Further results on F-Face magic mean graphs

[1]A.Meena Kumari and [2]S.Arockiaraj
[1]Research Scholar,Department of Mathematics,Bharathiar Univeristy,Coimbatore and Faculty in department of Mathematics,Mepco Schlenk Engineering College
Sivakasi - 626 005, Tamilnadu, India.
[2]Department of Mathematics
Government Arts and Science College
Sivakasi - 626 124, Tamilnadu, India.
Email: [1]meenakumarialagu@gmail.com, [2]psarockiaraj@gmail.com

Abstract

An (0,1,0)-F-Face magic mean labeling is an assignment of labels to the edges of planar graph such that the mean weight of each face is constant. An (1,1,0)- F-Face magic mean labeling is an assignment of labels to the vertices and edges of planar graph such that the mean weight of each face is constant. In this paper, the existence of (0,1,0)-F-Face magic mean labeling and (1,1,0)-F-Face magic mean labeling is proved for ladder graph, globe graph and new graphs obtained by applying graph operations.

Keywords: Labeling,(0,1,0)-F-Face magic mean labeling, (0,1,0)-F-Face magic mean graph, (1,1,0)-F-Face magic mean labeling, (1,1,0)-F-Face magic mean graph.

1. Introduction

Throughout this paper, a graph G is a finite, connected, undirected planar graph having neither loops nor multiple edges. A planar graph is a graph which can be drawn in a plane such that no two edges intersect.

A graph labeling is a mapping defined from graph elements to numbers(usually to the positive or non-negative integers). Different types of labelings have been studied and a survey of graph labeling can be found in[5]. A function ψ is called a mean labeling of graph G if $\psi: V(G) \to \{0,1,2,\dots,|E(G)|\}$ is injective and the inducededge function $\psi^*: E(G) \to \{1,2,\dots,|E(G)|\}$ defined as follows is bijective.

$$\psi^*(uv) = \begin{cases} \dfrac{\psi(u)+\psi(v)}{2} & if\ \psi(u)+\psi(v) \text{ is } even \\ \dfrac{\psi(u)+\psi(v)+1}{2} & if\ \psi(u)+\psi(v) \text{ is } odd \end{cases}$$

The graph which admits mean labeling is called a mean graph[8]. A graph G is magic if the edges of G can be labeled by the numbers $1,2,3,\dots,|E(G)|$, so that the sum of the labels of all the edges incident with any vertex is the same[6]. Motivated by these works, the authors of this paper introduce (0,1,0)-F-Face magic mean labeling and (1,1,0)-F-Face magic mean labeling of graphs as follows:

A bijection $\phi: E(G) \to \{1,2,\dots,|E(G)|\}$ is called a (0,1,0)-F-Face magic mean labeling ((0,1,0)-F-FMML) of G if the induced face labeling

$$\phi^*(f_i) = \left\lfloor \frac{sum\ of\ the\ labels\ of\ the\ edges\ in\ the\ boundary\ of\ f_i}{deg(f_i)} \right\rfloor$$

$$= \left\lfloor \frac{\sum_{e_j \epsilon f_i} \phi(e_j)}{deg(f_i)} \right\rfloor = k, \text{a constant}$$

for each face f_i, including the exterior face of G, where $deg(f_i)$ is the degree of the face f_i, that is the number of edges in the region of the face. The graph which posses (0,1,0)-F-Face magic mean labeling is called a (0,1,0)-F-Face magic mean graph((0,1,0)-F-FMMG) [7].

A bijection $\phi: V(G) \cup E(G) \to \{1,2,\dots,|V(G)|+|E(G)|\}$ is called an (1,1,0)- F-Face magic mean labeling ((1,1,0)-F-FMML) of G if the induced face labeling

ISSN: 2005-4238 IJAST

$$\phi^*(f_i) = \left\lfloor \frac{sum\ of\ the\ labels\ of\ the\ vertices\ and\ edges\ in\ the\ boundary\ of\ f_i}{deg(f_i)} \right\rfloor$$

$$= \left\lfloor \frac{\sum_{v_j \epsilon f_i} \phi(v_j) + \sum_{e_j \epsilon f_i} \phi(e_j)}{deg(f_i)} \right\rfloor = k, \text{a constant}$$

for each face f_i of G including the exterior face, where $deg(f_i)$ is the degree of the face f_i, that is the number of edges in the region of the face. The graph which posses an (1,1,0)-F-Face magic mean labeling is called a (1,1,0)-F-Face magic mean graph ((1,1,0)-F-FMMG).The weight of the face f of a graph G is defined as the sum of all the vertex labels and edge labels on the face and the mean weight of the face f is defined as the induced labeling on the face.[3]

A globe graph is a graph produced by joining two isolated vertices by n paths of length two. It is isomorphic to $K_{2,n}$. The square graph of G denoted by G^2 is a graph having vertex set as same as G and the two vertices u and v are adjacent if and only if the distance between them is atmost 2 in G. The $(m, n) -$ tadpole graph G is a graph obtained by identifying a vertex of a cycle of length m and a vertex of a path of length n.

Duplication of a vertex v of a graph G by a vertex v' produces a new graph G' with $N(v') = N(v)$ where $N(v)$ is the set of all neighbors of v. Duplication of an edge $e = uv$ of a graph G by an edge $e' = u'v'$ produces a new graph G' where $N(u$â€™$) = N(u) \cup \{v'\} - \{v\}$ and $N(v') = N(v) \cup \{u'\} - \{u\}$.

In this paper, the existence of (0,1,0)-F-FMML and (1,1,0)-F-FMML is proved for ladder graph, globe graph and new graphs obtained by applying graph operations.

2. Main Results

1. (0,1,0)-F-Face magic mean graphs

Theorem 1.1. *The globe graph $G \simeq K_{2,n}$ is a (0,1,0)-F-FMMG, for all $n \geq 2$.*

Proof.

Let $V(G) = \{u_i : i = 1,2\} \cup \{v_i : 1 \leq i \leq n\}$ and $E(G) = \{u_1v_i : 1 \leq i \leq n\} \cup \{u_2v_i : 1 \leq i \leq n\}$ Then the face set of G is $F(G) = \{f_i = (u_1v_i, u_1v_{i+1}, u_2v_i, u_2v_{i+1}) : 1 \leq i \leq n-1\} \cup \{f_0 = (u_1v_1, u_1v_n, u_2v_1, u_2v_n)\}$.

In G, $|V(G)| = n+2, |E(G)| = 2n, |F(G)| = n$ and $deg(f_i) = 4, 0 \leq i \leq n-1$.

Define $\phi : E(G) \to \{1,2, \ldots, 2n\}$ as follows:

$$\phi(u_1v_i) = i, 1 \leq i \leq n \text{ and}$$
$$\phi(u_2v_i) = 2n+1-i, 1 \leq i \leq n.$$

Then the labeling induced on each face $\phi^*(f_i)$,$1 \leq i \leq n-1$ in G is obtained as

$$\phi^*(f_i) = \left\lfloor \frac{1}{4}\{\phi(u_1v_i) + \phi(u_1v_{i+1}) + \phi(v_2v_i) + \phi(v_2v_{i+1})\} \right\rfloor$$
$$= \left\lfloor \frac{1}{4}\{4n+2\} \right\rfloor = n \text{ and}$$
$$\phi^*(f_0) = \left\lfloor \frac{1}{4}\{\phi(u_1v_1) + \phi(u_2v_1) + \phi(u_1v_n) + \phi(u_2v_n)\} \right\rfloor$$
$$= \left\lfloor \frac{1}{4}\{4n+2\} \right\rfloor = n.$$

Thus ϕ is a (0,1,0)-F-FMML of G. So, $G \simeq K_{2,n}$ is a (0,1,0)-F-FMMG having face constant n for all $n \geq 2$. A (0,1,0)-F-FMML of $K_{2,7}$ is shown in Figure 1.

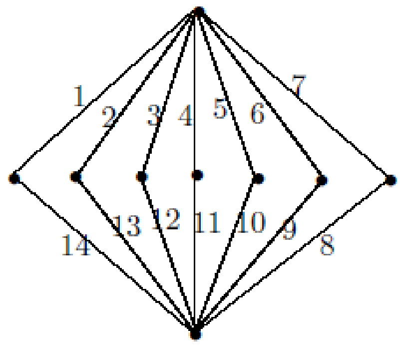

Figure 1: (0,1,0)-F-FMML of $K_{2,7}$ with face constant 7.

Theorem 1.2. *The square graph of a path P_n is a (0,1,0)-F-FMMG if n is odd.*
Proof.
Let $G \simeq P_n^2$ and $V(G) = \{u_i: 1 \le i \le n\}$ and
$E(G) = \{u_i u_{i+1}: 1 \le i \le n\} \cup \{u_i u_{i+2}: 1 \le i \le n-2\}$.
Then the face set of G is $F(G) = \{f_i = (v_i v_{i+1}, v_{i+1} v_{i+2}, v_i v_{i+2}): 1 \le i \le n-2\} \cup$

$$\left\{f_0 = (\bigcup_{i=1}^{n-2} u_i u_{i+2} u_1 u_2, u_{n-1} u_n)\right\}$$

In G, $|V(G)| = n, |E(G)| = 2n-3, |F(G)| = n-1, deg(f_0) = n$ and $deg(f_i) = 3, 1 \le i \le n-2$.
Define $\phi: E(G) \to \{1,2,\ldots,2n-3\}$ as follows:
$For\ 1 \le i \le n-1,$

$$\phi(u_i u_{i+1}) = \begin{cases} \frac{3n-5+i}{2}, & i \equiv 0(mod2) \\ \frac{i+1}{2}, & i \equiv 1(mod2) \end{cases}$$

$For\ 1 \le i \le n-2, \phi(u_i u_{i+2}) = \frac{3n-3-2i}{2}.$

Then the labeling induced on each face $\phi^*, 1 \le i \le n-2$ in G is obtained as

$$\phi^*(f_i) = \left\lfloor \frac{1}{3}\{\phi(u_i u_{i+1}) + \phi(u_{i+1} u_{i+2}) + \phi(u_i u_{i+2})\} \right\rfloor$$
$$= \left\lfloor \frac{1}{3}\{3n-3\} \right\rfloor = n-1 \text{ and}$$
$$\phi^*(f_0) = \left\lfloor \frac{1}{n}\left\{\sum_{i=1}^{n-2} \phi(u_i u_{i+2}) + \phi(u_1 u_2) + \phi(u_{n-1} u_n)\right\} \right\rfloor$$
$$= \left\lfloor \frac{1}{n}\{n^2 - n\} \right\rfloor = n-1.$$

Thus ϕ is a (0,1,0)-F-FMML of G. So, $G \simeq P_n^2$ is a (0,1,0)-F- FMMG having face constant $n-1$ if n is odd. A (0,1,0)-F- FMML of P_7^2 is shown in Figure 2.

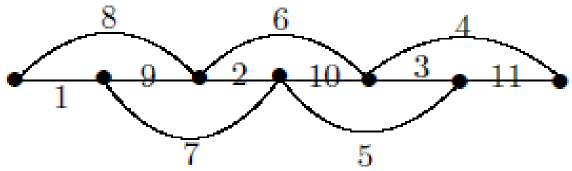

Figure 2: (0,1,0)-F-FMML of P_7^2 with face constant 6.

Theorem 1.3. *The graph G isomorphic to (m, n)-Tadpole graph is a (0,1,0)-F-FMMG, for all $m \ge 3, n \ge 2$.*
Proof.

ISSN: 2005-4238 IJAST

Let $\{v_1, v_2, \ldots, v_m\}$ be the vertices on cycle and $\{u_1, u_2, \ldots, u_n\}$ be the vertices on path where $v_{\lfloor\frac{m}{2}\rfloor} = u_1$. In G, $|V(G)| = m+n-1, |E(G)| = m+n-1$ and $F(G) = \{f_0, f_1\}$ where f_0 is the outer face bounded by all edges of G and f_1 is the cycle C_m.
So $deg(f_1) = m, \deg(f_0) = m+n-1$
Define $\phi: E(G) \to \{1,2,\ldots,m+n\}$ as follows:

$$\phi(v_i v_{i+1}) = \begin{cases} i+1, & 1 \le i \le \lfloor\frac{m}{2}\rfloor - 1 \\ \lfloor\frac{3m}{2}\rfloor + n - i - 1, & \lfloor\frac{m}{2}\rfloor \le i \le m-1 \end{cases}$$

$\phi(v_m v_1) = 1$ and
$\phi(u_i u_{i+1}) = \lfloor\frac{m}{2}\rfloor + i, 1 \le i \le n-1.$

When n is even, the labeling induced on each face ϕ^* in G is obtained as follows

$$\phi^*(f_1) = \left\lfloor \frac{1}{m}\left\{ \sum_{i=1}^{\lfloor\frac{m}{2}\rfloor-1} \phi(v_i v_{i+1}) + \sum_{i=\lfloor\frac{m}{2}\rfloor}^{m-1} \phi(v_i v_{i+1}) + \phi(v_m v_1) \right\} \right\rfloor$$
$$= \left\lfloor \frac{1}{m}\left\{\frac{m(m+n)}{2}\right\}\right\rfloor$$
$$= \begin{cases} \frac{m+n}{2}, & n \equiv 0 (mod\ 2) \\ \frac{m+n-1}{2}, & n \equiv 1 (mod\ 2) \end{cases}$$ and

$$\phi^*(f_0) = \left\lfloor \frac{1}{m+n-1}\left\{ \sum_{i=1}^{m-1} \phi(v_i v_{i+1}) + \phi(v_m v_1) + \sum_{i=1}^{n-1} \phi(u_i u_{i+1}) \right\} \right\rfloor$$
$$= \left\lfloor \frac{1}{m+n-1}\left\{\frac{(m+n)(m+n-1)}{2}\right\}\right\rfloor$$
$$= \begin{cases} \frac{m+n}{2}, & n \equiv 0 (mod 2) \\ \frac{m+n-1}{2}, & n \equiv 1 (mod 2). \end{cases}$$

When n is odd, the labeling induced on each face ϕ^* in G is obtained as follows

$$\phi^*(f_1) = \left\lfloor \frac{1}{m}\left\{ \sum_{i=1}^{\lfloor\frac{m}{2}\rfloor-1} \phi(v_i v_{i+1}) + \sum_{i=\lfloor\frac{m}{2}\rfloor}^{m-1} \phi(v_i v_{i+1}) + \phi(v_m v_1) \right\} \right\rfloor$$
$$= \left\lfloor \frac{1}{m}\left\{\frac{(m+1)(m+n-1)}{2}\right\}\right\rfloor$$
$$= \begin{cases} \frac{m+n-1}{2}, & n \equiv 0 (mod 2) \\ \frac{m+n}{2}, & n \equiv 1 (mod 2) \end{cases}$$ and

$$\phi^*(f_0) = \left\lfloor \frac{1}{m+n-1}\left\{ \sum_{i=1}^{m-1} \phi(v_i v_{i+1}) + \phi(v_m v_1) + \sum_{i=1}^{n-1} \phi(u_i u_{i+1}) \right\} \right\rfloor$$
$$= \left\lfloor \frac{1}{m+n-1}\left\{\frac{(m+n)(m+n-1)}{2}\right\}\right\rfloor$$
$$= \begin{cases} \frac{m+n-1}{2}, & n \equiv 0 (mod 2) \\ \frac{m+n}{2}, & n \equiv 1 (mod 2). \end{cases}$$

Thus ϕ is a (0,1,0)-F-FMML of G.
So, G is a (0,1,0)-F-FMMG. A (0,1,0)-F-FMML of (9,6) −tadpole graph and (6,4) −tadpole graph is shown in Figure 3.

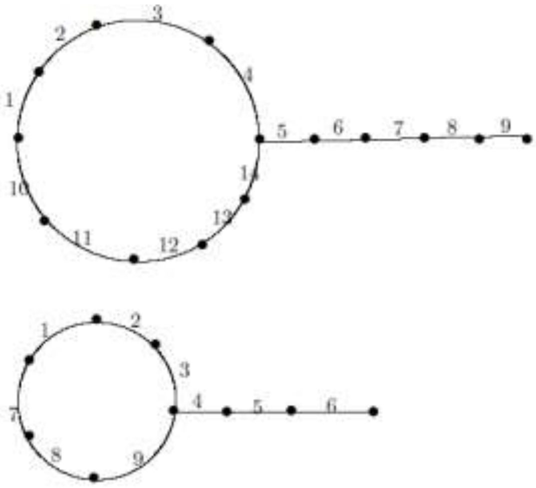

Figure 3: A (0,1,0)-F-FMML of $(9,6) -$ tadpole graph and $(6,4) -$tadpole graph with face constant 7 and 5 respectively.

2. *(1,1,0)-F-face magic mean graphs*

Theorem 2.1 *The Ladder graph $G \simeq L_n$ is a (1,1,0)-F-FMMG with face constant $5n-1$, for all $n \geq 2$.*

Proof.

Let $V(G) = \{u_i : 1 \leq i \leq n\} \cup \{v_i : 1 \leq i \leq n\}$ and $E(G) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{v_i v_{i+1} : 1 \leq i \leq n-1\}$.Then the face set of G is $F(G) = \{f_i = (u_i v_i, u_i u_{i+1}, v_i v_{i+1}, u_{i+1} v_{i+1}) : 1 \leq i \leq n-1\} \cup \{f_n = (\bigcup_{i=1}^{n-1} u_i u_{i+1}, \bigcup_{i=1}^{n-1} v_i v_{i+1}, u_1 v_1, u_n v_n)\}$.

In G, $|V(G)| = 2n, |E(G)| = 3n-2, |F(G)| = n, deg(f_i) = 4, 1 \leq i \leq n$ and $deg(f_n) = 2n$.

Define $\phi : V(G) \cup E(G) \to \{1,2,\ldots,5n-2\}$ as follows:

$$\phi(v_i) = 2i-1, 1 \leq i \leq n,$$

$$\phi(u_2) = \begin{cases} \frac{5n-1}{2}, & n \equiv 1 (mod\ 2) \\ \frac{5n-2}{2}, & n \equiv 0 (mod\ 2) \end{cases}$$

$$\phi(v_i) = 5n-2i, 1 \leq i \leq n,$$

$$\phi(u_i u_{i+1}) = 2i, 1 \leq i \leq n-1,$$

$$\phi(v_i v_{i+1}) = 5n-1-2i, 1 \leq i \leq n-1,$$

For $n \equiv 0 (mod\ 2)$,

$$\phi(u_i v_i) = \begin{cases} \frac{5n-1+i}{2}, & i \equiv 1 (mod\ 2) \\ \frac{5n-i}{2}, & i \equiv 0 (mod\ 2), i \neq 2 \end{cases}$$

For $n \equiv 1 (mod\ 2)$,

$$\phi(u_i v_i) = \begin{cases} \frac{5n+2-i}{2}, & i \equiv 1 (mod 2) \\ \frac{5n-1+i}{2}, & i \equiv 0 (mod 2), i \neq 2 \end{cases} \text{ and}$$

$$\phi(u_2v_2) = 3.$$

Then the labeling induced on each face ϕ^* in G is obtained as

$$\phi^*(f_i) = \left\lfloor \frac{1}{4}\{\phi(u_i) + \phi(u_{i+1}) + \phi(v_i) + \phi(v_{i+1}) + \phi(u_iu_{i+1}) + \phi(v_iv_{i+1})\} \right\rfloor + \left\lfloor \frac{1}{4}\{\phi(u_iv_i) + \phi(u_{i+1}v_{i+1})\} \right\rfloor$$

$$= 5n - 1, for\ all\ 1 \leq i \leq n-1\ and$$

$$\phi^*(f_0) = \left\lfloor \frac{1}{2n}\left\{ \sum_{i=1, i\neq 2}^{n} \phi(u_i) + \phi(u_2) + \sum_{i=1}^{n} \phi(v_i) + \sum_{i=1}^{n-1} \phi(u_iu_{i+1}) \right\} \right\rfloor + \left\lfloor \frac{1}{2n}\left\{ \sum_{i=1}^{n-1} \phi(v_iv_{i+1}) + \phi(u_1v_1) + \phi(u_nv_n) \right\} \right\rfloor = 5n - 1.$$

Thus $\phi^*(f) = 5n - 1$ for each face f of G.
So, $G \simeq L_n$ is a (1,1,0)-F-FMMG for all $n \geq 1$. A (1,1,0)-F-FMML of L_8 is shown in Figure 4.

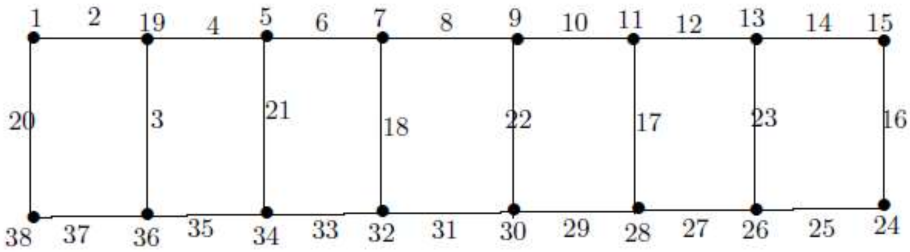

Figure 4: A (1,1,0)-F-FMML of L_8 with face constant 39.

Theorem 2.2 *The globe graph $G \simeq K_{2,n}$ is a (1,1,0)-F-FMMG, for all $n \geq 2$.*

Proof.
Let $V(G) = \{u_i : i = 1,2\} \cup \{v_i : 1 \leq i \leq n\}$ and
$E(G) = \{u_1v_i : 1 \leq i \leq n\} \cup \{u_2v_i : 1 \leq i \leq n\}$.
Then the face set of G is $F(G) = \{f_i = (u_1v_i, u_1v_{i+1}, u_2v_i, u_2v_{i+1}) : 1 \leq i \leq n-1\} \cup \{f_0 = (u_1v_1, u_1v_n, u_2v_1, u_2v_n)\}$.

In G, $|V(G)| = n + 2, |E(G)| = 2n, |F(G)| = n$ and $deg(f_i) = 4, 0 \leq i \leq n-1$.

Define $\phi: V(G) \cup E(G) \to \{1,2, \dots, 3n+2\}$ as follows:

$\phi(u_i) = i, i = 1,2,$

$\phi(v_i) = i + 2, 1 \leq i \leq n,$

For $n \equiv 0 (mod\ 2)$,

$$\phi(u_1v_i) = \begin{cases} \frac{4n+5-i}{2}, & i \equiv 1 (mod\ 2) \\ \frac{3n+6-i}{2}, & i \equiv 0 (mod\ 2) \end{cases}$$

$$\phi(u_2v_i) = \begin{cases} \frac{5n+5-i}{2}, & i \equiv 1 (mod\ 2) \\ \frac{6n+6-i}{2}, & i \equiv 0 (mod\ 2) \end{cases}$$

For $n \equiv 1 (mod\ 2)$,

$$\phi(u_1v_i) = \begin{cases} \dfrac{3n+6-i}{2}, & i \equiv 1(mod\ 2) \\ \dfrac{4n+6-i}{2}, & i \equiv 0(mod\ 2) \end{cases}$$

$$\phi(u_2v_i) = \begin{cases} \dfrac{6n+5-i}{2}, & i \equiv 1(mod\ 2) \\ \dfrac{5n+5-i}{2}, & i \equiv 0(mod\ 2) \end{cases}$$

Then the labeling induced on each face $\phi^*, 0 \leq i \leq n-1$ in G is obtained as

$$\phi^*(f_i) = \left\lfloor \frac{1}{4}\{\phi(u_1) + \phi(u_2) + \phi(v_i) + \phi(v_{i+1}) + \phi(u_1v_i)\} \right\rfloor + \left\lfloor \frac{1}{4}\{\phi(u_1v_{i+1}) + \phi(u_2v_i) + \phi(u_2v_{i+1})\} \right\rfloor$$

$$= \left\lfloor \frac{1}{4}\{9n+18\} \right\rfloor$$

$$= \begin{cases} \dfrac{9n+16}{4}, & n \equiv 0(mod\ 4) \\ \dfrac{9n+15}{4}, & n \equiv 1(mod\ 4) \\ \dfrac{9n+18}{4}, & n \equiv 2(mod\ 4) \\ \dfrac{9n+17}{4}, & n \equiv 3(mod\ 4). \end{cases}$$

Thus ϕ is a (1,1,0)-F-FMML of G.
So, $G \simeq K_{2,n}$ is a (1,1,0)-F-FMMG for all $n \geq 2$.
A (1,1,0)-F-FMML of $K_{2,7}$ and $K_{2,8}$ are shown in Figure 5.

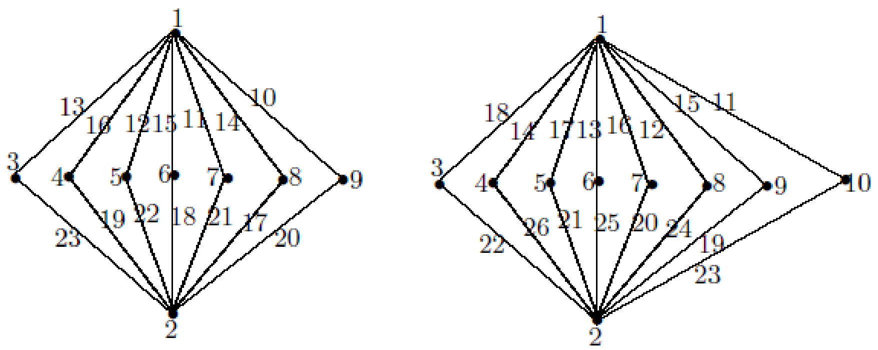

Figure 5: A (1,1,0)-F-FMML of $K_{2,7}$ and $K_{2,8}$ with face constant 20 and 22 respectively.

Theorem 2.3. *The graph G obtained by duplication of any vertex of cycle C_n by a vertex, is a (1,1,0)-F-FMMG with face constant $2n+3$, for $n \geq 3$.*

Proof.
Let $v_1, v_2, \ldots, v_n$ be the vertices of the cycle C_n, for $n \geq 3$. By the graph isomorphism, duplicate the vertex v_1 by a vertex v_1'.

Let the resultant graph be G whose vertex set is $V(C_n) \cup \{v_1'\}$,edge set is $E(C_n) \cup \{v_1'v_n, v_1'v_2\}$ and face set is $\{f_0, f_1, f_2\}$ where f_1 is the cycle C_n, f_2 is the face bounded by the edges $v_1v_2, v_2v_1', v_1'v_n, v_nv_1$ and f_0 is the outer face bounded by all the edges of G except v_1v_2, v_nv_1.
Define a labeling $\phi: V(G) \cup E(G) \to \{1,2,3, \dots 2n+3\}$ as follows.

$$\phi(v_1) = n+1,$$
$$\phi(v_1') = 1,$$
$$\phi(v_i) = i, 2 \le i \le n,$$
$$\phi(v_iv_{i+1}) = 2n+3-i, 1 \le i \le n-1,$$
$$\phi(v_1'v_2) = 2n+3,$$
$$\phi(v_1'v_n) = n+3 \text{ and}$$
$$\phi(v_1v_n) = n+2.$$

Then the labeling induced on each face ϕ^* in G is obtained as

$$\phi^*(f_1) = \left\lfloor \frac{1}{n}\left\{\sum_{i=1}^{n} \phi(v_i) + \sum_{i=1}^{n-1} \phi(v_iv_{i+1}) + \phi(v_nv_1)\right\}\right\rfloor$$
$$= \left\lfloor \frac{1}{n}\{2n^2+4n-1\}\right\rfloor = 2n+3,$$

$$\phi^*(f_0) = \left\lfloor \frac{1}{n}\left\{\sum_{i=2}^{n} \phi(v_i) + \phi(v_1') + \sum_{i=2}^{n-1} \phi(v_iv_{i+1}) + \phi(v_1'v_2) + \phi(v_1'v_n)\right\}\right\rfloor$$
$$= \left\lfloor \frac{1}{n}\{2n^2+3n+1\}\right\rfloor = 2n+3 \text{ and}$$

$$\phi^*(f_2) = \left\lfloor \frac{1}{4}\{\phi(v_1) + \phi(v_2) + \phi(v_n) + \phi(v_1') + \phi(v_1v_2) + \phi(v_1v_n) + \phi(v_1'v_2) + \phi(v_1'v_n)\}\right\rfloor$$
$$= \left\lfloor \frac{1}{4}\{8n+14\}\right\rfloor = 2n+3.$$

Thus ϕ is a (1,1,0)-F-FMML with face constant $2n+3$.
So, G is a (1,1,0)-F-FMMG.

A (1,1,0)-F-FMML of a graph obtained by duplication of any vertex in cycle C_9 by a vertex is shown in Figure 6.

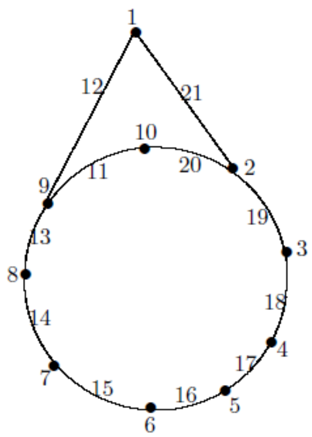

Figure 6: A (1,1,0)-F-FMML of a graph obtained by duplication of any vertex in cycle C_9 by a vertex with face constant 21.

Theorem 2.4. *The graph G obtained by duplication of any edge of cycle C_n by an edge, is a (1,1,0)-F-FMMG with face constant $2n+5$, for $n \ge 3$.*

ISSN: 2005-4238 IJAST

Proof.

Let $v_1, v_2, \ldots, v_n$ be the vertices of the cycle C_n, for $n \geq 3$. By the graph isomorphism, duplicate the edge v_1v_2 by an edge uv. Let the resultant graph be G whose vertex set is $V(C_n) \cup \{u, v\}$, edge set is $E(C_n) \cup \{uv, uv_n, uv_3\}$ and face set is $\{f_0, f_1, f_2\}$ where f_1 is the cycle C_n, f_2 is the face bounded by the edges uv, uv_1, vv_1 and f_0 is the outer face bounded by all the edges of G.

Define a labeling $\phi: V(G) \cup E(G) \rightarrow \{1,2,3,\ldots,2n+5\}$ as follows.

$\phi(u) = 1, \phi(v) = 2, \phi(v_1) = n+1, \phi(v_2) = n+2, \phi(v_i) = 3, 3 \leq i \leq n,$
$\phi(uv) = 2n+4, \phi(uv_n) = 2n+5, \phi(vv_3) = 2n+3, \phi(v_nv_1) = n+6$ and

$$\phi(v_iv_{i+1}) = \begin{cases} n+4, & i=1 \\ n+3, & i=2 \\ 2n+5-i, & 3 \leq i \leq n-2 \\ n+5, & i=n-1. \end{cases}$$

Then the labeling induced on each face is given by

$$\phi^*(f_1) = \left\lfloor \frac{1}{n} \left\{ \sum_{i=1}^{n} \phi(v_i) + \sum_{i=1}^{n-1} \phi(v_iv_{i+1}) + \phi(v_nv_1) \right\} \right\rfloor$$

$$= \left\lfloor \frac{1}{n} \{2n^2 + 5n\} \right\rfloor = 2n+5,$$

$$\phi^*(f_2) = \left\lfloor \frac{1}{6} \left\{ \phi(u) + \phi(v) + \sum_{i=1}^{3} \phi(v_i) + \phi(v_n) + \phi(uv_n) + \phi(vv_3) + \phi(uv) \right\} \right\rfloor$$
$$+ \left\lfloor \frac{1}{6} \{\phi(v_nv_1) + \phi(v_1v_2) + \phi(v_2v_3)\} \right\rfloor$$
$$= \left\lfloor \frac{1}{6} \{12n + 34\} \right\rfloor = 2n+5 \text{ and}$$

$$\phi^*(f_0) = \left\lfloor \frac{1}{n} \left\{ \sum_{i=3}^{n} \phi(v_i) + \phi(u) + \phi(v) + \sum_{i=3}^{n-1} \phi(v_iv_{i+1}) + \phi(uv) \right\} \right\rfloor$$
$$+ \left\lfloor \frac{1}{n} \{\phi(uv_n) + \phi(vv_3)\} \right\rfloor$$
$$= \left\lfloor \frac{1}{n} \{2n^2 + 6n - 1\} \right\rfloor = 2n+5.$$

Thus ϕ is a (1,1,0)-F-FMML with face constant $2n+5$.

So, G is a (1,1,0)-F-FMMG.

A (1,1,0)-F-FMML of a graph obtained by duplication of any edge in cycle C_9 by an edge is shown in Figure 7.

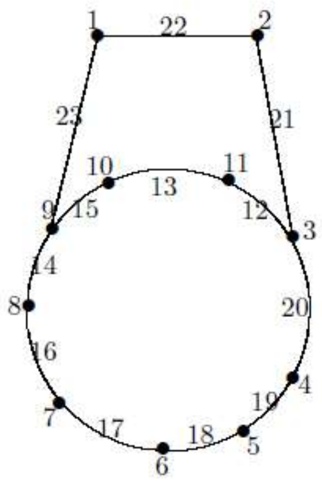

Figure 7: A (1,1,0)-F-FMML of a graph obtained by duplication of any edge in cycle C_9 by an edge with face constant 23.

REFERENCES

[1] A.Durai Baskar and S.Arockiaraj, "Further Results on Super Geometric Mean Graphs", International J.Math. Combin., vol.4, pp. 104–128, 2015.
[2] A.Durai Baskar, S.Arockiaraj and B.Rajendran, "Geometric meanness of graphs obtained from paths", Utilitas Mathematica, vol.101, pp.45–68, 2016.
[3] A.Meena Kumari and S.Arockiaraj, "On (1,1,0)-F-face magic mean labeling of some graphs", Utilitas Mathematica, vol.103, pp. 139–159, 2017.
[4] A.Meena Kumari and S.Arockiaraj, "On (1,0,0)-F-face magic mean labeling of some graphs", Journal of Physics: Conference Series 1139(2018) 012049, 2018.
[5] J. A. Gallian, "A dynamic survey of graph labeling", The Electronic Journal of Combinatorics,vol.5, # DS6, 2005.
[6] N. Hartsfield and G. Ringel, "Pearls in Graph Theory", Academic Press, San Diego, 1990.
[7] S.Arockiaraj and A.Meena Kumari, "On F-face magic mean labeling of some duplicated graphs", International Journal of Applied Mathematics, Vol.31, No.2, pp. 231–240, 2018.
[8] R.Vasuki and S.Arockiaraj, "On mean graphs", International Journal of Mathematical Combinatorics, vol.3 , pp.22–34, 2013.
[9] S.Arockiaraj and A.Meena Kumari, "On F-face magic mean labeling of some planar graphs", International Journal of Pure and Applied Mathematics, vol.117, No.6 , pp.1–8, 2017.
[10] W. D. Wallis, "Magic Graphs", Birhauser, Borton-Bacel-Berlin, 2001.

ISSN: 2005-4238 IJAST