International Journal of Software Engineering and Its Applications
Vol. 11, No. 3 (2017), pp. 21-30

A Novel Two Dimensional Adaptive Filtering Algorithm for Image De-Noising via Fractional Gradient

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Abstract

In the recent decade it has been witnessed that raster images are the primary source of information for numerous applications such as bio-medical, law enforcement, *geographical information system* (GIS), photography, astronomy, etc. Primarily, the quality of raster images compromises due to the surrounding factors of these applications. Because, it is very difficult to control surrounding parameter (light, motion, distance) while acquiring images. Therefore, the image acquisition in these applications is very much prone to the noise. In the literature, researchers have targeted this issue and have already devised classical image filters for image de-noising. Afterwards, in the recent years the performance of classical filtering was further improved by employing *two dimensional adaptive filters* (2-DAF) for image de-noising and enhancement. In the literature, researchers have reported the performance comparisons of various 2-DAF specifically for image restoration, enhancement, estimation, and de-noising. In this paper an extended version of *one dimensional fractional least mean square* (1-DFLMS) to *two dimensional fractional least mean square* (2-DFLMS) is presented. Moreover the performance of the proposed algorithm has been rigorously compared with the existing and most employed 2-DAF algorithm namely, *two dimensional least mean square* (2-DLMS), *two dimensional variable step size least mean square* (2-DVSSLMS). The simulation results illustrate the notable performance edge of the proposed algorithm with the existing approaches.

***Keywords**:* Image de-noising, two-dimensional adaptive filtering, least mean square (LMS), variable step size least mean square (VSSLMS), fractional least mean square (FLMS)

1. Introduction

The tremendous growths of information and communication technology have supported the use of images as a primary source of information for various applications. This includes, but not limited to, the *content based image retrieval* (CBIR) [1], *biomedical imaging* [2], GIS [3], *photography* [4] etc. The image acquisition in these applications is mainly based on raster imaging which is very prone to the surrounding noise and system distortion [5]. In the literature researchers have devised classical filters for image de-noising. Afterwards, the performance of image filtering is further improved by employing 2-DAF [6]. The use of 1-DAF has already been intensively investigated on numerous engineering and scientific applications [7-10]. This is because of its ability to deal with the inherent non-stationary statistical properties of data and its statistical correlations. Likewise, in the recent years the 2-DAF have also been emerged for the numerous image processing applications such as image de-noising, image enhancement, image restoration *etc*.

ISSN: 1738-9984 IJSEIA

In 1988, a researcher has extended 1-DLMS to 2-DLMS for estimation of non-stationary images [11]. This was the major breakthrough in the field of 2-DAF but, performance of this 2-DLMS was found to be constrained due to its very low convergence speed and high *mean square error* (MSE). The convergence issue was further resolved in the same year with an improved algorithm which uses McClellan transformation [12]. However, no significant improvement in MSE has been reported. These two publications open the door for investigating the performance of different variants of 2-DAF for image de-noising, image enhancement, image restoration *etc*.

In the literature the 2-DAFs are reported as an attractive adaptation algorithm, however, they are highly sensitive to *Eigen-value* disparity, high computational cost and low convergence speed. These constraints limit the use of 2-DAF for many applications [13, 14]. Researchers have targeted this issue and have devised some more variants of AF such as *affine projection algorithm* (APA) [15], *two dimensional recursive least squares* (2-DRLS) [16] *etc*. These approaches have relatively improved convergence speed and tracking ability. However, it results into significantly higher complexity and computational cost. This is because the complexity and computational cost for such algorithm is already higher in one dimensional and if its scope is extended for two dimensional data, the cost will increase significantly. It has been inferred logically there is a trade-off between AF data dimensions, convergence speed, tracking ability and AF system complexity and computational cost. This relation was further investigated and found step-sizes as a significant key factor. This is because step-size is strongly correlated with the system convergence speed and computational cost.

Therefore, there is a pressing need to optimize the step-size in variants of AF. In this regards new methods were presented such as *variable step-size APA* (VSS-APA), and *variable step-size NLMS* (VSS-NLMS) [17]. The performances of these methods were encouraging in adaptive noise cancellation but on the contrary due to time varying variable step-size the computational complexity increase significantly. This computational complexity was further handled in some more variants of AF filters. In these methods filter coefficient were partially updated and optimally selected during adaptation. These algorithms include but not limited to, *selective partial update NLMS* (SPU-NLMS) and SPU-APA [18–22]. In these methods researchers have reported a marginal improvement in MSE but with the relatively higher computational cost. With the above literature review it can be inferred that significant improvement in MSE of AF algorithm is desired but with low computational cost.

In the literature of 1-D adaptive filter, researchers have reported that FLMS has a performance edge in terms of MSE and computational cost with the other variants of AF such as LMS, VSSLMS *etc.*, [10]. This fact creates a rationale to derive a 2-DFLMS from 1-DFLMS for relatively improved MSE at low computational cost. In this paper an extended version of 1-DFLMS is proposed. Moreover, the performance in terms of MSE and computational cost is compared with the existing and most employed AF algorithm namely LMS and VSSLMS.

Another rationale to investigate this area is the ascent publication growth in the area of 2-D adaptive filtering application for image processing. It has also been inferred that relatively very low publication count has been reported in this area. The same can be illustrated in Figure 1. This figure shows a research growth in terms of number of research publications each year. It can be inferred that relatively a very few work has been reported on Google Scholar with the keywords "two dimensional" adaptive filter or "2 D" adaptive filter.

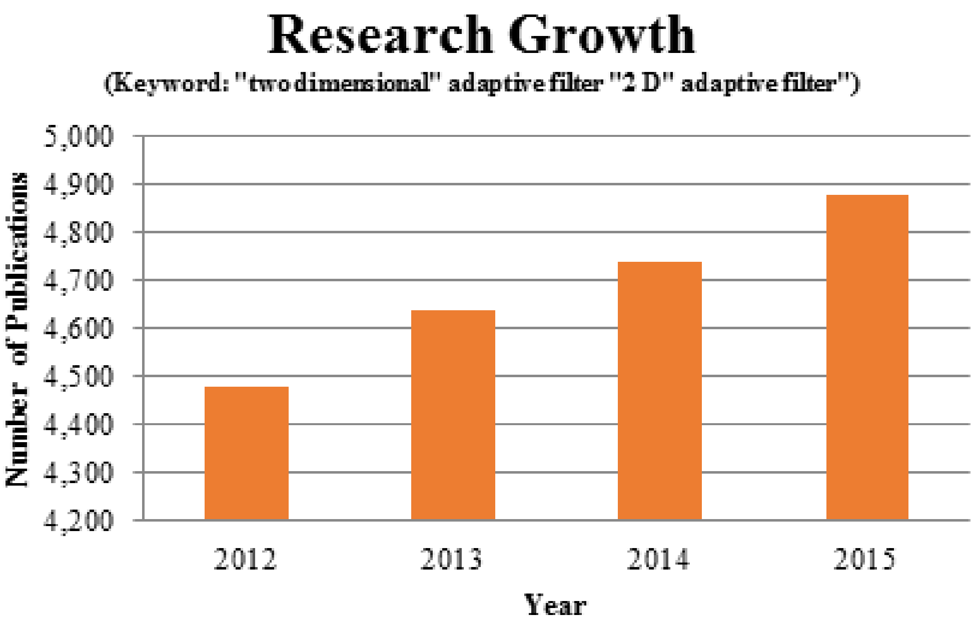

Figure 1. Research Growth (Keyword: "two dimensional" adaptive filter "2 D" adaptive filter")

2. 1-D Adaptive Filtering Algorithms

Adaptive filtering is a system that primarily comprises of a linear filter that has a intigrated transfer function which is further controlled by variable distinct parameters and also means to adjust these parameters according to the optimization algorithm. Due to the complexity of these optimization algorithms, almost all adaptive filters are digital in nature.

In this section of the paper the concise description of 1-DLMS, 1-DVSSLMS and 1-DFLMS are presented for the ready reference. This description includes the theoretical background along with the numerical relationships of their weight update equations, output equation and step-size.

2.1 Least Mean Square

Least mean squares (LMS) is one of the variant of adaptive filter which is used to represent a desired filter type by computing the filter coefficients. This filter coefficient is further translated to produce the least mean square of the error signal. This is in principle the difference between the desired information and the computed signal. In 1959, Windrow and Hoff have presented a least mean square (LMS) algorithm [23, 24]. This AF algorithm is primarily based on gradient computation through steepest descent method. The LMS algorithm is very simple in structure and it does not require any matrix inversion or any correlation matrix. Figure 2 illustrates the block diagram of classical LMS. In this figure an input signal is first be transform into an output signal by applying initial weights. Afterwards, the error is computed with an absolute difference between computed output *y(k)*and the desired output *d(k).* Finally, the adaptive weight update mechanism, which is the function of the mean square error *e(k),* will update the weight factor for the subsequent output. This process repeats it-self iteratively until the absolute error is minimal or a marginal change in the weight update is been observed.

The empirical relationship of the output *y(k)*, error *e(k)* and updated weight *w(k+1)* is shown in Equation 1, Equation 2 and Equation 3, respectively. The output equation of 1-DLMS is the function of updated weights and the input signal *x(k)*. Subsequently, an error could be computed with an absolute difference of output signal *y(k)* and the desired signal *d(k)*. Finally, this error and the computed output will correspond to the new updated weight *w(k+1)*, which is the function of previous weight *w(k)*, static step size μ, input signal *x(k)* and an absolute error *e(k)*.This process repeats it-self iteratively until the absolute error is minimal or a marginal change in the weight update is been observed.

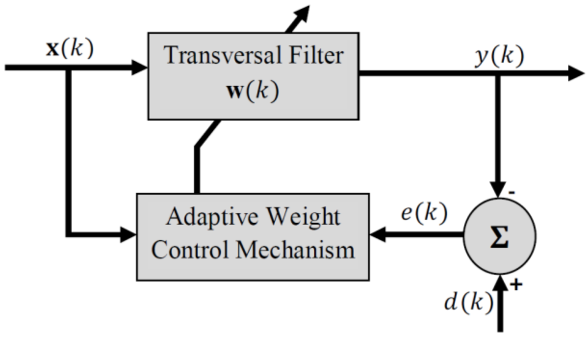

Figure 2. Block Diagram of LMS Algorithm

$$y(k) = w^H(k)x(k) \quad (1)$$
$$e(k) = d(k) - y(k) \quad (2)$$
$$w(k+1) = w(k) + \mu x(k)e^*(k) \quad (3)$$

2.2 1-D Variable Step Size Least Mean Square

In the 1-DLMS method, the weight update can be computed with an adjustment in step size μ. It has been observed that the large step size eventually results in high steady state error while the small step size gives slow convergence. Researchers have observed this behaviors and proposed to initially keep the step size larger and then gradually reducing it in order to have effective and reasonably fast convergence. The *variable step size least mean square* (VSSLMS) is an algorithm which control the step size for effective and fast convergence. The output equation of 1-DVSS LMS *y(k)* and the error equation *e(k)* is identical to 1-DLMS, Equation 1 and Equation 2. However, the weight update equation of VLMS is slightly different than LMS weight update equation. The weight update equation of 1-DVSSLMS s show in Equation 4. In this equation the step size $\mu(k)$ varies with respect to the iteration *k*. for effective and fast convergence. Moreover, the step size μ(k) can be updated using Equation 5.

$$w(k+1) = w(k) + \mu(k)x(k)e^*(k) \quad (4)$$

where $\mu(k)$ is the step size at time k. The update in step size can be obtained as:

$$\mu(k+1) = \alpha\mu(k) + \gamma|e(k)|^2 \quad (5)$$

where $0 < \alpha < 1$ and the updating parameter $\gamma > 0$. This updating parameter plays an important role in step size adjustment.

2.3 Fractional Least Mean Square

In the literature of 1-D adaptive filtering researchers have reported that FLMS has a performance edge in terms of MSE and computational cost with the other variants of AF such as LMS, VSSLMS *etc*. [10]. This is because the *fractional least mean square* (FLMS) algorithm is primarily based on fractional order calculus [26]. Like, 1-VSSLMS, the output equation of 1-DFLMS *y(k)* and the error equation *e(k)* is same as to 1-DLMS, Equation 1 and Equation 2. However, the weight update equation of 1-DFLMS is far improved and relatively complex then classical LMS methods. Equation 6 illustrates the weight update equation of 1-DFLMS.

$$w_k(n+1) = w_k(n) - \mu_1 \frac{ðJ(n+1)}{ðw_k} - \mu_2 \frac{ð^v J(n+1)}{ðw_k^v} \quad (6)$$

In this equation $w_k(n)$ represents k^{th} filter weight at time$n$. The notation ð is used for the fractional derivative. The value of v is a real number which lies between 0 and 1. This v also serves as a tuning parameter for fast convergence.

3. 2-D Adaptive Filtering Algorithm

Adaptive filtering is an The 2-DLMS Adaptive filters are extension of 1-DLMS adaptive filters. The principle objective of 2-DAF is to optimize MSE by adjusting filter coefficients. In this section a brief description of existing 2-DAF (2-DLMS and 2-DVSSLMS) is presented. Moreover, the extension of 1-DFLSM to 2-DFLMS is explained in this section, which is the main contribution of this paper.

3.1 2D Least Mean Square (2D LMS) FIR Adaptive Filters

The structure of 1-DLMS is identical to 2-DLMS with the change in the parameter dimension and equation domain and ranges. The output equation of 2-DLMS is shown in Equation 7. In this equation w_j is the initial weight in 2-D domain and x is the input noisy images. Afterwards, the absolute difference between the computed output $\mathrm{y_j(n_1, n_2)}$ and the desired image (noise less image) $\mathrm{d_j(n_1, n_2)}$ is then computed using Equation 8 with helps in update the weight in the nest iteration. The weight update function for 2-DLMS is show in Equation 9.

$$y_j(n_1, n_2) = \sum_{m_1=0}^{N-1} \sum_{m_2=0}^{N-1} w_j(m_1, m_2)\, x(n_1 - m_1, n_2 - m_2) \quad (7)$$

where j is the iteration number for adaptive filters. The error signal e_j at the j^{th} iteration is defined as

$$\begin{aligned} e_j(n_1, n_2) &= d_j(n_1, n_2) - y_j(n_1, n_2) \\ &= d_j(n_1, n_2) \\ &\quad - \sum_{m_1=0}^{N-1} \sum_{m_2=0}^{N-1} w_j(m_1, m_2)\, x(n_1 - m_1, n_2 - m_2) \end{aligned} \quad (8)$$

$$w_{j+1}(n_1, n_2) = w_j(n_1, n_2) + 2\mu e_j(n_1, n_2) x(n_1, n_2) \quad (9)$$

In this equation μ is the scalar multiplier controlling which can control the rate of convergence and filter stability.

3.2 2D Variable Step Size Least Mean Square (2D VSSLMS) FIR Adaptive Filters

As discuss in Section 1, the VSSLMS is the one of the LMS variant in which the step size is adjusted for effective and reasonably fast convergence. The same approach is applied for 2-DVSSLMS adaptive filters. The 2-DVSSLMS is derived from the 1-DVSSLMS. The main difference between 2-DLMS and 2-DVSSLMS is the adaptive step size. As mentioned in Section 2 the output and error equation of LMS and VSSLMS is identical. However, the weight update equation has parametric difference. Equation 10 is the weight update equation for 2-DVSSLMS, where $\mu(n_1, n_2)$ is the step size. This step size can be updated by using Equation 11.

$$w_{j+1}(n_1, n_2) = w_j(n_1, n_2) + 2\mu(n_1, n_2)e_j(n_1, n_2)x(n_1, n_2) \quad (10)$$

$$\mu_{j+1} = \alpha\mu_j + \gamma e_j^2(n_1, n_2) \quad (11)$$

In this equation, $0 < \alpha < 1$, $\gamma > 0$ and μ_{j+1} is set to μ_{max} or μ_{min}. The μ_{max} constant is selected near the point of instability of the conventional 2-DLMS for maximum possible convergence speed. Likewise, the value of $\mu_{min}$is selected as a tradeoff between the desired level of steady-state mis-adjustment and the required tracking capabilities. The parameter γ controls the convergence time and the level of mis-adjustment of the algorithm.

3.3 2D Fractional Least Mean Square (2D FLMS) FIR Adaptive Filters

The principle contribution of this paper is the extension of 1-DFLMS to 2-DFLMS. The structure of 1-DFMS and 2-DFLMS is quite similar however; the empirical relationships are extended to two dimensions. In 2-DFLMS the output and error equations are similar to the 2-DFLMS as show in Equation 7 and Equation 8. But logically, the weight update equation is a major extension in scope and dimensions. The weight update equation is modified as:

$$w_{j+1}(n_1, n_2) = w_j(n_1, n_2) + 2\mu(n_1, n_2)e_j(n_1, n_2)x(n_1, n_2) + 2\mu(n_1, n_2)e_j(n_1, n_2)\frac{x(n_1, n_2) \odot w_j^{1-v}(n_1, n_2)}{\Gamma(2 - v)} \quad (12)$$

where μ is the scalar multiplier controlling which can control the rate of convergence and filter stability while v is the fractional power.

4. Simulation Results and Analysis

The multi-dimensional performance evaluation of the extended 2-DFLMS is presented in this paper. The criterion for performance evaluation is three folded. First, MSE of LMS, VSSLMS and FLMS has been computed. Second, the *peak signal to noise ratio* (PSNR) of the same approaches is then compared. Finally, the application layer output of these approaches is contrasted. Figure 3 illustrates the MSE of the proposed extension (2-DFLMS) and 2-DLMS and 2-DVSSLMS. The MSE wave forms of 2-DLMS, 2-DVSSLMS, and 2-DFLMS are represented with pink, blue and red color respectively. In this figure x-axis represents the number of iterations and *y*-axis represents the amplitude of MSE in dB. The average MSE of 2-DLMS, 2-DVSSLMS, and 2-DFLMS are -4.2 dB, -6.8 dB and -8.4 dB respectively. Therefore, the net improvement of approximately 1.6 dB is observed between 2-DVSSLMS and 2-DFLMS.

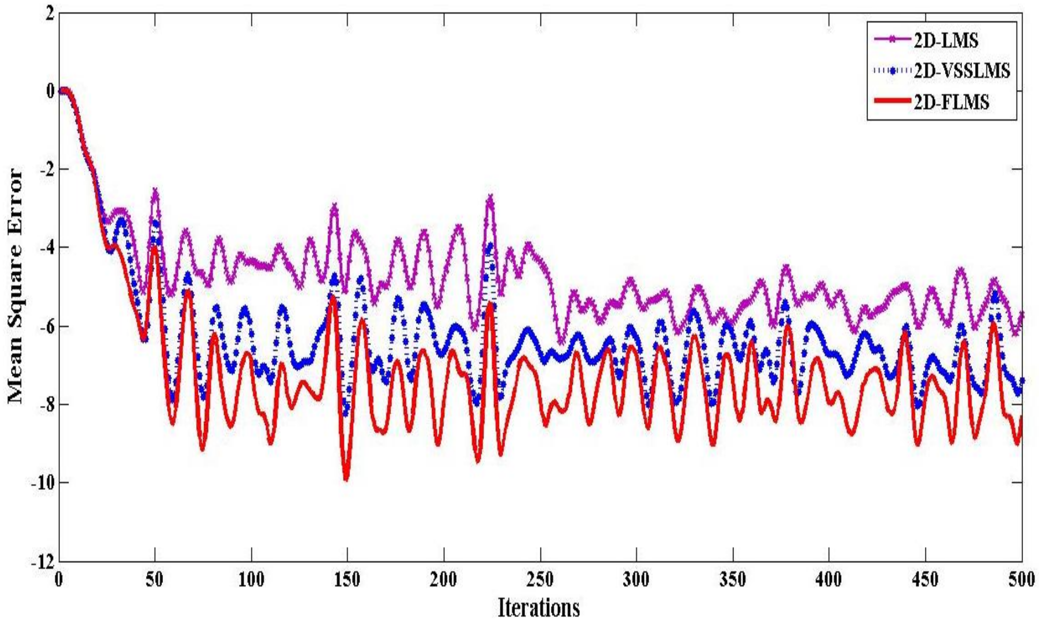

Figure 3. Comparison of MSE of LMS, VSSLMS, and FLMS

Moreover, the performance edge of 2-DFLMS can also be observed at the application layer. Figure 4 represents the pictorial representation of original noiseless image, noisy image, and filtered image using 2-DLMS, 2-DVSSLMS, and 2-DFLMS. It is observed that the image de-noising using 2-DFLMS is visually far improved and clear then the other considered approaches.

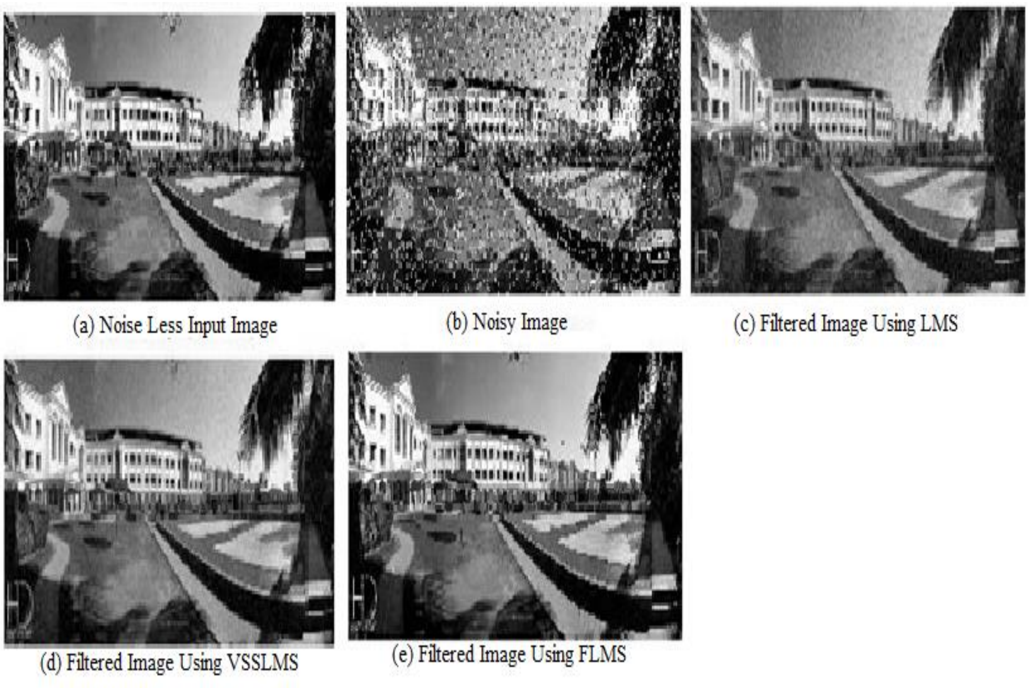

Figure 4. Pictorial Representation of Original Noiseless Image (a), Noisy Image (b), Filtered Image using 2-DLMS (c), 2-DVSSLMS (d), and 2-DFLMS (e)

5. Conclusion

In this paper an extended version of *one dimensional fractional least mean square* (1-DFLMS) is presented. We call it a *two dimensional fractional least mean square* (2-DFLMS). In addition, the performance measure of the proposed algorithm has been rigorously compared with the existing and most employed 2-DAF algorithm namely, *two dimensional least mean square* (2-DLMS), *two dimensional variable step size least mean square* (2-DVSSLMS). In the light of above simulation result it is concluded that FLMS is far improved 2-D adaptive algorithm for image de-noising. It is also a logical rationale that 1-DFLMS has proved the significant improvement in term of quality of solution other the LMS and VSSLM. Likewise, the presented simulation results prove that the proposed 2-DFLMS is a good candidate for image de-noising.

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