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Subclasses of BI-Univalent Functions Associated with the Multiplier Transformation

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Abstract

In this paper, we introduced two new subclasses of analytic bi-univalent functions by using multiplier transformation in the unit disc $U = \{z \in C : |z| < 1\}$. *The coefficient bounds of* $|a_2|$ and $|a_3|$ *for functions in these two new subclasses are obtained.*

Keywords— *Bi-univalent, Salagean operator, Multiplier transformation, Analytic functions.*

I. Introduction

Let A denote the class of analytic functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

defined in the unit disc U . The class of functions belongs to A and univalent in U is denoted by S. We know, for every $f(z)$ belongs to S has an inverse $f^{-1}(z)$ exist. Inverse function defined as

$$f^{-1}(f(z)) = z,\ z \in U \quad \text{and} \quad f(f^{-1}(w)) = w,\ w \in U \ \left(|w| < r_0(f),\ r_0(f) \geq \frac{1}{4}\right),$$

where
$$f^{-1}(w) = g(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots \tag{1.2}$$

If f^{-1} and f are univalent in U then $f \in A$ is called bi-univalent in U . Symbol Σ denote the class of bi-univalent functions in U . In 1967, Lewin [7] was investigated the class of bi-univalent functions and showed that $|a_2| < 1.51$. Afterward, Brannan and Clunie [1] conjectured that $|a_2| \leq \sqrt{2}$. Brannan and Taha [2] defined the certain subclasses of bi-univalent functions as follows:

A function $f(z)$ of the form (1.1) said to be in the class $S_{\Sigma}^{*}(\alpha)(0 < \alpha \leq 1)$ if

$$f \in \Sigma,\ \left|\arg\left(\frac{zf'(z)}{f(z)}\right)\right| < \frac{\alpha\pi}{2} \quad (z \in U) \quad \text{and} \quad \left|\arg\left(\frac{wg'(w)}{g(w)}\right)\right| < \frac{\alpha\pi}{2} \quad (w \in U),$$

where the function g given by (1.2). Denote $S_{\Sigma}^{*}(\alpha)$ the class of strongly bi-starlike functions of order $\alpha(0 < \alpha \leq 1)$. Similarly, a function $f(z)$ of the form (1.1) said to be in the class $K_{\Sigma}(\alpha)(0 < \alpha \leq 1)$ if

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$$f\in\Sigma,\ \left|\arg\left(1+\frac{zf''(z)}{f'(z)}\right)\right|<\frac{\alpha\pi}{2}\ (z\in U)\quad\text{and}\quad\left|\arg\left(1+\frac{wg''(w)}{g'(w)}\right)\right|<\frac{\alpha\pi}{2}\ (w\in U),$$

where the function g given by (1.2). Denote $K_\Sigma(\alpha)$ the class of strongly bi-convex functions of order $\alpha(0<\alpha\le 1)$. Now, A function $f(z)$ of the form (1.1) said to be in the class $S_\Sigma^*(\beta)(0\le\beta<1)$ if

$$f\in\Sigma,\ \Re\left(\frac{zf'(z)}{f(z)}\right)>\beta\ \ (z\in U)\quad\text{and}\quad\Re\left(\frac{wg'(w)}{g(w)}\right)>\beta\ \ (w\in U),$$

where the function g given by (1.2). Denote $S_\Sigma^*(\beta)$ the class of strongly bi-starlike functions of order $\beta(0\le\beta<1)$. Similarly, define $K_\Sigma(\beta)$ the class of bi-convex functions of order $\beta(0\le\beta<1)$. Recently, bounds of various subclasses of bi-univalent functions have been investigated by several authors (see [3], [4], [8]-[10], [13]).

Cho and Strivastava [5] introduced the operator $I_\gamma^k : A\to A$ defined as

$$I_\gamma^k f(z)=z+\sum_{n=2}^{\infty}\left(\frac{n+\gamma}{1+\gamma}\right)^k a_n z^n,\ \gamma\ge 0, k\in ¥_0=¥\cup\{0\}. \qquad (1.3)$$

For $\gamma=0$ the operator I_γ^k reduced to the Salagean operator introduced by Salagean [12].

In 2015, J. Jothibasu [6] defines the subclass $S_\Sigma^{k,\lambda}(\alpha)$ consisting of analytic functions $f(z)$ of the form (1.1) and $f(z)$ satisfies the following conditions:

$$f\in\Sigma,\left|\arg\left(\frac{D^{k+1}f(z)}{(1-\lambda)D^k f(z)+\lambda D^{k+1}f(z)}\right)\right|<\frac{\alpha\pi}{2}, 0<\alpha\le 1, 0\le\lambda<1, z\in U \qquad (1.4)$$

and

$$\left|\arg\left(\frac{D^{k+1}g(w)}{(1-\lambda)D^k g(w)+\lambda D^{k+1}g(w)}\right)\right|<\frac{\alpha\pi}{2}, 0<\alpha\le 1, 0\le\lambda<1, w\in U, \qquad (1.5)$$

Where the function g of the form (1.2) and D^k is the differential operator introduced by Salagean [12] and defined as

$$D^k f(z)=z+\sum_{n=2}^{\infty}n^k a_n z^n,\ k\in ¥_0=¥\cup\{0\}.$$

Also, J. Jothibasu [6] define the subclass $M_\Sigma^{k,\lambda}(\beta)$ consisting of analytic functions $f(z)$ of the form (1.1) and $f(z)$ satisfies the following conditions:

$$f\in\Sigma,\ \Re\left(\frac{D^{k+1}f(z)}{(1-\lambda)D^k f(z)+\lambda D^{k+1}f(z)}\right)>\beta, 0\le\beta<1, 0\le\lambda<1,\ z\in U \qquad (1.6)$$

and

$$\Re\left(\frac{D^{k+1}g(w)}{(1-\lambda)D^k g(w)+\lambda D^{k+1}g(w)}\right)>\beta, 0\le\beta<1, 0\le\lambda<1,\ w\in U \qquad (1.7)$$

Where the function g of the form (1.2) and D^k is the differential operator introduced by Salagean [12].

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Motivated by this aforementioned work, we introduced two new subclasses of analytic and bi-univalent functions associated with multiplier transformation. Also, obtain the coefficient bounds of $|a_2|$ and $|a_3|$ for functions in these two new subclasses.

Lemma1.1.[11] Let $\wp$ be the family of all analytic functions $h(z)$ of the form $h(z)=1+c_1z+c_2z^2+c_3z^3+\ldots$ and $\Re(h(z))>0$ defined in U. If $h\in\wp$ then $|c_n|\le 2$ for each n.

II. COEFFICIENT BOUNDS FOR THE FUNCTION CLASS $S_{\Sigma}^{k,\lambda,\gamma}(\alpha)$

Definition 2.1. A function $f(z)$ of the form (1.1) is said to be in the class $S_{\Sigma}^{k,\lambda,\gamma}(\alpha)$, $0<\alpha\le 1, 0\le\lambda<1, \gamma\ge 0, k\in\mathbb{N}_0$ if

$$f\in\Sigma,\left|\arg\left(\frac{I_{\gamma}^{k+1}f(z)}{(1-\lambda)I_{\gamma}^{k}f(z)+\lambda I_{\gamma}^{k+1}f(z)}\right)\right|<\frac{\alpha\pi}{2}\quad(z\in U)\tag{2.1}$$

and

$$\left|\arg\left(\frac{I_{\gamma}^{k+1}g(w)}{(1-\lambda)I_{\gamma}^{k}g(w)+\lambda I_{\gamma}^{k+1}g(w)}\right)\right|<\frac{\alpha\pi}{2}\quad(w\in U),\tag{2.2}$$

where g is the function of the form (1.2).

For particular values of k,λ and γ, the class $S_{\Sigma}^{k,\lambda,\gamma}(\alpha)$ reduce to varies subclasses as:

(1) For $\gamma=0$ the class $S_{\Sigma}^{k,\lambda,\gamma}(\alpha)$ reduce to $S_{\Sigma}^{k,\lambda}(\alpha)$, studied by Jothibasu [6], (2) For $\gamma=0,\lambda=0,k=0$ the class $S_{\Sigma}^{k,\lambda,\gamma}(\alpha)$ reduce to $S_{\Sigma}^{*}(\alpha)$, studied by Brannan and Taha[2], (3) For $\gamma=0,\lambda=0,k=1$ the class $S_{\Sigma}^{k,\lambda,\gamma}(\alpha)$ reduce to $K_{\Sigma}(\alpha)$, studied by Brannan and Taha[2], (4) For $\gamma=0,k=0$ the class $S_{\Sigma}^{k,\lambda,\gamma}(\alpha)$ reduce to $G_{\Sigma}(\alpha,\lambda)$, studied by G. Murugusundaramoorthy et.al [10].

Theorem 2.1. Let function $f(z)$ of the form (1.1) be in the class $S_{\Sigma}^{k,\lambda,\gamma}(\alpha), 0<\alpha\le 1, 0\le\lambda<1$ and $\gamma\ge 0$, then

$$|a_2|\le\frac{2\alpha(1+\gamma)^{k+1}}{\sqrt{\left|4\alpha(1+\gamma)^{k+1}(1-\lambda)(3+\gamma)^{k}+\left[2\alpha\left((\lambda^2-1)+(\lambda-1)\gamma\right)-(\alpha-1)(1-\lambda)^2\right](2+\gamma)^{2k}\right|}}\tag{2.3}$$

and

$$|a_3|\le\frac{\alpha(1+\gamma)^{k+1}}{(1-\lambda)(3+\gamma)^{k}}+\frac{4\alpha^2(1+\gamma)^{2k+2}}{(1-\lambda)^2(2+\gamma)^{2k}}.\tag{2.4}$$

Proof. From conditions (2.1) and (2.2), we have

$$\frac{I_{\gamma}^{k+1}f(z)}{(1-\lambda)I_{\gamma}^{k}f(z)+\lambda I_{\gamma}^{k+1}f(z)}=[p(z)]^{\alpha}\tag{2.5}$$

and

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$$\frac{I_{\gamma}^{k+1}g(w)}{(1-\lambda)I_{\gamma}^{k}g(w)+\lambda I_{\gamma}^{k+1}g(w)}=\left[q(w)\right]^{\alpha}. \quad (2.6)$$

Where functions $p(z)$ and $q(w)$ are in $\wp$ and have the forms:

$$p(z)=1+p_1z+p_2z^2+p_3z^3+\ldots, \quad (2.7)$$

$$q(w)=1+q_1w+q_2w^2+q_3w^3+\ldots \quad (2.8)$$

Now, equating the coefficients in (2.5) and (2.6), we get

$$\left(\frac{1-\lambda}{1+\gamma}\right)\left(\frac{2+\gamma}{1+\gamma}\right)^{k}a_2=\alpha p_1, \quad (2.9)$$

$$\left(\frac{(\lambda^2-1)+(\lambda-1)\gamma}{(1+\gamma)^2}\right)\left(\frac{2+\gamma}{1+\gamma}\right)^{2k}a_2^2+\left(\frac{2-2\lambda}{1+\gamma}\right)\left(\frac{3+\gamma}{1+\gamma}\right)^{k}a_3=\frac{1}{2}\left[\alpha(\alpha-1)p_1^2+2\alpha p_2\right], \quad (2.10)$$

$$-\left(\frac{1-\lambda}{1+\gamma}\right)\left(\frac{2+\gamma}{1+\gamma}\right)^{k}a_2=\alpha q_1 \quad (2.11)$$

and

$$\left(\frac{(\lambda^2-1)+(\lambda-1)\gamma}{(1+\gamma)^2}\right)\left(\frac{2+\gamma}{1+\gamma}\right)^{2k}a_2^2+\left(\frac{2-2\lambda}{1+\gamma}\right)\left(\frac{3+\gamma}{1+\gamma}\right)^{k}(2a_2^2-a_3)=\frac{1}{2}\left[\alpha(\alpha-1)q_1^2+2\alpha q_2\right]. \quad (2.12)$$

From (2.9) and (2.11), we get

$$p_1=-q_1 \quad (2.13)$$

and

$$2\left(\frac{1-\lambda}{1+\gamma}\right)^{2}\left(\frac{2+\gamma}{1+\gamma}\right)^{2k}a_2^2=\alpha^2(p_1^2+q_1^2). \quad (2.14)$$

From (2.10),(2.12) and (2.14), we get

$$a_2^2=\frac{(1+\gamma)^{2k+2}\alpha^2(p_2+q_2)}{4\alpha(1+\gamma)^{k+1}(1-\lambda)(3+\gamma)^k+\left[2\alpha\left((\lambda^2-1)+(\lambda-1)\gamma\right)-(\alpha-1)(1-\lambda)^2\right](2+\gamma)^{2k}}.$$

By Lemma 1.1, $|p_n|\le 2$ and $|q_n|\le 2$. Hence

$$|a_2|\le\frac{2\alpha(1+\gamma)^{k+1}}{\sqrt{\left|4\alpha(1+\gamma)^{k+1}(1-\lambda)(3+\gamma)^k+\left[2\alpha\left((\lambda^2-1)+(\lambda-1)\gamma\right)-(\alpha-1)(1-\lambda)^2\right](2+\gamma)^{2k}\right|}}.$$

Now, subtracting equation (2.12) from equation (2.10), we obtain

$$\left(\frac{4-4\lambda}{1+\gamma}\right)\left(\frac{3+\gamma}{1+\gamma}\right)^{k}a_3-\left(\frac{4-4\lambda}{1+\gamma}\right)\left(\frac{3+\gamma}{1+\gamma}\right)^{k}a_2^2=\frac{1}{2}\left[\alpha(\alpha-1)(p_1^2-q_1^2)+2\alpha(p_2-q_2)\right]. \quad (2.15)$$

From (2.13), (2.14) and (2.15), we get

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$$a_3 = \frac{\alpha(p_2 - q_2)(1+\gamma)^{k+1}}{(4-4\lambda)(3+\gamma)^k} + \frac{\alpha^2(p_1^2 + q_1^2)(1+\gamma)^{2k+2}}{2(1-\lambda)^2(2+\gamma)^{2k}}. \quad (2.16)$$

By Lemma 1.1, $|p_n| \le 2$, $|q_n| \le 2$ and apply on (2.16), we obtain

$$|a_3| \le \frac{\alpha(1+\gamma)^{k+1}}{(1-\lambda)(3+\gamma)^k} + \frac{4\alpha^2(1+\gamma)^{2k+2}}{(1-\lambda)^2(2+\gamma)^{2k}}.$$

The proof is completed.

If we take $\gamma = 0$ in Theorem (2.1), then Corollary (2.2) is obtained.

Corollary 2.2.([6]) Let function $f(z)$ of the form (1.1) be in the class $S_\Sigma^{k,\lambda}(\alpha), 0 < \alpha \le 1, 0 \le \lambda < 1$, then

$$|a_2| \le \frac{2\alpha}{\sqrt{\left|4\alpha(1-\lambda)3^k + \left[2\alpha(\lambda^2 - 1) - (\alpha - 1)(1-\lambda)^2\right]2^{2k}\right|}}$$ and

$$|a_3| \le \frac{\alpha}{(1-\lambda)3^k} + \frac{4\alpha^2}{(1-\lambda)^2 2^{2k}}.$$

If we take $\gamma = 0$ and $\lambda = 0$ in Theorem (2.1), then Corollary (2.3) is obtained.

Corollary 2.3 ([6]). Let function $f(z)$ of the form (1.1) be in the class $S_\Sigma^{k}(\alpha)$, then

$$|a_2| \le \frac{2\alpha}{\sqrt{4\alpha 3^k + (1-3\alpha)2^{2k}}} \text{ and } |a_3| \le \frac{\alpha}{3^k} + \frac{4\alpha^2}{2^{2k}}.$$

III. COEFFICIENT BOUNDS FOR THE FUNCTION CLASS $M_\Sigma^{k,\lambda,\gamma}(\beta)$

Definition 3.1. A function $f(z)$ of the form (1.1) is said to be in the class $M_\Sigma^{k,\lambda,\gamma}(\beta),\ 0 \le \beta < 1, 0 \le \lambda < 1, \gamma \ge 0$,if

$$f \in \Sigma,\ \Re\left(\frac{I_\gamma^{k+1} f(z)}{(1-\lambda)I_\gamma^k f(z) + \lambda I_\gamma^{k+1} f(z)}\right) > \beta\ (z \in U), \quad (3.1)$$

and

$$\Re\left(\frac{I_\gamma^{k+1} g(w)}{(1-\lambda)I_\gamma^k g(w) + \lambda I_\gamma^{k+1} g(w)}\right) > \beta\ (w \in U), \quad (3.2)$$

where g is the function of the form (1.2).

For particular values of k, λ and γ, the class $M_\Sigma^{k,\lambda,\gamma}(\beta)$ reduce to varies subclasses as:

(1) For $\gamma = 0$ the class $M_\Sigma^{k,\lambda,\gamma}(\beta)$ reduce to $M_\Sigma^{k,\lambda}(\beta)$, studied by Jothibasu [6], (2) For $\gamma = 0, \lambda = 0, k = 0$ the class $M_\Sigma^{k,\lambda,\gamma}(\beta)$ reduce to $S_\Sigma^*(\beta)$, studied by Brannan and Taha [2], (3) For $\gamma = 0, \lambda = 0, k = 1$ the class $M_\Sigma^{k,\lambda,\gamma}(\beta)$ reduce to $K_\Sigma(\beta)$, studied by Brannan and Taha[2], (4) For $\gamma = 0, k = 0$ the class $M_\Sigma^{k,\lambda,\gamma}(\beta)$ reduce to $M_\Sigma(\beta, \lambda)$, studied by G. Murugusundaramoorthy et.al [10].

Theorem 3.1. Let function $f(z)$ of the form (1.1) be in the class $M_\Sigma^{k,\lambda,\gamma}(\beta),\ 0 \le \beta < 1, 0 \le \lambda < 1, \gamma \ge 0$, then

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$$|a_2| \le \sqrt{\frac{2(1-\beta)(1+\gamma)^{2k+2}}{\left|2(1-\lambda)(1+\gamma)^{k+1}(3+\gamma)^k + \left[(\lambda^2-1)+(\lambda-1)\gamma\right](2+\gamma)^{2k}\right|}} \quad (3.3)$$

and

$$|a_3| \le \frac{4(1-\beta)^2(1+\gamma)^{2k+2}}{(1-\lambda)^2(2+\gamma)^{2k}} + \frac{(1-\beta)(1+\gamma)^{k+1}}{(1-\lambda)(3+\gamma)^k}. \quad (3.4)$$

Proof. From conditions (3.1) and (3.2) that there exist $p, q \in \wp$ such that

$$\frac{I_\gamma^{k+1} f(z)}{(1-\lambda)I_\gamma^k f(z) + \lambda I_\gamma^{k+1} f(z)} = \beta + (1-\beta)p(z) \quad (3.5)$$

and

$$\frac{I_\gamma^{k+1} g(w)}{(1-\lambda)I_\gamma^k g(w) + \lambda I_\gamma^{k+1} g(w)} = \beta + (1-\beta)q(w). \quad (3.6)$$

Where functions $p(z)$ and $q(w)$ belong to $\wp$ and have the forms:

$$p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \cdots, \quad (3.7)$$

$$q(w) = 1 + q_1 w + q_2 w^2 + q_3 w^3 + \cdots \quad (3.8)$$

Now, equating the coefficients in (3.5) and (3.6), we get

$$\left(\frac{1-\lambda}{1+\gamma}\right)\left(\frac{2+\gamma}{1+\gamma}\right)^k a_2 = (1-\beta)p_1, \quad (3.9)$$

$$\left(\frac{(\lambda^2-1)+(\lambda-1)\gamma}{(1+\gamma)^2}\right)\left(\frac{2+\gamma}{1+\gamma}\right)^{2k} a_2^2 + \left(\frac{2-2\lambda}{1+\gamma}\right)\left(\frac{3+\gamma}{1+\gamma}\right)^k a_3 = (1-\beta)p_2, \quad (3.10)$$

$$-\left(\frac{1-\lambda}{1+\gamma}\right)\left(\frac{2+\gamma}{1+\gamma}\right)^k a_2 = (1-\beta)q_1 \quad (3.11)$$

and

$$\left(\frac{(\lambda^2-1)+(\lambda-1)\gamma}{(1+\gamma)^2}\right)\left(\frac{2+\gamma}{1+\gamma}\right)^{2k} a_2^2 + \left(\frac{2-2\lambda}{1+\gamma}\right)\left(\frac{3+\gamma}{1+\gamma}\right)^k (2a_2^2 - a_3) = (1-\beta)q_2. \quad (3.12)$$

From (3.9) and (3.11), we get

$$p_1 = -q_1 \quad (3.13)$$

and

$$2\left(\frac{1-\lambda}{1+\gamma}\right)^2\left(\frac{2+\gamma}{1+\gamma}\right)^{2k} a_2^2 = (1-\beta)^2(p_1^2 + q_1^2). \quad (3.14)$$

From (3.10), (3.12) and (3.14), we get

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$$a_2^2 = \frac{(1-\beta)(p_2+q_2)(1+\gamma)^{2k+2}}{4(1-\lambda)(1+\gamma)^{k+1}(3+\gamma)^k + 2\left[(\lambda^2-1)+(\lambda-1)\gamma\right](2+\gamma)^{2k}}.$$

By Lemma 1.1, $|p_n| \le 2$ and $|q_n| \le 2$. Hence

$$|a_2| \le \sqrt{\frac{2(1-\beta)(1+\gamma)^{2k+2}}{\left|2(1-\lambda)(1+\gamma)^{k+1}(3+\gamma)^k + \left[(\lambda^2-1)+(\lambda-1)\gamma\right](2+\gamma)^{2k}\right|}}.$$

Now, subtracting equation (3.12) from equation (3.10), we obtain

$$\left(\frac{4-4\lambda}{1+\gamma}\right)\left(\frac{3+\gamma}{1+\gamma}\right)^k a_3 - \left(\frac{4-4\lambda}{1+\gamma}\right)\left(\frac{3+\gamma}{1+\gamma}\right)^k a_2^2 = (1-\beta)(p_2-q_2). \quad (3.15)$$

From (3.13), (3.14) and (3.15), we get

$$a_3 = \frac{(1-\beta)(p_2-q_2)(1+\gamma)^{k+1}}{(4-4\lambda)(3+\gamma)^k} + \frac{(1-\beta)^2(p_1^2+q_1^2)(1+\gamma)^{2k+2}}{2(1-\lambda)^2(2+\gamma)^{2k}}. \quad (3.16)$$

By Lemma 1.1, $|p_n| \le 2$, $|q_n| \le 2$ and apply on (3.16), we obtain

$$|a_3| \le \frac{4(1-\beta)^2(1+\gamma)^{2k+2}}{(1-\lambda)^2(2+\gamma)^{2k}} + \frac{(1-\beta)(1+\gamma)^{k+1}}{(1-\lambda)(3+\gamma)^k}.$$

The proof is completed.

If we take $\gamma = 0$ in Theorem (3.1), then Corollary (3.2) is obtained.

Corollary 3.2.([6]) Let function $f(z)$ of the form (1.1) be in the class $M_\Sigma^{k,\lambda}(\beta)$, $0 \le \beta < 1, 0 \le \lambda < 1$, then

$$|a_2| \le \sqrt{\frac{2(1-\beta)}{\left|2(1-\lambda)3^k + (\lambda^2-1)2^{2k}\right|}} \quad \text{and} \quad |a_3| \le \frac{4(1-\beta)^2}{(1-\lambda)^2 2^{2k}} + \frac{(1-\beta)}{(1-\lambda)3^k}.$$

If we take $\gamma = 0$ and $\lambda = 0$ in Theorem (3.1), then Corollary (3.3) is obtained.

Corollary 3.3([6]). Let function $f(z)$ of the form (1.1) be in the class $M_\Sigma^k(\beta)$, then

$$|a_2| \le \sqrt{\frac{1-\beta}{3^k - 2^{2k-1}}} \quad \text{and} \quad |a_3| \le \frac{4(1-\beta)^2}{2^{2k}} + \frac{(1-\beta)}{3^k}.$$

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