

Reduction of Pilot Contamination with Maximum Likelihood Estimation for Massive MIMO Systems

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Abstract

To estimate the channel in a time division duplex (TDD) mode of a large-scale multi cell system in the uplink, a maximum likelihood estimator (MLE) is proposed for reduction of pilot contamination problem and compared with the traditional estimators; least squares (LS) and minimum mean square error (MMSE). An i.i.d Rayleigh channel model is considered for the fast fading channels. Simulations have been performed in MATLAB with Monte-Carlo Simulations to observe that the proposed estimator MLE performs better than the traditional channel estimators for complex systems.

Keywords: Pilot Contamination, Maximum Likelihood Estimation, Massive MIMO, Channel Estimation

1. Introduction

Wireless technology has become the primary enabler of progressive and pervasive network access over the years. With the upsurge in technology, various gadgets are being enhanced each day with wireless capabilities to fulfill the needs and practices of the world's population with major challenges in data throughput, latency, and coverage. To meet the requirements of users, one way is to increase the amount of base stations and densify the network. However, this will increase interference and deployment costs. Another option is to increase the number of antennas and introduce large-scale antennas which will decrease deployment cost and will achieve high performance. Over the past several years large scale MIMO has come forth as being among the core innovations and key aspects of all developed and enhanced cellular wireless systems [1-7].

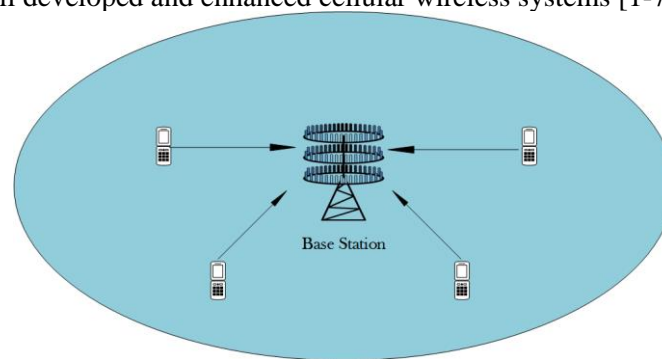


Figure 1 Massive MIMO Uplink System

In the modern communication system, the massive MIMO antenna technology has become a very important and effective innovation. Multiple input single output (MISO) technology and single input multiple output (SIMO) technology were used with a single antenna at transmitter and receiver but it caused problems with multipath effects [8]. While these traditional technologies are having problems in providing high data

throughputs, massive MIMO with large-scale antennas provide 10s of Gbps data rates for real-time multimedia wireless services. Massive MIMO attained much attention as it provides truly broadband wireless networks [9]. The time division duplex (TDD) feature is generally used in large scale MIMO systems. BS receives orthogonal pilot signals from respective users in the uplink to estimate the channel [10].

Large scale antennas gained substantial attention as it is one of the 5th Generation physical layer candidates. Due to these large scale antennas at the base station, higher multiplexing and diversity gains can be gained for uplink and downlink training. The usage of large scale antennas provides several other advantages too like a better signal to noise ratio, coverage, capacity, reliability, and energy-efficient system [11]. Hence, the influence of noise and fast fading is tremendously decreased. The BS needs to know the responses of the channel estimated in the uplink training by transmitting noise and inter-cell interference contaminated pilot signals. In pilot patterns, several adjacent cells can use the same pilot sequence and may overlap due to extensive network deployment and limitation of channel coherence interval. This overlapping of pilot positions and inter-cell interference will have a direct impact on channel estimation performance, generating pilot contamination [12].

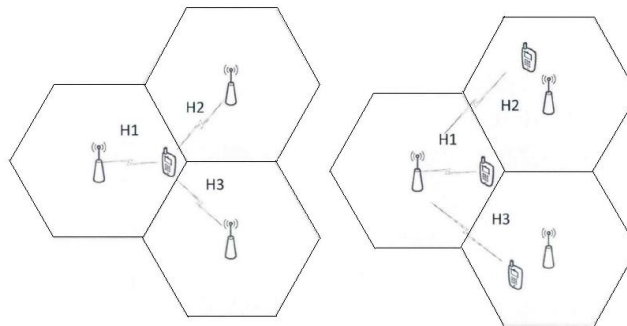


Figure 2 Pilot Contamination Scenario

The augmentation of massive MIMO encounters many problems and pilot contamination is among some of the biggest challenges faced by massive MIMO. It is a key factor to limit the efficiency and potential capacity gain of massive MIMO due to deteriorated channel estimation and restricted orthogonal pilots at limited cohesive intervals and bandwidth [13]. Various schemes have been developed to alleviate this pilot contamination challenge. Among different existing schemes, pilot assignment, precoding process, and channel estimation are key aspects. Most popular among these are channel estimation methods. Channel estimation is a very crucial component of cellular transmission since the precision of estimation can influence the efficiency of data transmission.

Channel estimation plays a vital role in massive MIMO in reducing pilot contamination and increasing system efficiency. Training sequence algorithms, blind channel estimation algorithms, and semi-blind estimation algorithms are some of the categories used for channel estimation. Training sequence-based linear algorithms perform much better than latter estimation techniques. Non-linear algorithm eigenvalue-decomposition based approach can also help to reduce pilot contamination [14]. However, utilizing practical training based channel estimator for multi cell massive MIMO systems will result in very high throughput with less pilot contamination. In TDD systems, the general methods for evaluation are uplink and downlink calculation [15-20]. However, due to pilot contamination and hardware, the downlink measurement is limited by channel reciprocity [21].

In this article, we consider the challenges of pilot pollution in Rayleigh fading channels with uplink training and consider its effect on our system. We present and

analyze an effective and realistic channel estimator Maximum Likelihood which does not require prior information of large scale noise power, inter and intra cell interference. We demonstrate that this estimator is unbiased and attains Cramér–Rao bound. Being classic estimators, both LS estimator and ML estimator is equal in a linear model where we assume an unknown channel coefficient in additive white Gaussian noise (AWGN). Simulation result demonstrates that MLE works better than LS and MMSE when done with Monte-Carlo Simulations in calculating both bit and mean square error rates. The modulation of the data sequence is performed in QAM and QPSK scheme at the transmitter for multi-cell massive MIMO TDD network.

The rest of this article is split into four sections: in the first section, we demonstrate the system model for multi cell massive MIMO system, second, we present the popular estimation techniques LS and MMSE for estimating channel. In the third section, we propose a maximum likelihood estimator for Rayleigh fading channels. The fourth section displays simulation results to prove the efficiency of the proposed channel estimator and finally, we will present the conclusion of this article.

2. System Model

We suppose a massive MIMO multi cell uplink network from single antenna terminals M_t to one base station in each cell with M_r transceiver antennas. The system includes cell 1 as a desired cell. In the desired cell, the generated signal consists of the required symbols from L interfering cells. In the present scenario of massive MIMO configuration, $M_r \gg M_t$ is satisfied.

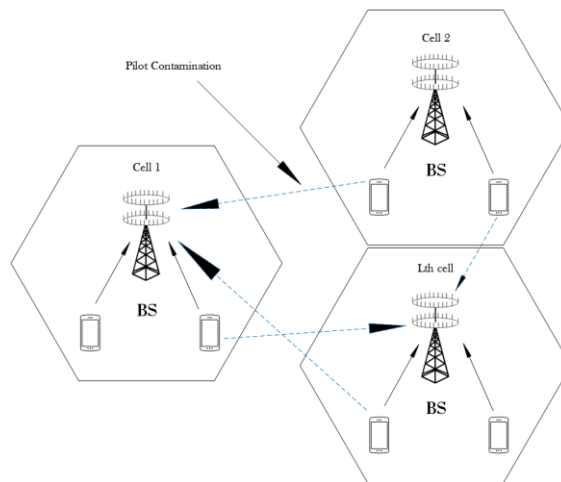


Figure 3 System Model of Multi cell Uplink Massive MIMO

2.1. Signal Model

The pilot signal is given at the required BS as

$$Y = JX + \sum_{l=1}^L J_l X_l + N \quad (1)$$

Where $Y \in C^{M_r \times 1}$, $J \in C^{M_r \times 1}$, $J_l \in C^{M_r \times 1}$, represent received vector, transmitted vector of required cell and the l^{th} interfering cell accordingly and J_l represents the corresponding cell. $N \in C^{M_r \times 1}$ is the AWGN vector.

$$W = \sum_{l=1}^L J_l X_l + N \quad (2)$$

By considering all these vectors as independent and identical complex random variables and noise and interference as a unique matrix, we can write the above equation (1) as

$$Y = JX + W \quad (3)$$

2.2. Uplink Training

The multi cell system contains L cells and every cell in the system is arranged with a BS with M_r antennas and M_t randomly located single-antenna users. We can represent that the received pilot signal in l^{th} cell as

$$Y_l = (J_1 X_1 + J_2 X_2 + \dots + N) \quad (4)$$

It contains pilots from other users which are present in other cells. Thus, pilot contamination cannot be avoided as the same pilots are being used in different cells. For estimation in the uplink, the l^{th} BS will receive the uplink training sequence as

$$Y_u = \sqrt{q} \sum_{l=1}^L J_u X_u^H + W_u \quad (5)$$

Where \sqrt{q} represents uplink power, X_u is the pilot transmission matrix, Y_u is received signal matrix and W_u represents the AWGN matrix with I.I.D entities $CN(0, \sigma^2)$.

2.3. Least Square Channel Estimation:

A conventional LS estimation algorithm is considered for estimation in uplink training. The receiver of the required cell first obtains α pilot symbols and then utilizes the α signals in pilot positions to generate a receiving signal matrix $y \in C^{M_r \times \alpha}$.

$$\bar{Y} = J \bar{X} + \bar{W} \quad (6)$$

Where $X \in C^{M_t \times \alpha}$, $W \in C^{M_r \times \alpha}$ and the correlative pseudo-inverse matrix is defined as $X \in C^{M_t \times \alpha}$.

The received output of LS channel estimation is

$$\hat{J} = \overline{y} \overline{x}^\dagger = \overline{y} \overline{x}^H (\overline{x} \overline{x}^H)^{-1} \quad (7)$$

Taking Hermitian on both sides

$$\overline{y}^H = \overline{x}^H J^H + \overline{W}^H \quad (8)$$

$$\| \overline{y}^H - \overline{x}^H J^H \|^2 \quad (9)$$

Again taking Hermitian on both sides

$$\hat{J}_{LS} = (\hat{J})^H = \overline{y} \overline{x}^H (\overline{x} \overline{x}^H)^{-1} \quad (10)$$

Mean Square Error of LS can be given as

$$MSE_{LS} = E\{(J - \hat{J}_{LS})^H (\hat{J}_{LS})\} \quad (11)$$

By putting the value of \hat{H}_{LS} in equation (11), we get

$$MSE_{LS} = E\{(J - X^{-1}Y)^H (J - X^{-1}Y)\} \quad (12)$$

Now, inserting the value of Y from equation (3) in equation (12)

$$MSE_{LS} = E\{(J - X^{-1}(XJ + W))^H (J - X^{-1}(XJ + W))\} \quad (13)$$

By solving equation (13), we get

$$MSE_{LS} = E\{(X^{-1}W)^H (X^{-1}W)\} \quad (14)$$

$$MSE_{LS} = \frac{\sigma_w^2}{\sigma_x^2} \quad (15)$$

2.4. Minimum Mean Square Channel Estimation:

Bayesian channel estimator MMSE is a more realistic approach than LS channel estimator to reduce noise enhancement. LS approach is simple in computation but produces lesser MSE than MMSE. The MMSE estimator requires information of statistical details of parameters to be estimated. To find the MSE of MMSE, we will use the general equation (6)

$$\bar{Y} = J\bar{X} + \bar{W}$$

Where \bar{X} is the pilot vector, \bar{Y} is the observation vector, \bar{W} is noise vector and J is a complex channel coefficient that means it is complex Gaussian. By obtaining a strong linear estimate in form of weight vector M and LS value, we will define $\hat{J} \cong M\bar{J}$, the MMSE estimator reduces the mean square error (MSE) for both the true channel J and the MMSE estimated channel \hat{J}^{MMSE} . The MMSE channel estimate for this scenario is given as:

$$Z(\hat{J}) = E\{\|e\|^2\} \quad (16)$$

$$Z(\hat{J}) = E\{\|e\|^2\} \quad (17)$$

The estimation vector $v = J - \hat{J}$ is orthogonal to \hat{J} as per the principle of orthogonality

$$E\{v\hat{J}^H\} = E\{(J - \hat{J})\hat{J}^H\} \quad (18)$$

After inserting the value of \hat{J} in equation (18), we get

$$E\{v\hat{J}^H\} = E\{(J - M\hat{J})\hat{J}^H\} \quad (19)$$

$$E\{v\hat{J}^H\} = E\{\hat{J}\hat{J}^H\} - ME\{\hat{J}\hat{J}^H\} \quad (20)$$

$$E\{v\hat{J}^H\} = 0 \quad (21)$$

As J is a symmetric complex Gaussian parameter, it will have two parts; real and imaginary with mean μ_j and variance σ_j^2 . MMSE channel estimate of both real and imaginary parts will always be equal in the complex parameters scenario.

The MSE of MMSE is given as:

$$MSE_{MMSE} = R_{JJ}^{-1} (R_{JJ} + \frac{\sigma_w^2}{\sigma_x^2} I)^{-1} J \quad (22)$$

2.5. Maximum Likelihood Channel Estimation:

To perform parameter estimations by ML, we will first form a log-likelihood function in terms of the parameter to be estimated. Applying the ML method for a complex parameter, we find the following result

$$\hat{j} = \frac{\overline{x}^H y}{\overline{x}^H x} \quad (23)$$

$$\hat{j} = \frac{\overline{x}^H y}{\|x\|^2} \quad (24)$$

This estimator has $E\{\hat{j}\} = j$ which describes that ML estimator is unbiased and variance of ML estimator can be given as

$$\text{var}(j) = \frac{\sigma^2}{\|x\|^2} \quad (25)$$

More efficiency of the estimator can be achieved by deriving Cramer-Rao lower bound as

$$\text{var}(j) \geq \frac{\sigma^2}{\|x\|^2} \quad (26)$$

In general, for a complex parameter, the MSE of ML Channel Estimate can be given as

$$MSE_{MLE} = \frac{\sigma^2}{\|x\|^2} + (\frac{\overline{x}^H y}{\|x\|^2})^2 \quad (27)$$

Where $\frac{\sigma^2}{\|x\|^2}$ is variance and $(\frac{\overline{x}^H y}{\|x\|^2})^2$ is the squared bias relationship of the estimator. As

MLE is an unbiased estimator so in this case, the MSE of MLE will be equivalent to its variance and the squared bias.

3. Simulation Results

To obtain simulation results for the decontamination of pilot contamination through channel estimation techniques, we used MATLAB for massive MIMO TDD systems. BER and MSE performances over SNR has been achieved for LS, MMSE and ML estimation methods. We assume a simple but realistic approach to prove the efficiency of our proposed estimator with Rayleigh fading model. We simulated our

results with 4 antenna terminals and 100 receiver antennas with 4 number of pilot sequences. SNR value for each simulation result used is 30dB. A simple comparison of BER and MSE has been done between LS and MMSE methods in figure 4 and figure 5.

Table 1 Simulation Parameters

Serial #	Parameters	Values
1	Mr	100
2	Mt	4
3	Monte-Carlo Simulations	10e2
4	Num_pilot	4
5	Constellation	QAM,QPSK
6	SNR	30dB
7	Fading Model	Rayleigh

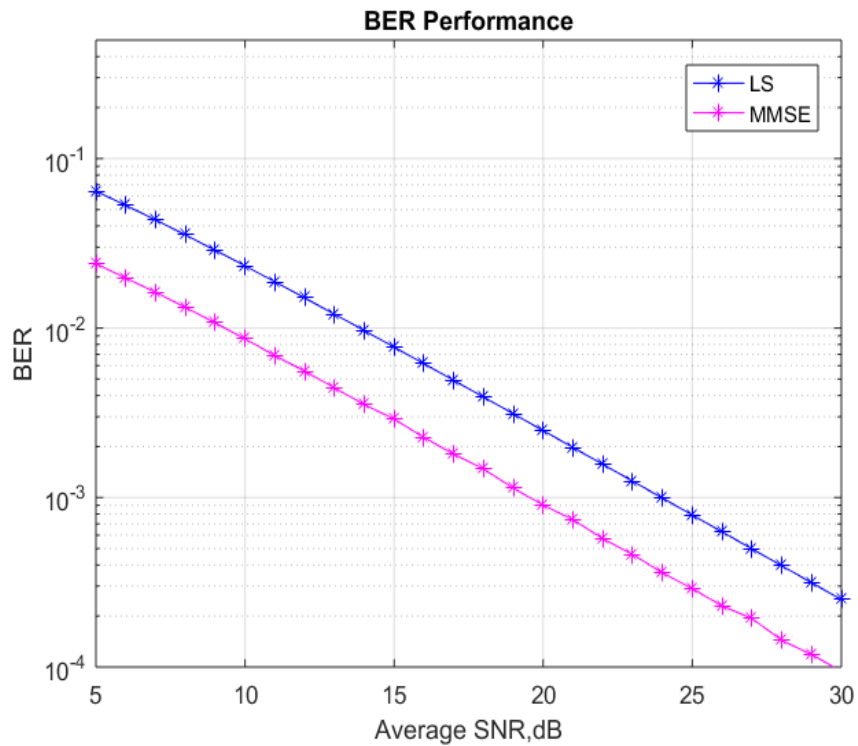


Figure 4 BER vs SNR for LS, MMSE

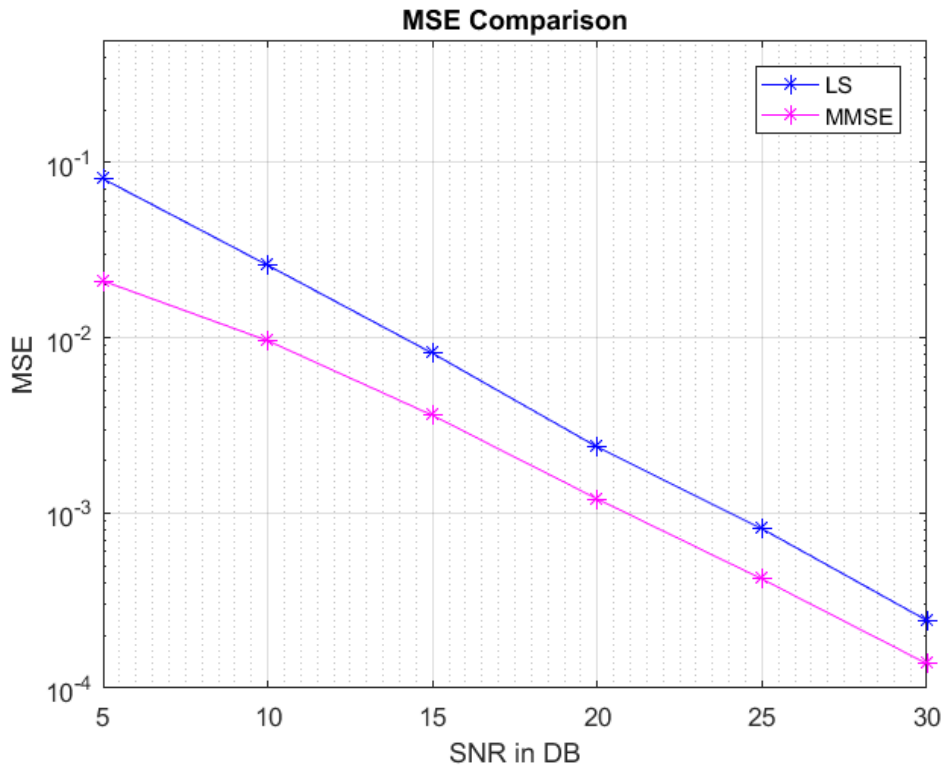


Figure 5 MSE vs SNR for traditional LS, MMSE

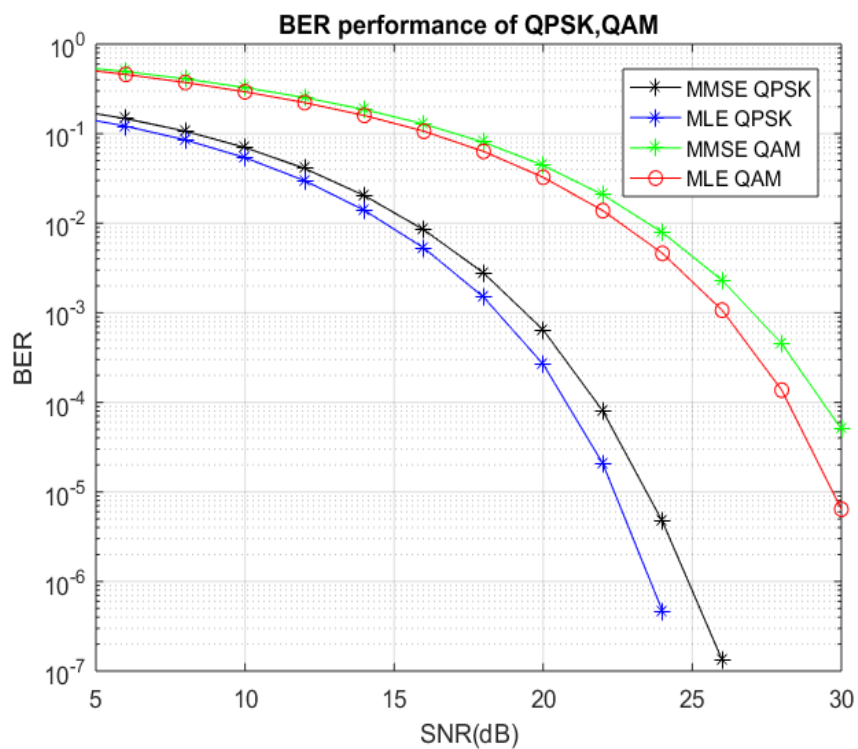


Figure 6 BER vs SNR for QPSK, QAM

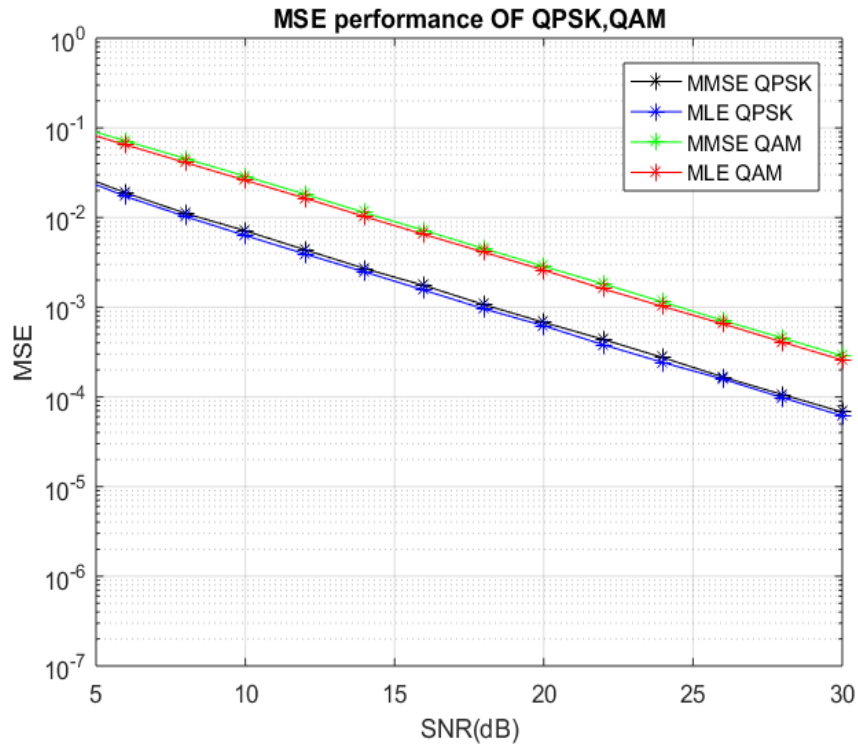


Figure 7 MSE vs SNR for QPSK, QAM

BER and MSE performance for Bayesian estimator MMSE and classic estimator MLE has been done with both QAM and QPSK modulation schemes in figure 6 and figure 7 and proved that QPSK produces better results than QAM.

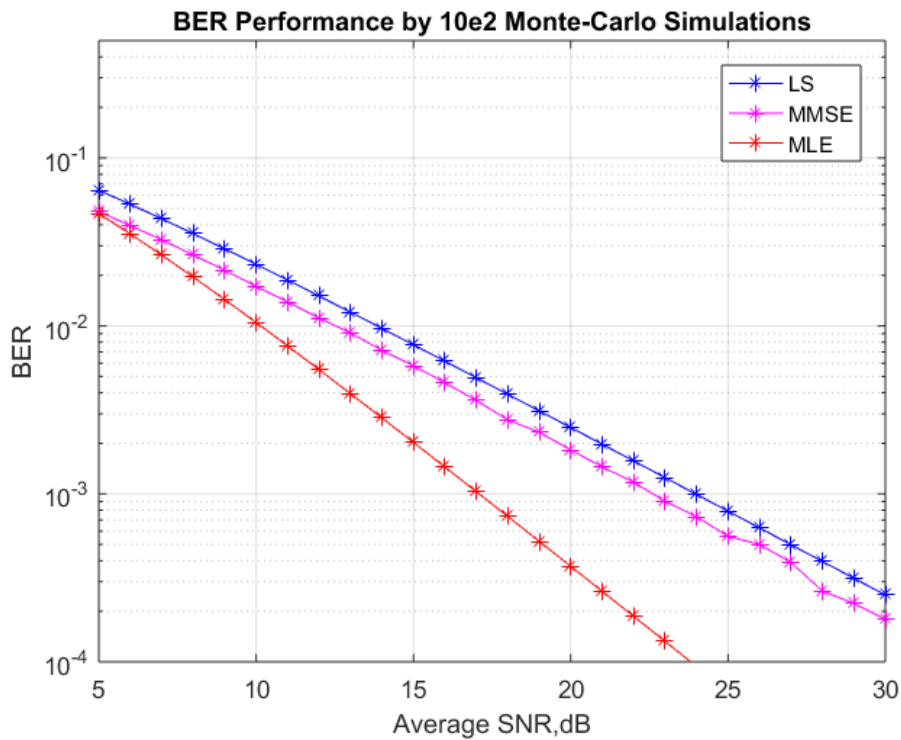


Figure 8 BER vs SNR for LS, MMSE and MLE

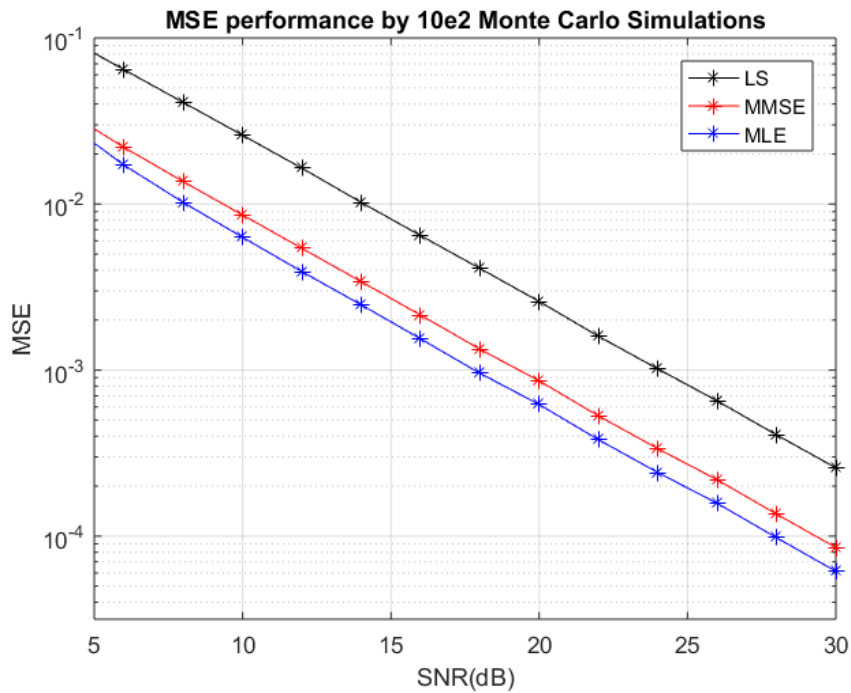


Figure 9 MSE vs SNR by Monte-Carlo Simulations method for LS, MMSE, and MLE

Monte-Carlo simulations method with 10e2 iterations is used to evaluate BER and MSE performance to show significant achievement of the proposed estimator as compared to other estimation techniques in figure 8 and figure 9. As we discussed earlier, LS and MLE are equal in terms of unknown channel coefficient with AWGN. The use of Monte-Carlo Simulations cross-validated the same errors numerically.

4. Conclusions

Pilot Contamination can substantially reduce the functionality of massive MIMO system and a major challenge in 5G massive MIMO networks. Traditional channel estimation methods used to mitigate this challenge cannot achieve very high performance due to the complexity of large number of unknown channel coefficients. However, to alleviate the effect of pilot contamination we implemented ML Estimate to ease the pilot multiplexing and to reduce this effect of pilot contamination. Improved results by simulation work proved that ML Estimation done with Monte-Carlo Simulations provides better results than conventional LS and MMSE estimation.

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