

Fuzzy Logistics -Delay using with the subsidiary frame work.

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ABSTRACT

In many real life problems most of the times we comes across the discussion on expenditure minimization and Profit maximization. This is the all time business algorithm to sustain in the present competitive world online or offline. the cost of transporting some goods from one location to another cannot be determined precisely with out any past background numerical data. Means every developed Transportation problems is a mathematical model may be linear or Non-linear. This may be qualitatively studied into many reasons such as fluctions in the daily commodities like raw materials , Lubricant materials like Crude oil fuel prices in the international market scenario. Daily wages of Labour, ,man power utilization, optimality utilization of man power etc. The inexact costs can be conveniently modeled by coining the most importat numbered pattern called fuzzy numbers. Thus transportation problems with fuzzy costs are of a great importance and which are reliable also in all business fields. Our aim here is to optimize the cost, Labour in order to get Maximum gains as marketing strategy is concerned. This will be addressed by suitable logical constructed frame work with the virtue of Fuzzy sets and Fuzzy numbers.

Key Words: *Fuzzy logic, fuzzy set; fuzzy number; transport problems, fuzzy linear programming.*

Mathematical Subject Classification numbers: **90C70, 90C26.**

I. Introduction :

In the process exists in the assignment problem with fuzzy costs has been considered by Lin and Wen [2004]. They have modeled the problem into a combinational integer programming problem, using the fuzzy high resolution of Bellman and Zadeh [1970] and then converted it into a simple linear fractional programming . They have developed a suitable group algorithm ,to the solution of the resultant linear fractional programming problem.

In this present context of fuzzy logic and algorithm precision the historicity and Non-chaotic characteristic of the governed mathematical model considered. The introductory need for an best nested j solution, or the best solution, among those avai lablein a properly defined, problem is the rationale behind studying

the theories and proposing methodologies appropriate to the scientific field in which the problem arises. The aim of an optimal solution of a transportation problem with fuzzy cost coefficients is to optimize the objective function with prescribed conditions and a useful flow of computational algorithm. This kind of frame work we can come across in the literatre survey[1,2,3] for determining solution and the nature of the optimality feasible conditions was given by so many statisticians in past .

For example, some industries receiving coal may fix some limits on the quantity of sulphur in the coal supplied. A method of solution of a multiobjective time dependent transportation problem with non-quality restrictions was developed by eminent researchers like Chawla [2002].

The present insight is to aimed, at developing a solution methodology for a transportation problem involving both the above aspects i.e costs are fuzzy and also impurity restrictions are imposed. The demand and supply values are assumed to be hard numerals.

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II. Mathematical Modeling:

The mathematical aspect formulation for so called the fuzzy transportation problem by means of supplementary restrictions is defined as

$$\text{Minimize } \sum_{i=1}^m \sum_{j=1}^n C_{ij}x_{ij} \quad (1)$$

Subject to the given constraints

$$\sum_{j=1}^n x_{ji} = a_i, (I =1,2,\dots,m) : \sum_{i=1}^m x_{ji} = b_j, (J =1,2,\dots,n) : \sum_{i=1}^m f_i x_{ij} \leq p_j, (j =1,2,3,4 \dots,n)$$

$$x_{ij} \text{ all values are positive in nature \& which are integers, } i =1,2,\dots,m: j =1,2,\dots,n$$

Here a_i is the amount of commodity available at the j^{th} origin and b_j is the prerequisite of the product at the j^{th} destination. Single unit of the article of trade at the i^{th} origin contains f_i units of a certain adulteration and j^{th} destination should not receive more than p_i units of the dirtiness.

x_{ij} is the amount of commodity transported from the i^{th} origin to the j^{th} destination.

The costs $C_{ij} = (\alpha_{i^*j}, \beta_{ij})$ ($i =1,2,\dots,m, j =1,2,\dots,n$) are sub normal fuzzy numbers having strictly increasing linear membership functions defined as follows .

Monotony function :

$$\mu_{ij}(C_{ij}) = \begin{cases} \beta_{ij} & \text{if } C_{ij} = \beta_{ij} \text{ and } x_{ij} > 0 \\ \alpha_{ij}(C_{ij}-\alpha_{ij})/(\beta_{ij}-\alpha_{ij}) & \text{if } \alpha_{ij} \leq C_{ij} \leq \beta_{ij} : x_{ij} > 0 \end{cases}$$

$$0 : \text{Otherwise} \quad (2)$$

The condition $x_{ij} > 0$ is balanced at the point equation (2) because there is no precise expense, Provided $x_{ij} = 0$ in any sufficient solution x of (1). We use the notation $(\alpha_{ij}, \beta_{ij})$ to denote the unique type of fuzzy number C_{ij} used here. Matrix $[C_{ij}]$ is written as $[C_{ij}] = [\langle \alpha_{ij}, \beta_{ij} \rangle]_{m \times n}$

Matrix $[q_{ij}]$ is defined by $[q_{ij}] = [q_{ij}]_{m \times n}$

Let C_T denote the total cost and a, b be the lower as well as upper bounds of the total cost respectively. We define the membership function of C_T as linear decreasing function in (3) and use the notation (a, b) to denote fuzzy number C_T . Here a, b considered to be constants. The required task is to engage in recreation down cost for the defined transportation problem which is define in equation (1)..

$$\left\{ \begin{array}{l} 1, \text{ if } C_T \leq a \\ \mu_T(C_T) = \mu_{\tau\tau} \left(\sum_{i=1}^m \sum_{j=1}^n C_{ij}x_{ij} \right) = (b - \sum_{i=1}^m \sum_{j=1}^n C_{ij}x_{ij}) / (b - a) \\ \text{if } a \leq C_T \leq b \\ 0, \text{ if } C_T \geq b \end{array} \right. \quad (3)$$

III Algorithm for the problem and its solution:

With the Virtue of Bellman-Zadeh's fuzzy decision [1970], to Non- minimize the minimum of the hierarchy functions subsequent to that solution, in other words

$$\text{Max - Min } (\mu_{ij}(i=1,2,\dots,m, j=1,2,\dots,n), \mu_T(C_T)) \quad (4)$$

Where x_{ij} is the element of a feasible solution x of problem (1). Then one can embody the defined model (1) as below.

$$\text{IV. Objective function : } \{ \text{max-min. } (\mu_{ij}, \mu_T(C_T)) \} \quad (5)$$

X_{IJ} must be positive real number for feasible solution . .

Subject to the constraints

$$\sum_{j=1}^n X_{IJ} = A_I \quad , (I=1,2,\dots,m) : \quad \sum_{i=1}^m x_{ij} = B_j, (j=1,2,\dots,n)$$

$$\sum_{i=1}^m f_i x_{ij} \leq P_j, (j=1,2,\dots,n) ; \quad x_{ij} \geq 0 \text{ and are integer for } i=1,2,\dots,m, j=1,2,\dots,n$$

Applying the defined membership functions of the unit costs and , total transportation cost as defined in (2) and (3) respectively, we can further define (5) as the subsequent equivalent model.

$$\text{Maximize } \lambda \tag{6}$$

Subject to the bounded constraints

$$x_{ij} \leq \frac{(q_{ij}(C_{ij}\lambda - \alpha_{ij})x_{ij})}{(B_{ij} - \alpha_{ij})\lambda} \text{ here } i=1,2,\dots,m, j=1,2,\dots,n$$

$$(B - A) \lambda \leq (b - \sum_{i=1}^m \sum_{j=1}^n C_{ij}^\lambda x_{ij})$$

$$\sum_{j=1}^n x_{ij} = a_i, (i=1,2,\dots,m)$$

$$\sum_{i=1}^m x_{ij} = b_j, (j=1,2,\dots,n)$$

$$\sum_{i=1}^m f_i x_{ij} \leq p_j, (j=1,2,\dots,n)$$

$$C_{ij}^\lambda x_{ij} \leq \beta_{ij} x_{ij} : j=1,2,\dots,n, i=1,2,\dots,m$$

$$x_{ij} \geq 0 \text{ and integer for } i=1,2,\dots,m, : j=1,2,\dots,n$$

Where C_{ij}^λ denotes the λ -cut off C_{ij} . In equation (6), since x_{ij} , C_{ij}^λ , and λ are all decision variables, it can be considered as a assorted integer non-linear programming model

We define E as the set of all pairs (i, j) where x_{ij} is an element of the basic feasible solution x of equation (1) and , **confine** our discussion based on E. Then, we can define and rewrite Equation (6) as follows:

$$\text{Maximize } \lambda \tag{7}$$

Subject to the boundary restrictions as

$$\lambda \leq (q_{ij}(C_{ij}^\lambda - \alpha_{ij})) / (\beta_{ij} - \alpha_{ij}) \text{ for } (i, j) \in E$$

$$\lambda \leq (b - \sum_{(i,j) \in E} C_{ij}^\lambda x_{ij}) / (b-a),$$

$$C_{ij}^\lambda \leq \beta_{ij} \text{ for } (i, j) \in E$$

Let $d_{ij} = \beta_{ij} - C_{ij}^\lambda \geq 0$. Then (3.7) can be expressed as follows:

Optimization for the function :

$$\text{Maximize } \lambda \tag{8}$$

Subject to

$$\lambda \leq q_{ij}(\beta_{ij} - \alpha_{ij} - d_{ij}) / (\beta_{ij} - \alpha_{ij}) \text{ for } (i, j) \in E \tag{9}$$

$$\lambda \leq (b - \sum_{(i,j) \in E} (\beta_{ij} - d_{ij}) x_{ij}) / (b-a), \tag{10}$$

$$d_{ij}, \lambda \geq 0 \text{ for } (i, j) \text{ belongs to } E \tag{11}$$

Now, the necessary theory for the development of a linear fractional programming problem having the same optimal solution of problem equation (8) to constraints (11) is given which accept the basic principles of optimization which are existing in the literature.

V. The fractional programming model

problem (.6) can be restated as

$$\text{Maximize } (b - \sum_{i=1}^m \sum_{j=1}^n \alpha_{ij} x_{ij}) / (b-a + \sum_{i=1}^m \sum_{j=1}^n \gamma_{ij} x_{ij}) \tag{12}$$

With the given supportive conditions $\sum_{j=1}^n x_{ij} = a_i, (i=1,2,\dots,m); \sum_{i=1}^m x_{ij} = b_j, (j=1,2,\dots,n);$

$$\sum_{i=1}^m f_i x_{ij} \leq p_j, (j=1,2,\dots,n); \quad x_{ij} \geq 0 \text{ and integer for } i=1,2,\dots,m, j=1,2,\dots,n$$

Problem (12) is a linear fractional encoding problem and its optimal solution may be obtained by the algorithm due to Kantiswarup [1965]. The optimal value of the objective function of prople12) is λ_x .

d_{ij} for $(i, j) \in E$ can be obtained from

$$\lambda_x = (\beta_{ij} - \alpha_{ij} - d_{ij}) / \gamma_{ij} \text{ for } (i, j) \in E \quad (13)$$

Then the fuzzy costs corresponding to the maximal value of λ are given by

$$C_{ij}^\lambda = \beta_{ij} - d_{ij} \text{ for } (i, j) \in E \quad (14)$$

VI. Problems for discussion :

Let us consider the following example:

$$\text{Minimize } \sum_{i=1}^3 \sum_{j=1}^3 C_{ij} x_{ij} \quad (15)$$

“Subject to “

$$x_{13} - (-x_{23}) + x_{33} = 5 ; x_{11} + x_{12} + x_{13} = 4 , x_{11} + x_{21} + x_{31} = 5 ; \quad x_{31} + x_{32} + x_{33} = 6 ; x_{21} + x_{22} + x_{23} = 5 ; x_{12} + x_{22} + x_{32} = 5 ; 2x_{11} + x_{21} + 0.x_{31} \leq 4$$

$$20x_{12} + 10x_{22} + 0.x_{32} \leq 0.1 ; \quad 2x_{13} + x_{23} + 0.x_{33} \leq 9$$

$x_{ij} \geq 0$ and are integers for $i, j = 1, 2, 3, \dots, n$.

$$\text{Where } [C_{ij}] = \begin{bmatrix} \langle 4,13 \rangle & \langle 3,12 \rangle & \langle 2,6 \rangle \\ \langle 4,13 \rangle & \langle 6,14 \rangle & \langle 7,15 \rangle \\ \langle 7,10 \rangle & \langle 4,8 \rangle & \langle 6,11 \rangle \end{bmatrix}$$

$$q_{ij} = \begin{bmatrix} 0.9 & 0.6 & 0.8 \\ 0.9 & 0.8 & 0.8 \\ 0.6 & 0.4 & 0.4 \end{bmatrix} \quad \text{Then we have}$$

$$[\alpha_{ij}] = \begin{bmatrix} 4 & 3 & 2 \\ 4 & 6 & 7 \\ 7 & 4 & 6 \end{bmatrix} \quad [\beta_{ij}] = \begin{bmatrix} 13 & 12 & 6 \\ 13 & 14 & 15 \\ 10 & 8 & 12 \end{bmatrix} \quad [\gamma_{ij}] = \begin{bmatrix} 10 & 15 & 5 \\ 10 & 10 & 10 \\ 5 & 5 & 10 \end{bmatrix}$$

“ a “ is selected , as the minimum outlay of the transportation problem with costs as α_{ij} 's (a = 54) and b be in use as the , utmost(Non-minimum) cost of the transportation problem with costs as β_{ij} 's (b =191.98). Now, using (12), the following fractional programming problem is developed corresponding to problem defined as (15)

$$\text{Maximize } (192 - 4x_{11} - 3x_{12} - 2x_{13} - 4x_{21} - 6x_{22} - 7x_{23} - 7x_{31} - 4x_{32} - 6x_{33}) / \quad (16)$$

$$(138 + 10x_{11} + 15x_{12} + 5x_{13} + 10x_{21} + 10x_{22} + 10x_{23} + 5x_{31} + 5x_{32} + 10x_{33})$$

Subject to the supporting conditions

$$x_{11} + x_{12} + x_{13} = 4 ; \quad x_{21} + x_{22} + x_{23} = 5 ; \quad x_{31} + x_{32} + x_{33} = 6$$

$$x_{11} + x_{21} + x_{31} = 5 ; \quad x_{12} + x_{22} + x_{32} = 5 ; \quad x_{13} + x_{23} + x_{33} = 5$$

$$2x_{11} + x_{21} + 0.x_{31} \leq 3.99 ; \quad 2x_{12} + x_{22} + 0.x_{32} \leq 1 ; \quad 2x_{13} + x_{23} + 0.x_{33} \leq 9$$

$$x_{ij} \geq 0 \text{ and are positive integers for } i, j = 1,2,3,4,\dots,n.$$

The best solution of problem (16) is obtained by using the method developed K'antiswarup [1965] as follows,

$$x_{13} = 4, x_{21} = 4, x_{23} = 1, x_{31} = 1, x_{32} = 5 \text{ with } \max \lambda_x = 0.563$$

for $(i, j) \in E$, we have

$$\lambda_x = (\beta_{ij} - \alpha_{ij} - d_{ij})/\gamma_{ij} \text{ so that } d_{ij} = \beta_{ij} - \alpha_{ij} - \lambda_x \gamma_{ij}$$

∴ We have

$$d_{13} = 4 - (0.563)(5) = 1.185 ; d_{21} = 9 - (0.563)(10) = 3.37 ; d_{23} = 8 - (0.563)(10) = 2.37$$

$$d_{31} = 3 - (0.563)(5) = 0.185 ; d_{32} = 4 - (0.563)(5) = 1.185$$

The fuzzy costs corresponding to $\lambda = 0.562.99$ will be

$$C_{ij}^\lambda = \beta_{ij} - d_{ij} \text{ for } (i,j) \in E \text{ with } \therefore \text{ We have}$$

$$C_{13}^{0.563} = 6 - 1.185 = 4.815 ; C_{21}^{0.563} = 13 - 3.37 = 9.63 ; C_{23}^{0.563} = 15 - 2.37 = 12.63$$

$$C_{31}^{0.563} = 10 - 0.185 = 9.815 ; C_{32}^{0.563} = 8 - 1.185 = 6.815$$

$$\therefore \text{Total transportation cost} = \sum_{(i,j) \in E} C_{ij}^{0.563} x_{ij} = 114.29 \approx 114 \text{ Approximately .}$$

VII. observations and Discussions:

The discussed and logically developed procedure for obtaining an optimal solution of a transportation problem with fuzzy costs and involving impurity restrictions is computationally efficient and gives us most accurate reliable values. The optimal values of x_{ij} 's can be obtained by the method developed by Kantiswarup in the year 1965 which is totally sequence input data collected model. A sophisticated computer program can be developed with ease in any programming language for this erudite work. The optimal values of d_{ij} 's and hence the unit costs can be obtained using expressions given in the method.

The fuzzy transportation problem selected in this research paper may not possess an optimal solution, in some cases. This may be attributed to the presence of the impurity restrictions. In such cases, a feasible solution and therefore an optimal solution may be obtained by relaxing the impurity restrictions of the destinations to some extent. By enlarge this method is most suitable for modernized technological firms with great accurate results indeed.

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