thus the cycle C_n is an exponential mean graph, for $n\ge 3$. □

Observation 2.3 The line graph of the cycle C_n is an exponential mean graph, for $n\ge 3$.

Proof. Since $L(C_n)$ is again a cycle, for $n\ge 4$ and $L(C_3)=P_2$ by Theorem 2.2 and Theorem 1.1, the result follows.

Theorem 2.4 The graph $P_n\circ S_1$ is an exponential mean graph, for $n\ge 1$.

Proof. Let $v_1,v_2,v_3,\ldots,v_n$ be the vertices of P_n and $u_1^{(\alpha)}$ be the pendant vertices at each v_α, for $1\le\alpha\le n$.

Define $\chi:V(P_n\circ S_1)\to\{1,2,3,\cdots,2n\}$ by

$\chi(v)$	$\alpha=1$	$2\le\alpha\le n$
$\chi(v_\alpha)$	1	2α
$\chi(u_1^{(\alpha)})$	2	$2\alpha-1$

Then the induced edge labeling is as follows.

$\chi^*(e)$	$1\le\alpha\le n-1$	$\alpha=n$
$\chi^*(v_\alpha v_{\alpha+1})$	2α	-
$\chi^*(v_\alpha u_1^{(\alpha)})$	$2\alpha-1$	

Hence, χ is an exponential mean labeling of the graph $P_n\circ S_1$. Thus the graph $P_n\circ S_1$ is an exponential mean graph, for $n\ge 1$. □

Theorem 2.5 The line graph of $P_n\circ S_1$ is an exponential mean graph, for $n\ge 2$.

Proof. Let $V(P_n)=\{v_1,v_2,v_3,\cdots,v_n\}$ and $E(P_n)=\{e_\alpha=v_\alpha v_{\alpha+1}:1\le\alpha\le n-1\}$ be the vertex set and edge set of the path P_n. Then $V(L(P_n\circ S_1))=\{v_1,v_2,v_3,\ldots,v_n,e_1,e_2,e_3,\ldots e_{n-1}\}$ and $E(L(P_n\circ S_1))=\{v_\alpha e_\alpha,e_\alpha v_{\alpha+1}:1\le\alpha\le n-1\}\cup\{e_\alpha e_{\alpha+1};1\le\alpha\le n-2\}$.

Define $\chi:V(L(P_n\circ S_1))\to\{1,2,3,\cdots,3n-3\}$ by

$\chi(v)$	$\alpha=1$	$2\le\alpha\le 3$	$4\le\alpha\le n-1$	$\alpha=n$
$\chi(v_\alpha)$	$2\alpha-1$		$3\alpha-4$	
$\chi(e_\alpha)$	2	3α		-

Then the induced edge labeling is as follows.

$\chi^*(e)$	$\alpha=1$	$\alpha=2$	$3\le\alpha\le n-1$	$\alpha=n-2$
$\chi^*(v_\alpha e_\alpha)$	$3\alpha-2$		$3\alpha-3$	-

$\chi^*(e_\alpha v_{\alpha+1})$	$3\alpha-1$		-
$\chi^*(e_\alpha e_{\alpha+1})$	3	$3\alpha+1$	

Hence, χ is an exponential mean labeling of the graph $L(P_n \circ S_1)$. Thus the graph $L(P_n \circ S_1)$ is an exponential mean graph, for $n \geq 2$. □

Theorem 2.6 The graph $P_n \circ S_2$ is an exponential mean graph, for $n \geq 1$.

Proof. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of P_n and $u_1^{(\alpha)}, u_2^{(\alpha)}$ be the pendant vertices at each v_α, for $1 \leq \alpha \leq n$.

Define $\chi : V(P_n \circ S_2) \to \{1,2,3,\cdots,3n\}$ by

$\chi(v)$	$1 \leq \alpha \leq n$
$\chi(v_\alpha)$	$3\alpha-1$
$\chi(u_1^{(\alpha)})$	$3\alpha-2$
$\chi(u_2^{(\alpha)})$	3α

Then the induced edge labeling is as follows.

$\chi^*(e)$	$1 \leq \alpha \leq n-1$	$\alpha = n$
$\chi^*(v_\alpha v_{\alpha+1})$	3α	-
$\chi^*(v_\alpha u_1^{(\alpha)})$	$3\alpha-2$	
$\chi^*(v_\alpha u_2^{(\alpha)})$	$3\alpha-1$	

Hence, χ is an exponential mean labeling of the graph $P_n \circ S_2$. Thus the graph $P_n \circ S_2$ is an exponential mean graph, for $n \geq 1$. □

Theorem 2.7 The line graph of $P_n \circ S_2$ is an exponential mean graph, for $n \geq 2$.

Proof. Let $u_1, u_2, u_3, \ldots, u_n$ be the vertices of P_n and v_α, w_α be the pendant vertices attached at $u_\alpha, 1 \leq \alpha \leq n$ in $P_n \circ S_2$. The edge set of $P_n \circ S_2$ is $\{x_\alpha = u_\alpha u_{\alpha+1} : 1 \leq \alpha \leq n-1\}$ $\cup \{y_\alpha = u_\alpha v_\alpha : 1 \leq \alpha \leq n\} \cup \{z_\alpha = u_\alpha w_\alpha : 1 \leq \alpha \leq n\}$.
Let $V(L(P_n \circ S_2)) = \{x_\alpha : 1 \leq \alpha \leq n-1\} \cup \{y_\alpha, z_\alpha : 1 \leq \alpha \leq n\}$ and
$E(L(P_n \circ S_2)) = \{x_\alpha z_\alpha, x_\alpha y_{\alpha+1}, x_\alpha z_{\alpha+1}, x_\alpha y_\alpha ; 1 \leq \alpha \leq n-1\} \cup \{x_\alpha x_{\alpha+1} ; 1 \leq \alpha \leq n-2\} \cup$ $\{y_\alpha z_\alpha : 1 \leq \alpha \leq n\}$ be the vertex set and edge set of the graph $L(P_n \circ S_2)$.
Define $\chi : V(L(P_n \circ S_2)) \to \{1,2,3,\cdots,6n-5\}$ by

$\chi(v)$	$\alpha = 1$	$\alpha = 2$	$3 \leq \alpha \leq n-1$	$\alpha = n$
$\chi(x_\alpha)$	5	6α		-
$\chi(y_\alpha)$	1	$6\alpha-8$		
$\chi(z_\alpha)$	$6\alpha-4$		$6\alpha-5$	

Then the induced edge labeling is as follows.

$\chi^*(e)$	$\alpha = 1$	$2 \leq \alpha \leq n-2$	$\alpha = n-1$	$\alpha = n$
$\chi^*(x_\alpha x_{\alpha+1})$	$6\alpha+2$		-	-

$\chi^*(x_\alpha y_\alpha)$	2	$6\alpha-5$	-
$\chi^*(x_\alpha z_\alpha)$	$6\alpha-3$		-
$\chi^*(x_\alpha z_{\alpha+1})$	6α		-
$\chi^*(x_\alpha y_{\alpha+1})$	$6\alpha-2$		-
$\chi^*(y_\alpha z_\alpha)$	1	$6\alpha-7$	

Hence, χ is an exponential mean labeling of the graph $L(P_n \circ S_2)$. Thus the graph $L(P_n \circ S_2)$ is an exponential mean graph, for $n \geq 2$. □

Theorem 2.8 The graph $[P_n; S_1]$ is an exponential mean graph, for $n \geq 1$.

Proof. Let $u_1, u_2, u_3, \ldots, u_n$ be the vertices of P_n and $v_1^{(\alpha)}, v_2^{(\alpha)}, v_3^{(\alpha)}, \cdots, v_{m+1}^{(\alpha)}$, be the vertices of the star graph S_m such that $v_1^{(\alpha)}$ is the central vertex of the star graph S_m, for $1 \leq \alpha \leq n$.
Define $\chi : V([P_n; S_1]) \to \{1,2,3,\cdots,3n\}$ by

$\chi(v)$	$1 \leq \alpha \leq n$	
	n is odd	*n is even*
$\chi(u_\alpha)$	3α	$3\alpha-2$
$v_1^{(\alpha)}$	$3\alpha-1$	
$\chi(v_2^{(\alpha)})$	$3\alpha-2$	3α

Then the induced edge labeling is as follows.

		$\chi^*(e)$		
		$\chi^*(u_\alpha u_{\alpha+1})$	$\chi^*(u_\alpha v_1^{(\alpha)})$	$\chi^*(v_1^{(\alpha)} v_2^{(\alpha)})$
n *is odd*	$1 \leq \alpha \leq n-1$	3α	$3\alpha-1$	$3\alpha-2$
	$\alpha = n$	-		
n *is even*	$1 \leq \alpha \leq n-1$	3α	$3\alpha-2$	$3\alpha-1$
	$\alpha = n$	-		

Hence, χ is an exponential mean labeling of the graph $[P_n; S_1]$. Thus the graph $[P_n; S_1]$ is an exponential mean graph, for $n \geq 1$. □

Theorem 2.9 The line graph of $[P_n; S_1]$ is an exponential mean graph, for $n \geq 1$.
Proof. Let $V(L([P_n; S_1])) = \{u_\alpha, v_\beta, w_\beta : 1 \leq \alpha \leq n-1, 1 \leq \beta \leq n\}$ and
$E(L([P_n; S_1])) = \{u_\alpha u_{\alpha+1} : 1 \leq \alpha \leq n-2\} \cup \{v_\beta w_\beta : 1 \leq \beta \leq n\} \{u_\alpha v_{\alpha+1} : 1 \leq \alpha \leq n-1\} \cup$
$\{u_\beta v_\beta : 1 \leq \beta \leq n-1\}$ be the vertex set and edge set of the graph $L([P_n; S_1])$.
Assume that $n \geq 2$.
Define $\chi : V(L([P_n; S_1])) \to \{1,2,3,\cdots,4n-3\}$ by

$\chi(v)$	$\alpha = 1$	$2 \leq \alpha \leq n-1$	$\alpha = n$
$\chi(u_\alpha)$	4α		-
$\chi(v_\alpha)$	2	$4\alpha-3$	
$\chi(w_\alpha)$	1	$4\alpha-5$	

Then the induced edge labeling is as follows.

$\chi^*(e)$	$\alpha=1$	$2\le\alpha\le n-1$	$\alpha=n$
$\chi^*(u_\alpha u_{\alpha+1})$	$4\alpha+1$		-
$\chi^*(u_\alpha v_{\alpha+1})$	4α		-
$\chi^*(u_\alpha v_\alpha)$	$4\alpha-2$		-
$\chi^*(v_\alpha w_\alpha)$	1	$4\alpha-5$	

Hence, χ is an exponential mean labeling of the graph $L([P_n;S_1])$, for $n\ge 2$..
For $n=1$, the graph $L([P_n;S_1])$ is a path and by Theorem 1.1, the result follows. □

Theorem 2.10 The graph $S(P_n\circ K_1)$ is an exponential mean graph, for $n\ge 1$.

Proof. In $P_n\circ K_1$, let $u_\alpha, 1\le\alpha\le n$, be the vertices on the path P_n and v_α be the vertex attached at each vertex $u_\alpha, 1\le\alpha\le n$. Let x_α be the vertex which divides the edge $u_\alpha v_\alpha$, for $1\le\alpha\le n$ and y_α be the vertex which divides the edge $u_\alpha u_{\alpha+1}$, for $1\le\alpha\le n-1$.
Define $\chi: V(S(P_n\circ K_1))\to\{1,2,3,\cdots,4n-1\}$ by

$\chi(v)$	$1\le\alpha\le n-1$	$\alpha=n$
$\chi(u_\alpha)$	$4\alpha-3$	
$\chi(x_\alpha)$	$4\alpha-2$	
$\chi(y_\alpha)$	$4\alpha-1$	-
$\chi(v_\alpha)$	4α	$4n-1$

Then the induced edge labeling is as follows.

$\chi^*(e)$	$1\le\alpha\le n-1$	$\alpha=n$
$\chi^*(u_\alpha y_\alpha)$	$4\alpha-2$	-
$\chi^*(y_\alpha u_{\alpha+1})$	4α	-
$\chi^*(u_\alpha x_\alpha)$	$4\alpha-3$	
$\chi^*(x_\alpha v_\alpha)$	$4\alpha-1$	$4n-2$

Hence, χ is an exponential mean labeling of the graph $S(P_n\circ K_1)$. Thus the graph $S(P_n\circ K_1)$ is an exponential mean graph, for $n\ge 1$. □

Theorem 2.11 The line graph of $S(P_n\circ K_1)$ is an exponential mean graph, for $n\ge 1$.

Proof. The vertex set and edge set of the line graph of $S(P_n\circ K_1)$ are as given below.
$V(L(S(P_n\circ K_1)))=\{u_\alpha, u'_\beta, v_\alpha, w_\alpha : 1\le\alpha\le n, 1\le\beta\le n-2\}$ and
$E(L(S(P_n\circ K_1)))=\{u_\alpha v_\alpha, v_\alpha w_\alpha : 1\le\alpha\le n\}\cup\{u'_\alpha v_{\alpha+1} : 1\le\alpha\le n-2\}\cup$
$\{u_\alpha u'_{\alpha-1} : 2\le\alpha\le n-1\}\cup\{u'_\alpha u_{\alpha+2} : 1\le\alpha\le n-2\}\cup u_1u_2$.
Assume that $n\ge 3$.
Define $\chi: V(L(S(P_n\circ K_1)))\to\{1,2,3,\cdots,5n-4\}$ by

$\chi(v)$	$\alpha=1$	$2\le\alpha\le n-2$	$\alpha=n-1$	$\alpha=n$
$\chi(u_\alpha)$	3	$5\alpha-6$		$5n-4$
$\chi(u'_\alpha)$	$5\alpha+3$		-	

$\chi(v_\alpha)$	2	$5\alpha-4$		$5n-3$
$\chi(w_\alpha)$	1	5α	$5\alpha-6$	$5n-8$

Then the induced edge labeling is as follows.

$\chi^*(e)$	$\alpha=1$	$2\le\alpha\le n-2$	$\alpha=n-1$	$\alpha=n$
$\chi^*(u_\alpha v_\alpha)$	2	$5\alpha-6$		$5n-5$
$\chi^*(v_\alpha w_\alpha)$	1	$5\alpha-3$		$5n-7$
$\chi^*(u'_\alpha v_{\alpha+1})$	6	$5\alpha+1$	-	-
$\chi^*(u_\alpha u'_{\alpha-1})$	-	5α		-
$\chi^*(u'_\alpha u_{\alpha+2})$	$5\alpha+3$		$5\alpha-1$	-

and $\chi^*(u_1u_2)=3$.
Hence, χ is an exponential mean labeling of the graph $L(S(P_n\circ K_1))$, for $n\ge3$. For $1\le n\le2$, the $L(S(P_n\circ K_1))$ is a path and by theorem 1.1, the result follows. □

Theorem 2.12 The triangular snake T_n is an exponential mean graph, for $n\ge2$.

Proof. The vertex and edge set of the graph T_n are $V(T_n)=\{u_\alpha:1\le\alpha\le n-1\}\cup\{v'_\alpha:1\le\alpha\le n\}$ and $E(T_n)=\{u_\alpha v_\alpha, u_\alpha u_{\alpha+1}, v_\alpha v_{\alpha+1}:1\le\alpha\le n-1\}$.
Define $\chi:V(T_n)\to\{1,2,3,\cdots,3n-2\}$ by

$\chi(v)$	$1\le\alpha\le n-1$	$\alpha=n$
$\chi(u_\alpha)$	3α	-
$\chi(v_\alpha)$	$3\alpha-2$	

Then the induced edge labeling is as follows.

$\chi^*(e)$	$1\le\alpha\le n-1$
$\chi^*(u_\alpha v_\alpha)$	$3\alpha-2$
$\chi^*(u_\alpha v_{\alpha+1})$	3α
$\chi^*(v_\alpha v_{\alpha+1})$	$3\alpha-1$

Hence, χ is an exponential mean labeling of the graph triangular snake T_n. Thus the graph triangular snake T_n is an exponential mean graph, for $n\ge2$. □

Theorem 2.13 The line graph of triangular snake is an exponential mean graph, for $n\ge2$.

Proof. The vertex set and edge set of the graph $L(T_n)$ are $V(L(T_n))=\{x_\alpha:1\le\alpha\le n\}\cup\{x'_\alpha:1\le\alpha\le n-2\}\cup\{y'_\alpha:1\le\alpha\le n-1\}$ and $E(L(T_n))=\{y_\alpha y_{\alpha+1}, x_\alpha x'_{\alpha-1}:1\le\alpha\le n-2\}\cup\{x_\alpha x'_{\alpha+2}:1\le\alpha\le n-2\}\cup\{y_\alpha x_{\alpha+1}, y_\alpha x'_{\alpha-1}, x_\alpha y_\alpha:1\le\alpha\le n-1\}\cup\{x_1x_2, x_2x'_1\}$.
Assume that $n\ge3$.
Define $\chi:V(L(T_n))\to\{1,2,3,\cdots,7n-10\}$ by

$\chi(v)$	$\alpha=1$	$2\le\alpha\le n-2$	$\alpha=n-1$
$\chi(x'_\alpha)$	$7\alpha+3$		-
$\chi(y_\alpha)$	5	$7\alpha-6$	

and

$$\chi(x_\alpha)=\begin{cases}\alpha, & 1\le\alpha\le 2\\ 11, & \alpha=3 \text{ and } \alpha=n\\ 12, & \alpha\ne 3 \text{ and } \alpha=n\\ 7\alpha-9, & 4\le\alpha\le n-1\\ 7\alpha-10, & \alpha=n.\end{cases}$$

Then the induced edge labeling is as follows.

$\chi^*(e)$	$\alpha=1$	$\alpha=2$	$3\le\alpha\le n-1$	$\alpha=n-2$
$\chi^*(y_\alpha y_{\alpha+1})$	6	$7\alpha-3$		
$\chi^*(y_\alpha x_{\alpha+1})$	3	$7\alpha-5$		-
$\chi^*(x_\alpha x'_{\alpha-1})$	-	5	$7\alpha-7$	
$\chi^*(y_\alpha x'_{\alpha-1})$	-	$7\alpha-6$		-
$\chi^*(x_\alpha y_\alpha)$	2α		$7\alpha-8$	-
$\chi^*(x'_\alpha x_{\alpha+2})$	10	$7\alpha+3$		

$\chi^*(x_1x_2)=1$ and $\chi^*(x_2x'_1)=5.$

Hence, χ is an exponential mean labeling of the graph $L(T_n)$, for $n\ge 3.$

For $n=2,$ the graph $L(T_n)$ is a cycle C_3 and by Theorem 2.2, the result follows. □

3. Conclusion

In this paper, it is found that the line graph operation preserves the exponential mean property for some standard graphs. Further investigation can be done to analyze that line graph operation preserves the exponential mean property for other class of graphs.

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