Some Standard Results on Triple Connected Total Perfect Domination of Fuzzy Graph

P. Kathavarayan[1], T.Gunasekar[2*], and K.Elavarasan[3]
[3]Research Scholar,[2,3]Department of Mathematics, Vel Tech RangarajanDr.Sagunthala R&D Institute of Science and Technology, Chennai – 62, Tamil Nadu, India.
[1]Chennai Institute of Technology, Kundrathur, Chennai - 600069, Tamil Nadu, India
Email: [1]kathuray@gmail.com, [2]tguna84@gmail.com and [3]elavarasank2@gmail.com
[2]Corresponding Author

Abstract

In the present paper, we initiated the idea of a strong triple connected total perfect dominating set ($\dot{S}$), weak triple connected total perfect dominating set ($\grave{W}$) of a fuzzy graph. The new kinds of properties are initiated for $\dot{S}$, $\grave{W}$ of a fuzzy graph. Our result leads that the $\dot{S}$, $\grave{W}$ dominating number with examples are obtained.

***Keywords**: Fuzzy graph, Total perfect domination.*
***Mathematics Subject Classification:** 05C72, 05C17*

1. Introduction

In 1965, the concept of a fuzzy set was introduced by Zadeh [15] as a way of representing uncertainty and vagueness. In 1975, Rosenfeld [11] introduced the notion of a fuzzy graph and several fuzzy analogs of graph theoretical concepts such as paths, cycles, and connectedness. In 1998, A. Somasundaram and S. Somasundaram [14] begun the concept of domination in fuzzy graphs. The concept of perfect domination, total domination was introduced by Cockayne et al [2]. Revathi et al [10] initiated the concept of connected total perfect domination, strong and weak perfect domination in a fuzzy graph. Triple connected domination number introduced by G. Mahadevan, Selvam [3]. The purpose of the present paper is to initiate the new idea of a strong and weak triple connected total perfect dominating set and number of a fuzzy graph. Also, we prove some results with example diagrammatically.

2. Preliminaries

The fuzzy set of a base set or reference set V is specified by its function of membership σ, where $\sigma : V \to [0,1]$ assigning to each $u \in V$ the degree or grade to which u belongs to σ. There are two fuzzy sets σ and τ of a set V, then the set σ is called a fuzzy subset of τ, if $\sigma(u) \le \tau(u)$ each $u \in V$. A graph $G = (\sigma, \mu)$ is called a fuzzy graph, if there exist a set of functions of membership $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that $\mu(u,v) \le \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$. If $\tau(u) \le \sigma(u)$where $u \in V$ and $\rho(u,v) \le \mu(u,v)$ for every $u, v \in V$, then $H = (\tau, \rho)$ is called a fuzzy sub graph of G. If $\tau(u) = \sigma(u)$ where $u \in V$ and $\rho(u,v) \le \mu(u,v)$ each $u, v \in V$, then H is called a fuzzy spanning sub graph of G. Orderp $= \sum_{u \in V} \sigma(u)$ and size $q = \sum_{(u,v) \in E} \mu(u,v)$. If $\mu^{\infty}(u,v) \le \mu(u,v)$ for every $u, v \in V$, then arc (u,v) is called a strong arc. Where $\mu^{\infty}(u,v)$ be the strongest path strength and the vertex u is said to be a strong neighbor to v, otherwise it is called weak

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arc. The vertex u is called a isolated in G if $\mu(u,v) = 0$ every $v \neq u, v \in V$. $d_N(v) = \sum_{u \epsilon N_s(v)} \sigma(u)$, $\delta_N(G) = \min\{d_N(u): u \in V(G)\}$ and $\Delta_N(G) = \max\{d_N(u): u \in V(G)\}$. If $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for every $u, v \in V$, then the graph G is called a complete fuzzy graph. It is described by K_σ. There is a bipartition V_1 and V_2 of G. Every vertex in V_1 has a strong neighbor in V_2 and also V_2 has a strong neighbor in V_1, then the bipartition (V_1, V_2) is called a complete bipartite fuzzy graph of G. It is identified by $K_{m,n}$. If (u,v) be a strong arc, then the node u dominates the node v for every node $u, v \in V$ of G. If for every node v not in a subset P of Vwhich dominated by absolutely a node of P, then P is said to be a perfect dominating set of G. It is identified by P_D. If there is a subgraph P_C of G which is connected and induced by $P_D$of G, then P_C is said to be connected P_D. If for each node of G be dominates to at lest a node of $P_t$of G, then P_t is said to be a total P_D of G. There is a sub graph P_D of G induced by total P_D which is connected, then the total P_D is called a connected total P_D of. It is identified by $ctp(G)$. A ctp of G is called a minimal $ctp(G)$ if for all node in, $ctp - \{v\}$ is not $ctp(G)$. $\gamma_{ctp}(G) = \min\{ctp(G)\}$ and $\Gamma_{ctp}(G) = \max\{ctp(G)\}$. If there are three nodes connected and lying on a path$T_C$of G, then $T_C(G)$ called triple connected fuzzy graph. A $ctp(G)$ is called a triple connected total P_D of G if the induced sub graph $< ctp(G) >$ is triple connected. It is identified by $Tctp(G)$.If there exists $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ and $d_N(u) \geq d_N(v)$ for all $u, v \in V$ then u is strongly dominated by vof G. If there is a set $S \in V - D$ which is strongly dominated by at least one vertex in D then the fuzzy sub set D of V is called a strong dominating set(S_D) of G. The fuzzy cardinality which is minimum of S_D is said to be a strong domination number(γ_S) of G. If there exists $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ and $d_N(u) \leq d_N(v)$ for all $u, v \in V$ then u is weakly dominated by vof G. If there is a set $W \in V - D$ which is weakly dominated by at least one vertex in D then the fuzzy sub set D of V is called a weak dominating set(W_D)of G. The fuzzy cardinality which is minimum of W_D is said to be a weak domination number(γ_W)of G. There is a fuzzy subset F of V of a nontrivial G is defined to be Strong(Weak) triple connected perfect dominating set if there is strong(Weak) perfect dominating set which induced fuzzy sub graph is triple connected.

3. Main Result

Strong and Weak Tctp Domination of Fuzzy Graph

In the present section, we initiate the new idea of the strong triple connected total P_D which is($\dot{S}$) of G and define the concept of a minimal strong triple connected total perfect dominating set as well as introducing a strong triple connected total perfect dominating number($\gamma_{\dot{S}}$). Also, this idea is discussed for $\dot{W}$ dominating set.

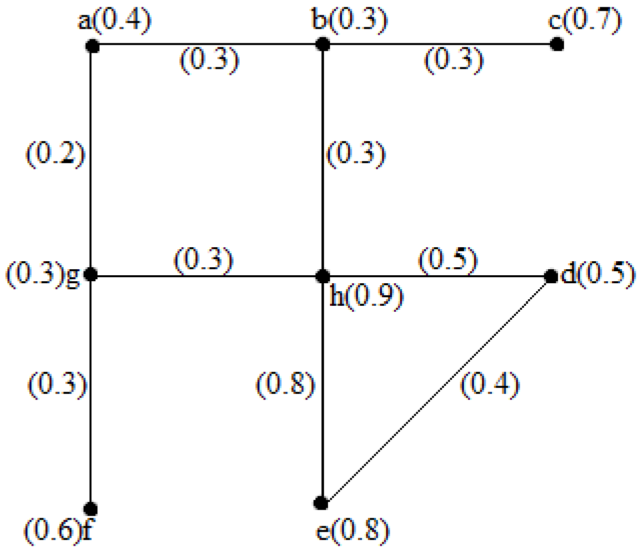
Definition 3.1

If there is Tctp and the fuzzy subgraph induced by <Tctp> which is strongly triple connected then the sub graph of G is called $\dot{S}$ dominating set. There exist a minimum fuzzy cardinality taken from all the $\dot{S}$ which is said to be a $\dot{S}$ domination number ($\gamma_{\dot{S}}$).

Definition 3.2

If there is Tctp and the fuzzy sub graph induced by <Tctp> which is weakly triple connected then the sub graph of G is called $\dot{W}$ dominating set. There exists a minimum fuzzy cardinality taken from all the $\dot{W}$ which is said to be a $\dot{W}$ domination number($\gamma_{\dot{W}}$).

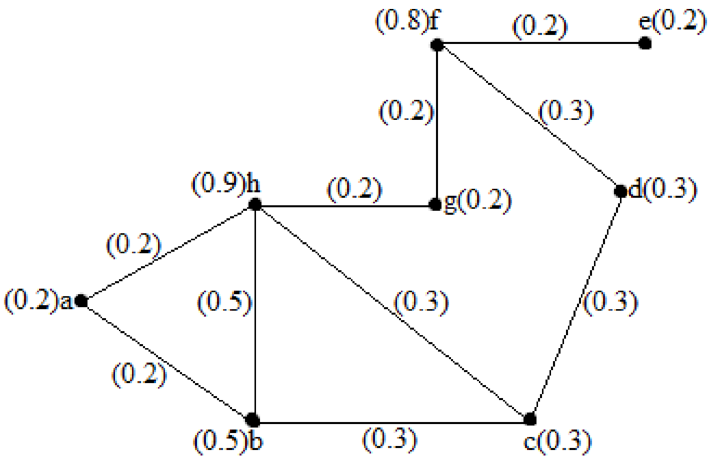
Example 3.3



Fuzzy Graph G

Here$d_N(a) = 0.3, d_N(b) = \{0.4 + 0.9 + 0.7\} = 2, d_N(c) = 0.3, d_N(d) = 0.9, d_N(e) = 0.9, d_N(f) = 0.3, d_N(g) = \{0.9 + 0.6\} = 1.5, d_N(h) = \{0.3 + 0.5 + 0.8 + 0.3\} = 1.9.$
Hence $\dot{S} = \{b, h, g\}$ and $\gamma_{\dot{S}} = \{0.3 + 0.9 + 0.3\} = 1.5.$

Example 3.4



Fuzzy Graph G

Here $d_N(a) = \{0.9 + 0.5\} = 1.4, d_N(b) = \{0.2 + 0.9 + 0.3\} = 1.4, d_N(c) = \{0.5 + 0.3 + 0.9\} = 1.7, d_N(d) = \{0.3 + 0.8\} = 1.1, d_N(e) = 0.8, d_N(f) = \{0.2 + 0.3 + 0.2\} = 0.7,\ d_N(g) = \{0.8 + 0.9\} = 1.7, d_N(h) = \{0.2 + 0.5 + 0.3 + 0.2\} = 1.2.$
Hence $\dot{W} = \{f, g, h\}$ and $\gamma_{\dot{W}} = \{0.8 + 0.2 + 0.9\} = 1.9.$

Theorem 3.5

Let $G = (\sigma, \mu)$ be a fuzzy graph then $\dot{S}$ and $\dot{W}$ of G does not exist for all fuzzy graph.

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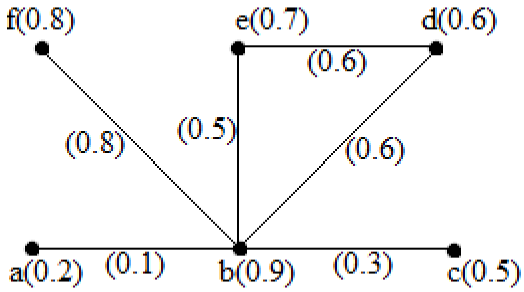
Proof

Now we consider the fuzzy set F which is a total P_D of Gin the induced subgraph $< ctp(G) >$ is not triple connected. So F does not satisfy that If there exists$\mu(u,v) = \sigma(u) \wedge \sigma(v)$,$d_N(u) \geq d_N(v)$ and $d_N(u) \leq d_N(v)$ for all $u, v \in V$ of G. There fore F is not a $\dot{S}$ and $\dot{W}$ of G. Hence $\dot{S}$ and $\dot{W}$ of G does not exist for all fuzzy graphs.

Example 3.6



Fuzzy Graph G

Here the total $P_D = \{b, d\} = F$ which is a connected total $P_D$of G, but F is not Tctp dominating set and also not a strong and weak Tctp of G.

Observation 3.6

Let G be a fuzzy graph and if there are a $\dot{S}$ and $\dot{W}$ of G then it is a $P_D$of G.

Observation 3.7

There is a $\gamma_{\dot{S}\,\dot{W}}(G)$, then $\gamma_{\dot{S}\,\dot{W}}(G) \leq p - 1$.

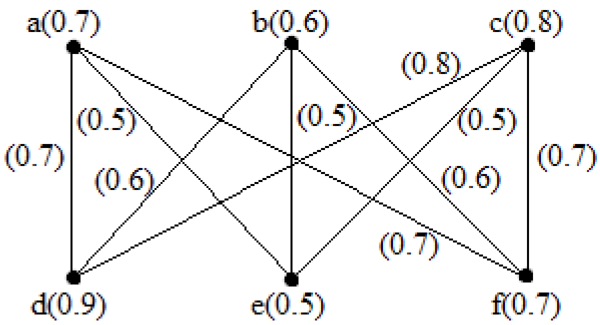
Observation 3.8

There is no $\dot{S}$ and $\dot{W}(G)$ if G is a K_σ.

Observation 3.9

There is no $\dot{S}$ and $\dot{W}(G)$ if G is a $K_{m,n}$.

Example 3.10



Complete bipartite fuzzy graph *G*

Here $F = \{a, e\}$ is a connected total P_D but not aTctp(G) also not $\dot{S}$ and $\dot{W}$ of G.

Theorem 3.11

If there exist $\dot{S}$ and $\dot{W}$ of the fuzzy graph G then it is a total P_D of G.

Proof

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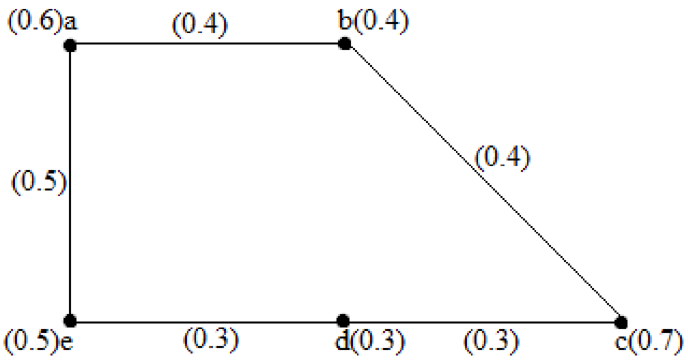
Consider the fuzzy subset F of V which contained G induced triple connected subgraph that is $\dot{S}$ and $\dot{W}$ dominating set of G. Here the set of all nodes of F is connected which satisfies all nodes of $V - F$ dominates exactly one node of F and exists Tctp of G. Therefore the fuzzy subset F of G is total P_D.

Example 3.12



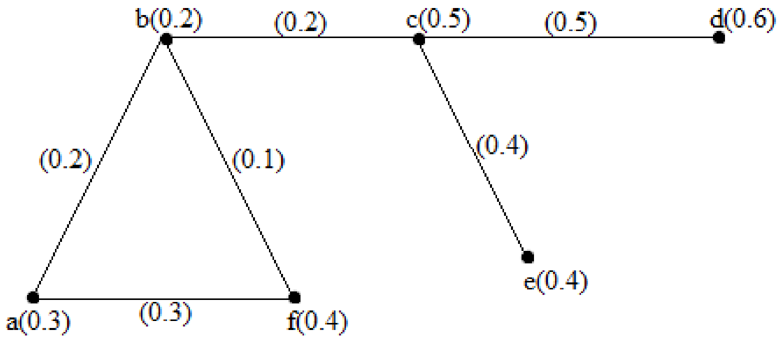
Fuzzy Graph G

Here $d_N(a) = \{0.4 + 0.5\} = 0.9, d_N(b) = \{0.7 + 0.6\} = 1.3, d_N(c) = \{0.4 + 0.3\} = 0.7$, $d_N(d) = \{0.7 + 0.5\} = 1.2$, $d_N(e) = \{0.3 + 0.6\} = 0.9$. $\dot{S}(G) = \{b, c, d\}$,$\gamma_{\dot{S}}(G) = \{0.4 + 0.7 + 0.3\} = 1.4$ and $\dot{W}(G) = \{a, b, c\}$, $\gamma_{\dot{W}}(G) = \{0.6 + 0.4 + 0.7\} = 1.7$.

Observation 3.13

There are two fuzzy set $\dot{S}(G)$ and $\dot{W}(G)$ of fuzzy graph G then Tctp exist in both sets.

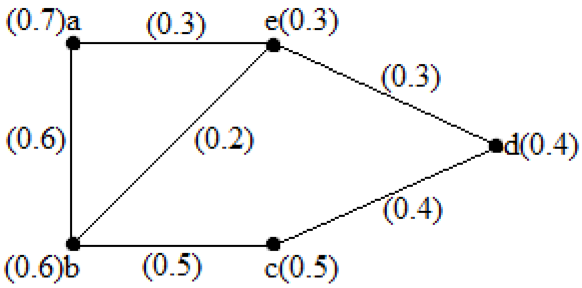
Example 3.14



Fuzzy Graph G

Here $d_N(a) = \{0.4 + 0.2\} = 0.6, d_N(b) = \{0.3 + 0.5\} = 0.8, d_N(c) = \{0.2 + 0.4 + 0.6\} = 1.2$,
$d_N(d) = 0.5, d_N(e) = 0.5, d_N(f) = 0.3$. $\dot{S}(G) = \{a, b, c\}$ which is also a $Tctp(G)$ and $\gamma_{\dot{S}}(G) = \{0.3 + 0.2 + 0.5\} = 1$.

Example 3.15



Fuzzy Graph G

$d_N(a) = 0.9, d_N(b) = 1.2, d_N(c) = 1, d_N(d) = 0.8, d_N(e) = 1.1.$ $\grave{W}(G) = \{a, e, d\}$ is also a $Tctp(G)$. $\gamma_{\grave{W}}(G) = \{0.7 + 0.3 + 0.4\} = 1.4$.

Remark

Converse of the observation 3.6, 3.13 and theorem3.11 need not be true.

Theorem 3.16

If there exist a minimum fuzzy cardinality of Tctp(G) in $\dot{S}$ and $\grave{W}(G)$ then $\gamma_{\dot{S}\,\grave{W}}(G) = \gamma_{Tctp}(G)$.

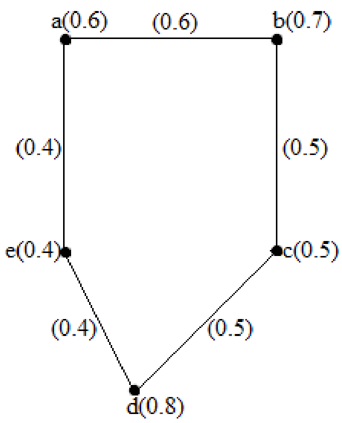
Proof

Let we take $G = (\sigma, \mu)$ be a fuzzy graph and consider the fuzzy set which is Tctpof $\dot{S}(G)$. it satisfies the condition $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ and $d_N(u) \geq d_N(v)$. Also Tctp is strongly triple connected. Therefore the nodes of $\dot{S}(G)$ and $Tctp(G)$ are same. Similarly for $\grave{W}(G)$. Hence $\gamma_{\dot{S}\,\grave{W}}(G) = \gamma_{Tctp}(G)$.

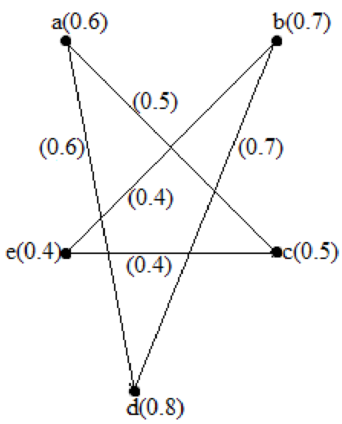
Observation 3.17

The complement of $\dot{S}$ and $\grave{W}(G)$ need not be a $\dot{S}$ and $\grave{W}(G)$.

Example 3.18



Fuzzy Graph G



Complement of Fuzzy Graph G

Now $Tctp(G) = \{e, a, b\}, \{a, b, c\}, \{b, c, d\}$and $\{c, d, e\}$ are triple connected, but complement of $Tctp(G)$ is not a triple connected.

4. Conclusion

We initiated the idea of strong triple connected total perfect ($\dot{S}$) dominating set, weak triple connected total perfect dominating set ($\grave{W}$) of fuzzy graph. The new kinds of

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observations are initiated for Ṡ and Ẇ dominating set of fuzzy graph. We conclude that the Ṡ and Ẇ dominating number are obtained and discussed some results with example.

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