Multi-Objective Multi-Period Fixed Charge Solid Transportation Problem With Externalities Using Extended Vikor Approach Under Pythagorean Fuzzy Environment

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Abstract

Transportation sector generates largest share of Greenhouse gas emissions. Spills and accidents in crude oil transportation also generate environmental damages. These are the major causes that affect human health. The externalities due to these environmental damages are not borne by the user. If firms had to pay the full social costs of moving goods by rail and truck, short and long-term changes would probably occur. In this paper focuses on the formulations of the mathematical model of multi-objective multi time fixed charge solid transportation problem with externalities .The parameters are considered as Pythagorean fuzzy number and the optimal solution is obtained by an Enhanced VIKOR method using Lingo software. The numerical example is given to validate this model.

***Keywords**: Multi-objective, Solid transportation problem, Enhanced VIKOR method, Pythagorean fuzzy number,*

1. Introduction

The conventional transportation problems (TP) have a aspiration to solve the most advantageous way to minimize the transportation cost, time, deterioration rate, and total external cost. This was introduced by Hitchcock(1941) The generalized transportation problem which takes into account three types of constraints, namely, source, destination, and conveyance constraints instead of only source and destination constraints. This generalized problem is called the solid transportation problem. It was first stated by Schell(1955).

In a transportation problem when fixed charge is considered against transportation of units from a source to destination, the problem is transformed to Fixed charge TP(FCTP).This FCTP was booted by Hirsch and Dantzig(1968) many researches to solve FCTP. The Fixed Charge Solid Transportation Problem(FCSTP) is discussed by several authors in crisp as well as fuzzy environments. Yang and Liu(2007) presented fuzzy fixed charge STP. In a real life situation like an industrial problem single objective transportation problems are not succeed to handle the managerial decision creation requirements which demands the multi objective transportation problems. For an example ,the objective functions which are minimized may be the total (variable and fixed)cost, the delivery time of transportation, the deterioration rate of the product for summer and winter season during transportation, the external cost, etc. Again, in real-life situations, the multi-objective functions of FCSTP generally conflicting and non-commensurable in nature. Due to this regard, FCSTP can be more altered by adopting the multi objective functions into FCSTP. This converted model is called multi-objective fixed charge solid transportation problem (MFCSTP).Several authors discussed the multi objective TP(2016,2012).

In real- life Product blending is an important technique used in the refining industry such as Petroleum, Chemical and process industries it is the final stage in the conversion of crude oil into

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useful fuels. The blender mixes together several streams from various process units to provide fuel. Due to the fact that it is the final stage in a refinery process, the optimization of this process is vital. Many authors discussed the product blending in industries such as oil refining, chemical and others. The liquid products like Petroleum, Gasoline, Chemical product etc., are transported through different modes of transport with different quality levels from various sources received to different demand points. Further, each demand point has required the minimum quality level of the particular product. For this case, the product taken at each destination can be blended together to satisfy the required quality of the product to the destination. To deal with this type of industrial problem a blending constraint is incorporated .Also construction of new estimates of the seasonal deterioration which affects the human health . Air pollution, greenhouse gas, and spill and accident costs associated with the long-distance movement of petroleum products by truck and rail. Air pollution cost, spills and accidents cost encompass cleanups and efforts to deal with harm to human health, crop and timber yields, degradation of buildings and materials, and reduced visibility and recreation. Hence this type of industrial problem is called multi-objective fixed charge solid transportation problem with externalities and product blending (MFSTEPB).

Many researchers studied multi-objective TP/FCTP/STP/FCSTP in various uncertain environments. Midya and Roy(2014) discussed fixed charge multi-objective (multi-index)stochastic transportation problem using fuzzy programming approach. Mahapatra et al.(2010) formulated multi-objective stochastic transportation problem involving log-normal.

Traditional transportation problems are confined to a particular span. But in real life, the distribution decisions are prolonged for more than one time frame, because it provides a chance to take advantage in lot sizing. Hence the transportation of product in the multi time span has much practical significance. In real life due to uncontrollable factor all the parameters of the TP may not be known precisely. To tackle this situation Zadeh (1965) introduced fuzzy set theory. It is very useful to represent the uncertain data's by fuzzy numbers. Zadeh's ordinary fuzzy sets have been extended to Intuitionistic Fuzzy Set .This Intuitionistic Fuzzy Set was proposed by Atanassov(1986) are the generalization of ordinary fuzzy sets, including membership, non-membership, and hesitancy functions.In IFS theory the sum of membership and non-membership functions are almost 1(Peng and Yang 2015).Yager and Abbasov (2013) developed Pythagorean fuzzy sets which relax the condition of sum of their membership function to square sum of its non-membership functions is less than or equal to one. Therefore, PFSs present a larger area than intuitionistic fuzzy sets to model the real life problems.

VIKOR is one of the well-known classical Multi-criteria Decision Making Methods (MCDM) based upon a particular measure of ''closeness to the ideal/aspired level''. The main focus of this method is on the ranking of a set of choices in the presence of conflicting criteria. It was first proposed by Opricovic and Tzeng 2004, and Opricovic and Tzeng 2007 as an alternative method to TOPSIS, another common MCDM method. Ou Yang, Shieh et al. 2009 stated that it helps decision-makers to select the ''best'' compromise choice. VIKOR method was developed as a multiple attribute decision-making (MADM) method to solve the discrete decision problems with non-commensurable and conflicting criteria. Several researchers
discussed the application of this method in decision making problemsm such as Tzeng, Teng et al. 2002, Opricovic and Tzeng 2004, Tzeng, Lin et al. 2005, Opricovic and Tzeng 2007, Ou Yang, Shieh et al. 2009, Vahdani, Hadipour et al. 2010, Vinodh, Varadharajan et al. 2013, Anvari, Zulkifli et al. 2014). VIKOR method provides a tradeoff between the maximum ''group utility'' of the ''majority'' and the minimum of the individual regret of the ''opponent''. While the method structure and calculation is allegedly simple and straightforward, corresponding compromise solution has shown to be not only a feasible solution but also the closest to the ideal solution, and a compromise means an agreement established by mutual concessions.

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The core objective decision making problems are solved using MADM methods. but it is not adequate to tackle more than one objective .For this purpose of solving MODM the extended TOPSIS method is introduced by several researchers such as Lai.Liu et al.1994,Abo-sinna(2000),Abo-sinna and Amer (2005),Abo-sinna et al.2008,chen 2000,and Deng.Yeh et al.(2000). Enhanced VIKOR method for multi objective optimization problem was proposed by Seyed mohammad salehi, Maghsoud Amiri, Saeed Ramezanzadeh and Mohammadali Abedini(2018)shows that VIKOR solution is better than TOPSIS solution. To the best of our knowledge there is no significant contribution that may have developed Enhanced VIKOR method for Multi-objective transportation problem. Enhanced VIKOR method transfers k- objectives into two objectives where both are the shortest distances from the Positive Ideal Solution(PIS) and the longest distance from Negative Ideal Solution(NIS).This is equivalent to measuring the distance by two L_P-metrics where only p=1 and p=∞ are considered. In this method , a k-dimensional objective space is reduced to two dimensional objective with minimization objectives namely S and R .Then a single objective programming problems is constructed by using PIS and NIS. From solving the later problem final solution is obtained. For the first time Extended VIKOR method is used to solve the multi objective transportation problem with seasonal deterioration and externalities under Pythagorean fuzzy number. A comparison is made for VIKOR and TOPSIS approach from that we observe VIKOR solution have a better performance than TOPSIS solution.

The rest of the paper is as follows. In section 2 , basic concepts of Pythagorean trapezoidal fuzzy number is discussed. In section 3 notations and assumption are examined. Mathematical model is depicted in section 4.Solution methodology is derived in section 5. Numerical example is illustrated in section 6. Conclusion is discussed in section 7.

2. Preliminaries:

Pythagorean fuzzy numbers(PFS):

PFSs are an extension of intuitionistic fuzzy sets, whose sum of membership and non-membership degrees can exceed 1, but their squared sum cannot exceed. Therefore, PFS can carry more information than intuitionistic fuzzy sets.

Definition 2.1

Let Y be a universe of discourse. A PFS $\tilde{A}$ in Y is given by

$$\tilde{A}=\{y,\alpha_{\tilde{A}}(y),\beta_{\tilde{A}}(y)\mid y\in Y\} \qquad \text{…………..(2.1)}$$

where $\alpha_{\tilde{A}}(y)=Y\rightarrow[0,1]$ denotes the degree of membership and $\alpha_{\tilde{A}}(y)=Y\rightarrow[0,1]$ denotes the degree of non-membership $y\in Y$ to the set A, respectively, with the condition that $0\leq\alpha_{\tilde{A}}(y)^2+\beta_{\tilde{A}}(y)^2\leq1.$ The degree of indeterminacy $\pi_{\tilde{A}}(y)=\sqrt{1-\left(\alpha_{\tilde{A}}(y)^2-\beta_{\tilde{A}}(y)^2\right)}$.

Definition 2.2

Let $a\geq0$ and r_1,r_2,r_3,and r_4 be nonzero, then TrPFN $\tilde{A}=\{r_1,r_2,r_3,r_4;\alpha_{\tilde{A}},\beta_{\tilde{A}}\}$ is called a positive TrPFN .

Definition 2.3

Let $\tilde{A}_1=\left\{r_1,r_2,r_3,r_4;\alpha_{\tilde{A}_1},\beta_{\tilde{A}_1}\right\}$ and $\tilde{A}_2=\left\{p_1,p_2,p_3,p_4;\alpha_{\tilde{A}_2},\beta_{\tilde{A}_2}\right\}$ be two TrPFN and **λ**≥0 be a scalar. Then,

$$\tilde{A}_1+\tilde{A}_2=\left[(r_1+p_1,r_2+p_2,r_3+p_3,r_4+p_4);\sqrt{\left(\alpha_{\tilde{A}1}\right)^2+\left(\alpha_{\tilde{A}2}\right)^2-\left(\alpha_{\tilde{A}1}\right)^2\left(\alpha_{\tilde{A}2}\right)^2},\beta_{\tilde{A}_1}\beta_{\tilde{A}_2}\right] \qquad \text{………..(2.2)}$$

$$\tilde{A}_1\times\tilde{A}_2=\left[(r_1\times p_1,r_2\times p_2,r_3\times p_3,r_4\times p_4);\alpha_{\tilde{A}_1}\alpha_{\tilde{A}_2}\sqrt{\left(\beta_{\tilde{A}1}\right)^2+\left(\beta_{\tilde{A}2}\right)^2-\left(\beta_{\tilde{A}_1}\beta_{\tilde{A}_2}\right)}\right] \qquad \text{…………(2.3)}$$

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$$k\tilde{A}_2 = \left[\left(kr_1, kr_2, kr_3 kr_4, r_4 \times p_4\right); \sqrt{1-\left(1-\left(\alpha_{\tilde{A}1}\right)^2\right)^k}, \left(\beta_{\tilde{A}2}\right)^k \right] \quad \text{…………..(2.4)}$$

Definition 2.4

Let $\tilde{A} = \left\{ r_1, r_2, r_3, r_4; \alpha_{\tilde{A}_1}, \beta_{\tilde{A}_1} \right\}$ be a positive TrPFN, then the score function s can be defined as follows

$$s(\tilde{A}) = \left[\left(\frac{r_1+r_2+r_3+r_4}{4} \frac{\alpha^2_{\tilde{A}_1} - \beta^2_{\tilde{A}_1}}{2} \right) \right] s(\tilde{A}_1) \in [-1,1] \quad \text{…………(2.5)}$$

Definition 2.5

Let $\tilde{A}_1 = \left\{ r_1, r_2, r_3, r_4; \alpha_{\tilde{A}_1}, \beta_{\tilde{A}_1} \right\}$ be a positive TrPFN, then the accuracy function h can be defined as follows

$$h(\tilde{A}) = \left[\left(\frac{r_1+r_2+r_3+r_4}{4} \frac{\alpha^2_{\tilde{A}_1} + \beta^2_{\tilde{A}_1}}{2} \right) \right] h(\tilde{A}_1) \in [0,1]. \quad \text{…………(2.6)}$$

3. Notations and assumptions

The following notations and assumptions are used in this paper.

Notations:

m: number of origin

n: number of destination

k: number of conveyances

Dx_{IJK}: unit amount of the product to be transported from I th source to J th destination by K th conveyance,

$\alpha(Dx_{IJK})$:binary variable takes the value "1" if the source I is used ,and "0" otherwise,

$P\tilde{C}_{IJK}$: Pythagorean fuzzy transportation cost for the unit quantity of the product from I th source to J th destination by K th conveyance,

$P\tilde{F}_{IJK}$: Pythagorean fuzzy fixed cost associated with I th source to J th destination by K th conveyance,

$P\tilde{t}_{IJK}$: Pythagorean fuzzy time of transportation of the product from I th source to J th destination by K th conveyance,

$P\tilde{d}^{t}_{IJK}$: Pythagorean fuzzy deterioration rate of goods of the product from I th source to

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J^{th} destination by K^{th} conveyance in time period t ,

$P\tilde{e}_{IJK}$: Pythagorean fuzzy external cost of the product from I^{th} source to J^{th} destination by K^{th} conveyance,

$P\tilde{a}_{I}$: Pythagorean fuzzy availability of the product at I^{th} source,

$P\tilde{b}_{J}$: Pythagorean fuzzy demand of the product at J^{th} destination,

$P\tilde{e}_{K}$: Pythagorean fuzzy total capacity of the product which can be carried by K^{th} conveyance,

$P\tilde{Z}_{K}$: objective function in Pythagorean fuzzy nature(K=1,2,3),

DZ_{K} : objective function in deterministic form, where DZ_{K} =R[$D\tilde{Z}_{K}$] (K=1,2,3),

Dq_{I} : nominal quality of the product which is available from I^{th} source,

Dq_{J}^{min} : least quality of the product needs at destination J.

Assumptions:

1. $P\tilde{a}_{I} > 0$, $P\tilde{b}_{J} > 0, \forall I, J.$
2. Each chosen Pythagorean trapezoidal fuzzy number is positive in all of its components.

4.Mathematical formulation:

This paper explores on the four objective functions. The first objective function assumes the total transportation cost(the variable cost and the fixed cost),the second objective function considers the transporting time and the third objective function refers to the deterioration rate of goods in two different seasons and the last one refers to the external cost of goods (noise, air pollution, climate change, accidents) ; all of the four goals to be minimized. The second objective function is taken to maximize the customers' satisfaction level; in fact, to measure it, the total transportation time is taken. So, with respect to maximizing the customers' satisfaction level, the value of the objective function should be minimized. There are m factories(origin),n customers(destination) and K conveyances(different transportation modes such as trucks, air freight, goods trains, ships, etc,). Each of m factories can transport to any of the n customers by the K conveyances at a transporting cost of DC_{IJK} per unit commodity and a fixed cost of DF_{IJK} .The problem is to calculate the amount Dx_{IJK} ,for any I,J,K of the product conveyed from I^{th} source to J^{th} destination by K^{th} conveyance, in such a way that the overall value of four objective functions are minimized. In several practical situations such as gasoline, petroleum and others industries, blending of raw materials with different attributes and purities into analogous intermediate or final product is a common topic. Blending raw materials give an organization for the chance to perceive more cost savings, while meeting demand for an array to the end of the products and fulfilling pre-decided quality requirements for such kind of product .The intrinsic flexibility of the blending

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activity can be utilized to optimize the allotment and transportation of raw materials to the production facilities. So in MFSTEPB an additional proportionality requirement on the quality of the product is adopted. The additional constraints (4.5)in model-A is indicated as linear blending constraints. The average quality of all products received at destination J is as follows:

$\frac{\sum_{I=1}^{m}\sum_{K=1}^{p} Dq_I Dx_{IJK}}{\sum_{I=1}^{m}\sum_{K=1}^{p} Dx_{IJK}}, J = 1,2,......n$ we assume that the least quality(i.e., clarity) of the product at destination J is $Dq_J^{\min}$ So the constraints on the quality requirement of the product can be defined as :

$$\frac{\sum_{I=1}^{m}\sum_{K=1}^{p} Dq_I Dx_{IJK}}{\sum_{I=1}^{m}\sum_{K=1}^{p} Dx_{IJK}} \geq Dq_J^{\min}, J = 1,2,......n$$

i.e., $\sum_{I=1}^{m}\sum_{K=1}^{p}\left(Dq_I - Dq_J^{\min}\right) Dx_{IJK} \geq 0, J = 1,2,......n.$ Thus, the proposed problem including blending constraints can be formulated as follows:

Model-A

minimize $P\tilde{Z}_1 = \sum_{I=1}^{m}\sum_{J=1}^{n}\sum_{k=1}^{p} P\tilde{C}_{IJK} Dx_{IJK} + P\tilde{F}_{IJK}\alpha(Dx_{IJK})$ ………………(4.1)

minimize $P\tilde{Z}_2 = \sum_{I=1}^{m}\sum_{J=1}^{n}\sum_{k=1}^{p} P\tilde{t}_{IJK}\alpha(Dx_{IJK})$ ………………(4.2)

minimize $P\tilde{Z}_3 = \sum_{I=1}^{m}\sum_{J=1}^{n}\sum_{k=1}^{p}\sum_{T=1}^{t} P\tilde{d}_{IJK}^{T} Dx_{IJK}^{T}$ ………………(4.3)

minimize $P\tilde{Z}_4 = \sum_{I=1}^{m}\sum_{J=1}^{n}\sum_{K=1}^{p} P\tilde{e}_{IJK} Dx_{IJK}$ ………………(4.4)

subject to $\sum_{I=1}^{m}\sum_{K=1}^{p}\left(Dq_I - Dq_J^{\min}\right) Dx_{IJK} \geq 0, J = 1,2,......n.$ ………………(4.5)

$\sum_{J=1}^{n}\sum_{K=1}^{p} Dx_{IJK} \leq P\tilde{a}_I, I = 1,2,......m.$ ………………(4.6) (5.1)

$\sum_{I=1}^{m}\sum_{K=1}^{p} Dx_{IJK} \geq P\tilde{b}_J, J = 1,2,......n.$ ………………(4.7)

$\sum_{I=1}^{m}\sum_{J=1}^{n} Dx_{IJK} \leq P\tilde{e}_K, K = 1,2,......p.$ ………………(4.8)

$\alpha(Dx_{IJK}) = \begin{cases} 0, \text{if } Dx_{IJK} = 0 \\ 1, \text{if } Dx_{IJK} > 0 \end{cases}$ ………………(4.9)

$Dx_{IJK} \geq 0, (I = 1,2,.......m; J = 1,2,.......n; K = 1,2,.......p)$ ………………(4.10)

The feasibility condition is chosen as follows:

$$\sum_{I=1}^{m} P\tilde{a}_I \geq \sum_{J=1}^{n} P\tilde{b}_J, and \sum_{K=1}^{p} P\tilde{e}_K \geq \sum_{J=1}^{n} P\tilde{b}_J.$$

5.Solution methodology

5.1. Enhanced VIKOR method for MFCSTEPB:

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A vector of objective functions $pZ(Dx) = \left(pz_1(Dx), pz_2(Dx) \ldots\ldots pz_k(Dx) \right)$ exists in MFCSTEPB instead of single objective function. Consider MFCSTEPB stated in (4.1) to (4.10)

The positive and negative ideal solution of the objectives are denoted by the vector

$pZ^* = \left(pz_1^*, pz_2^* \ldots\ldots pz_k^* \right)$ and $pZ^- = \left(pz_1^-, pz_2^- \ldots\ldots pz_k^- \right)$ in which

$pz_I^* = \max_{Dx \in DX} pz_I(Dx)$ and $pz_I^- = \min_{Dx \in DX} pz_I(Dx)$, I=1,2,…..m respectively.

In case of minimization model $pz_I^* = \min_{Dx \in DX} pz_I(Dx)$ and $pz_I^- = \max_{Dx \in DX} pz_I(Dx)$. The Lp metric is used for the measure of "Closeness". The normal distance between $pZ(\mathrm{D}x)$ and Z^* are defined by Lp metric.

i.e
$$d_q = \left\{ \sum_{I=1}^{m} w_I \left(\frac{pz_I^* - pz_I}{pz_I^* - pz_I^-} \right)^q \right\}^{\frac{1}{q}}, q = 1, 2, \ldots\ldots \infty \qquad \ldots\ldots\ldots\ldots\ldots..(5.2)$$

where w_I ,I=1,2,…..m are the weights of the objectives. To obtain a compromise solution of MFCSTEPB problem (5.1) the ideal vector of objective functions $pZ(Dx) = \left(pz_1^*, pz_2^* \ldots\ldots pz_k^* \right)$ is considered to be the reference point. Equation (5.2) is used to find the distance to the reference point. This problem can be transformed to solve the following auxiliary problem

$$\min_{Dx \in DX} pZ = d_q^q = \left\{ \sum_{I=1}^{m} w_I \left(\frac{pz_I^* - pz_I}{pz_I^* - pz_I^-} \right)^q \right\}, q = 1, 2, \ldots\ldots \infty \qquad \ldots\ldots\ldots..(5.3)$$

In the VIKOR method for MADM, the distance is measured by Equation (5.2) for p=1 and p=∞.So to develop the Enhanced VIKOR method for solving MFCSTEPB problems, we rewrite (5.3) as the following two-objective programming model:

$$\min_{Dx \in DX} S = \sum_{I=1}^{m} w_I \left(\frac{pz_I^* - pz_I(Dx)}{pz_I^* - pz_I^-} \right)$$

$$\min_{Dx \in DX} R = \max_{I} \left\{ w_I \left(\frac{pz_I^* - pz_I(Dx)}{pz_I^* - pz_I^-} \right) : I = 1, 2, \ldots..m \right\} \qquad \ldots\ldots\ldots\ldots\ldots(5.4)$$

The equation (5.4) converts to the following problem:

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$$\min S = \sum_{I=1}^{m} w_I \left(\frac{pz_I^* - pz_I(Dx)}{pz_I^* - pz_I^-} \right)$$

$\min R$

s.t: constraints (4.5) to (4.10)

$$R \geq w_I \left(\frac{pz_I^* - pz_I(Dx)}{pz_I^* - pz_I^-} \right), I = 1, 2, ...m \quad ……………..(5.5)$$

$$Dx \in DX$$

Remark: In model (5.4), we can calculate the distance with formula (5.2) for $q \geq 2$.
Therefore, equation (5.5) can be rewritten as follows:

$$\min_{Dx \in DX} S = \sum_{I=1}^{m} w_I \left(\frac{pz_I^* - pz_I(Dx)}{pz_I^* - pz_I^-} \right)^q$$

$\min R$ …………(5.6)

s.t: (4.5) to (4.10)

$$R \geq w_I \left(\frac{pz_I^* - pz_I(Dx)}{pz_I^* - pz_I^-} \right), I = 1, 2, ...m$$

$$Dx \in DX, q = 1, 2, ...$$

By solving equation (5.5),the positive ideal (S^*, R^*) and negative ideal (S^-, R^-) solutions of the objective functions can be obtained ,in which $S^* = \min_{Dx \in DX} S, S^- = \max_{Dx \in DX} S, R^* = \min_{Dx \in DX} R$ and $R^- = \max_{Dx \in DX} R$.

Therefore, we can construct the following model by the Enhanced VIKOR methodology:

$$\min \phi = \eta \left(\frac{S^* - S(Dx)}{S^* - S^-} \right) + (1-\eta) \left(\frac{R^* - R(Dx)}{R^* - R^-} \right) \quad …………(5.7)$$

s.t: (4.5) to (4.10)

$Dx \in DX, \eta \in (0,1]$.

A weight function is denoted by η that acts as a preference model between "the majority of Criteria" and" the maximum group of utility ". Here for the ease of interpretation, we assumed that $\eta = 0.5$ or there is no preference between two factors. By solving the model (5.7) compromise solutions of MFCSTEPB problem (5.1) can be obtained.

6. Numerical example:

To illustrate the applicability of the proposed model for solving MFCSTEPB problem, the following numerical example is adopted from Sankar Kumar Roy (2019) with externality cost and seasonal deterioration under Pythagorean fuzzy environment.

A reputed oil production company in India (namely, IOC LTD) produces ethanol blended petrol (EPB) product.

Table:1 Transportation cost and fixed charge ($P\tilde{C}_{IJK}, P\tilde{F}_{IJK}$):

Factory Destination	Conveyance (K=1)		
	1	2	3

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1	(89,132,144,147; , .6.5) (61,66,68,73; .8,.4)	(71,114,126,129; .6,.5) (105,107,109, 111; .7,.5)	(107,150,162,165; .6,.5) (101,103,105,107; .7,.5)
2	(107,150,162,165; .6,.5) (69,74,76,81; .8, .4)	(61,63,65, 79; .7, .4) (73,78,80,85; .8,.4)	(71,114,126,129; .6.5) (63,68,70,75; .8,.4)
3	(71,114,126,129; .6,.5) (65,70,72,77; .8,.4)	(54,56,58, 72; .7, .4,) (58,61,63,68; .8, .4)	(63,65,67, 81;.7, .4) (100,101,102,105;.8, .5)
Conveyance (K=2)			
	1	2	3
1	(48,50,52, 66; .7, .4) (69,74,76,81; .8,.4)	(89,132,144,147; .6,.5) (58,61,63,68; .8, .4)	(54,56,58, 72; .7,.4,) (110,112,114,116; .7,.5)
2	(54,56,58,72; .7 .4) (100,101, 102,105;.8,.5)	(97,99, 101,103; .7, .5) (96,98, 118,120; .8, .5)	(61,63,65, 79; .7, .4) (67,72,74,79; .8,.4)
3	(107,150,162,165; .6,.5) (98, 100,102,120,; .8,.5)	(97,99, 101,103; .7, .5) (69,74,76,81; .8, .4)	(101,101,103,107;.7, .5) (98,100,120,122;.8,.5)

Table-2: Transportation time ($^{P}\tilde{t}_{IJK}$):

Factory Destination	**Conveyance**(K=1)		
	1	2	3
1	(48,50,52, 66; .7,.4)	(107,150,162,165; .6,.5)	(107,150,162,165; .6,.5)
2	(89,132,144,147; .6 .5)	(89,132,144,147; .6 .5)	(46,48,50, 64; .7,.4)
3	(54,56,58, 72; .7, .4)	(71,114,126,129; .6,.5)	(107,150,162,165; .6,.5)
Conveyance (K=2)			
	1	2	3
1	(61,63,65, 79; .7, .4)	(61,63,65, 79; .7, .4)	(52,54,56,70; .7,.4)
2	(48,50,52, 66; .7,.4)	(48,50,52, 66; .7,.4)	(63,65,67, 85.7, .4)
3	(105,107,109,111; .7,.5)	(107,150,162,165; .6,.5)	(54,56,58, 72; .7, .4)

Table-3: Deterioration rate for multi period(, $^{P}\tilde{d}^{T}_{IJK}$):

Factory Destination	**Conveyance**(K=1)		
	1	2	3
1	(11.5,11.8,12.9,13; .5,.3) (5.75,5.9,6.45,6.5; .5,.3)	(37,,41, 42,44;.6,.5) (5,6, 7,10; .5,.3)	(11.5,11.8,12.9,13; .5,.3) (5.75,5.9,6.45,6.5; .5,.3)
2	(9.5,9.8, 10,10.3;.5,.3) (3,3.6,4.5,5.5;.8,.5)	(40.2, 45, 46,48;.6,.5) (1.1,1.5,2.1,4.3; .8,.3)	(46,50, 51, 53;.6,.5) (1.5,1.5,3.1,4.3; .8,.3,)

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3	(11.5,11.8,12.9,13; .5,.3) (5.5,5.75,5.9,6.45;.5,.3)	(35, 39, 40,42; .6,.5) (5.75,5.9,7.5,7.65; .5,.3)	(39, 43, 45,46.2; .6,.5) (1,1.5,2.1,4.2; .8,.3)
Conveyance (K=2)			
	1	2	3
1	(38, 42, 43,45; .6,.5) (1.1,1.5,2.1,4.2; .8,.3)	(46,50, 51, 53;.6,.5) (1.5,1.5,3.1,4.3; .8,.3)	(49, 53, 54,56;.6,.5) (5.9,6.75, 7.65,8.5; .5,.3)
2	(11,11.5,11.8,12.9; .5,.3) (5.75,5.9,6.45,6.5; .5,.3)	(46,50, 51, 53;.6,.5) (1.2,1.6,3.1,4.1;.8,.3)	(55, 59, 60,62; .6,.5) (6, 7.2, 9,11;.8,.5)
3	(45,45.4,46, 48; .6,.5) (1.1,1.5,3,4; .8,.3)	(52, 56, 57,59; .6,.5) (5,7, 8,10;.8,.5)	(47, 51, 52,54; .6,.5) (1.5,1.5,3.1,4.3; .8,.3)

Table-4: Total External cost($P\tilde{e}_{IJK}$):

Factory Destination	**Conveyance**(K=1)		
	1	2	3
1	(27,28,30, 31; .9,.1)	(17.8,25, 27,27;.9,.1)	(8,11, 12,13;.7,.2)
2	(11,12, 13,14;.7,.2)	(20,21,21,22;.8,.2)	(36,37, 37,38;.9,.1)
3	(25, 26,27, 30;.9,.2)	(24,25,26, 31;.8,.1)	(31.2, 45,50,55; .9,.1)
Conveyance (K=2)			
	1	2	3
1	(16,18,20,22;.7,.2)	(15,15.2,17,18;.8,.4)	(6,8, 10,12; .6,.4)
2	(9.7, 10,11.5,14;.6,.4)	(8,9,11,12;.7,.2)	(20,21,21,22;.8,.2)
3	(16,18,20,22;.8,.4)	(26,27,28,29;.6,.3)	(16.2,18,19,20; .9,.1)

The company has three plants at Barauni (in Bihar), Haldia (in West Bengal), Paradip (in Odisha) and three distribution centers(n=3) situated at Kolkata, Jamshedpur ,Patna in the country. The company transports EBP from plants to demand points through tankers by two types of conveyances(p=2) namely, highways and railways. Decision maker (DM) desires that the total transporting cost (variable cost per unit and fixed cost),total transporting time to transport EBP from plants to distribution centers , deterioration rate of EBP for two different seasons and the total external cost (Noise, Air pollution, Climate change, Accidents) are to be minimized. In this real life problem fixed charge is considered in two ways. For highway transportation the oil company would pay a certain amount of toll charge to national Highway Authority of India for different types of tankers to move EBP and maintenance cost of tankers is also included. For railways transportation the oil company could pay a certain amount to Indian Rail Authority for booking train tankers. Furthermore, DM decides to find a pareto -optimal solution to the problem in which the values of the objective functions are to be minimized. The relative importance of four objective functions are considered as the weight factors which are specified by DM. The transportation cost in rupees per barrel, fixed-charge in rupees for an open route, time in hour ,deterioration rate in litre and total external cost in rupees per barrel are considered. Data for transportation cost and fixed –charge are shown in Table-1 transporting time are presented in table-2 deterioration rate of EPB for two different periods are given in Table-3 and total external cost are listed in Table-

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4;supply and demand parameters are shown in Table-5;capacity of the conveyance are present in Table-6.Also,each plant consist the formal quality

Table:5 Supply and demand and their deterministic value

I/J	$P\tilde{a}_I$	$P\tilde{b}_J$	$S(P\tilde{a}_I)$ $S(P\tilde{b}_J)$
1	(72,74,76,78; .9,.1)	(58,60,61,73; .9,.1)	30, 25
2	(60,62,63,75; .9,.1)	(65,67,68,80; .9,.1)	26, 28
3	(80,82,85,85; .9,.1)	(66,68,69,81; .9,.1)	33, 28.33

Table:6 Capacity of K^th conveyance $P\tilde{e}_K$ and its deterministic value $R(P\tilde{e}_K)$

	$P\tilde{e}_K$	$S(P\tilde{e}_K)$
K=1	(100,110, 110,120; .9,.1)	44
K=2	(70.2,98,107,118;.9,.1)	39.33

(i.e., purity) of the produced EBP which are Dq_1=0.85 (i.e.,85% clarity of the product), Dq_2=0.90, Dq_3=0.80 and the least quality of the produced EPB needs at each distribution center which are $Dq_1^{min}=0.90, Dq_2^{min}=0.80, Dq_3^{min}=0.85$. Solving the above problem by using Enhanced VIKOR method for multi objective optimization problem given in section5.They obtained Positive Ideal Solution (PIS) and Negative Ideal solution (NIS) that is given in Tables-7 & 8 .

Table-7: PIS pay off

	pz_1	pz_2	pz_3	pz_4
Min pz_1	893.75	68	279.0	294.441
Min pz_2	889	58	283.49	300.44
Min pz_3	869.95	52.5	259.28	302.67
Min pz_4	874.8	41	276.81	224.10

PIS pZ^* (869.95, 41, 259.28, 224.10)

Table-8: NIS pay off

	pz_1	pz_2	pz_3	pz_4
Max pz_1	953.61	74.5	289.84	303.31
Max pz_2	889.95	58	283.49	300.44
Max pz_3	905.13	49	293.48	264.9
Max pz_4	907.95	56	279.13	359.13

NIS pZ^* (953.61, 74.5, 293.48, 359.13)

$$\min S = w_1\left(\frac{869.95-pZ_1(Dx)}{-83.66}\right) + w_2\left(\frac{41-pZ_2(Dx)}{-33.5}\right) + w_3\left(\frac{259.28-pZ_3(Dx)}{-34.2}\right) + w_4\left(\frac{224.1-pZ_4(Dx)}{-135.03}\right)$$

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min R
s.t: (4.5) to (4.10)

$R \geq w_1 \left(\frac{869.95 - z_1(Dx)}{-83.66} \right),$

$R \geq w_2 \left(\frac{41 - z_2(Dx)}{-33.5} \right)$

$R \geq w_3 \left(\frac{259.28 - z_3(Dx)}{-34.2} \right)$

$R \geq w_4 \left(\frac{224.1 - z_4(Dx)}{-135.03} \right)$

D $x \in$ DX

The assumed unequal weights are w_1=0.2,w_2=0.4,w_3=0.3,w_4=0.1and the payoff is given in Table-9.

Table-9 solution for unequal weights.

	S	R
Min S	0.1846420	0.1194030
Min R	0.7034337	0.3104478

$\left(S^*, R^*\right) = (0.1846, 0.1194)$

$\left(S^-, R^-\right) = (0.7034, 0.3104)$

$$\min \phi = \eta \left(\frac{0.1846 - S(Dx)}{-0.5184} \right) + (1-\eta) \left(\frac{0.119 - R(Dx)}{-0.1914} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots(6.1)$$

s.t: (4.5) to (4.10)
D $x \in$ DX

By solving (6.1) we obtain the solution of Enhanced VIKOR method for MFCSTEPB take ($\eta = 0.5$) it is shown in table-10.

Table-10: Solution of Enhanced VIKOR method for MFCSTEPB ($\eta = 0.5$)

Weights	ϕ	pZ_1	pZ_2	pZ_3	pZ_4
w_1=0.2, w_2=0.4, w_3=0.3, w_4=0.1.	0.5746897	886.25	63.5	261.2541	293.2910

While comparing the solution of the existing Intuitionistic fuzzy TOPSIS method for multi-objective optimization problem by Sankar Kumar Roy(2019)) with the solution obtained by proposed Enhanced fuzzy VIKOR Method the following results were obtained.

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Table 11:

Method	pZ_1	pZ_2	pZ_3
Fuzzy TOPSIS (ExistingMethod)	852.66	40	176.80
Enhanced fuzzy VIKOR(Proposed Method)	767.42	56.5	176.80

This table shows that the proposed method would give the better result compared to the existing method as the objective function related to the transportation cost is minimum which plays the major role in logistics management.

7. Conclusion:

In the flied of research this study is the first attempt to apply Enhanced VIKOR method to solve multi-objective fixed charge solid transportation problem with externalities under Pythagorean fuzzy number. Pythagorean fuzzy number can carry more information than Intuitionistic fuzzy set. Because which relax the condition of sum of their membership function to square sum of its non-membership functions is less than one. Also Enhanced VIKOR method gives better performance than Extended TOPSIS method as shown in Table 11. Also in this study seasonal deterioration and external cost are included and solved by Enhanced VIKOR approach in Pythagorean fuzzy environment. In future Enhanced VIKOR method could be used to solve any multi-objective optimization problem.

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