Some New Class Of Generalized Closed And Open Mappings

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***ABSTRACT** : In This Paper We Introduce A Class Of Sets Called Greneralised $B\delta g$-Closed Sets In Topological Spaces. And We Introduce Basic Properties Of Generalised $B\delta g$-Closed Functions. Also We Investigate Contra-$B\delta g$-Closed Functions And Their Relationships To Other Functions.*
***Keywords**: $B\delta g$-Closed Map,$B\delta g$-Open Set,$B\delta g$-Irresolute,Weakly $B\delta g$-Closed,Contra-$B\delta g$-Opendness And $B\delta g$-Opendness .*

1 Introduction

Malghan [2] Introduced Generalised Closed Functions And Devi Et Al [4] Introduced Ag-Closed Functions. Noiri [3] And Veerakumar [1] Introduced Δ-Closed Functions And Gb-Closed Functions In Topological Spaces. In This Present Chapter We Use $B\delta g$-Closed Sets To Define A New Class Of Functions Called $B\delta g$-Closed Functions And Obtain Some Properties Of These Functions. We Further Introduce And Study A New Class Of Functions Namely Weakly $B\delta g$-Closed Functions And We Introduce A New Space Called $B\delta g$-Regular Space. Also We Define A New Class Of Generalized Closed Functions Called Contra-$B\delta g$-Closed Functions And Investigate Their Relationships To Other Functions.

2. Preliminaries

2.1 $B\delta g$-Closed Maps

Definition 2.1.1. A Map $F : (X, T) \to (Y, \Sigma)$ Is Called $B\delta g$-Closed If The Image Of Each Closed Set In (X, T) Is $B\delta g$-Closed In (Y, Σ).

Example 2.1.2. Let $X = \{A, B, C\}$ And $Y = \{P, Q, R\}$ With The Topologies

$T = \{\Phi, \{A\}, \{C\}, \{A, C\}, \{B, C\}, X\}$ And$\sigma = \{\Phi, \{P\}, \{Q, R\}, Y\}$. Define $F : (X, T) \to (Y, \Sigma)$ By $F(A) = Q$, $F(B) = R$ And $F(C) = P$. Then F Is $B\delta g$-Closed Map.

Remark 2.1.3. The Composite Mapping Of Two $B\delta g$-Closed Maps Is Not In $B\delta g$-Closed Maps As Shown In The Following Example.

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Example 2.1.4. Let $X = \{A, B, C\} = Y = Z$; $T = \{\Phi, \{A\}, X\}$, $\Sigma = \{\Phi, \{B\}, \{A, C\}, Y\}$ And $H = \{\Phi, \{A\}, \{C\}, \{A, B\}, \{A, C\}, Z\}$. Define A Map $F : (X, T) \to (Y, \Sigma)$ By $F(A) = A$, $F(B) = C$ And $F(C) = B$ And Let $G : (Y, \Sigma) \to (Z, H)$ By The Identity Function. Clearly F And G Are $B\delta g$-Closed Maps. But $G \circ F : (X, T) \to (Z, H)$ Is Not An $B\delta g$-Closed Map Because $(G\circ F)(\{B, C\}) = \{B, C\}$ Is Not An $B\delta g$-Closed Set Of (Z, H) Where $\{B, C\}$ Is A Closed Set Of (X, T).

Theorem 2.1.5. *If $F : (X, T) \to (Y, \Sigma)$ Is Closed And $G : (Y, \Sigma) \to (Z, H)$ Is $B\delta g$-Closed Map Then $G \circ F : (X, T) \to (Z, H)$ Is $B\delta g$-Closed.*

Proof. Let G Be A Closed Subset Of X. Since F Is Closed, $F(G)$ Is Closed Set Of (Y, Σ). On The Other Hand, $B\delta g$-Closeness Of G Implies $G(F(G))$ Is $B\delta g$-Closed In (Z, H). Hence $G \circ F$ Is $B\delta g$-Closed Map.

Remark 2.1.6. If $F : (X, T) \to (Y, \Sigma)$ Is $B\delta g$-Closed Map And $G : (Y, \Sigma) \to (Z, H)$ Is Closed Map Then
$G \circ F : (X, T) \to (Z, H)$ May Not Be $B\delta g$-Closed Map As Shown By The Following Example.

Example 2.1.7. Let $X = \{A, B, C\} = Y = Z$; $T = \{\Phi, \{B\}, X\}$, $\Sigma = \{\Phi, \{C\}, \{A, B\}, Y\}$ And $H = \{\Phi, \{B\}, \{C\}, \{A, B\}, \{B, C\}, Z\}$. Define A Map $F : (X, T) \to (Y, \Sigma)$ By $F(A) = B$, $F(B) = A$ And $F(C) = C$ And Let
$G : (Y, \Sigma) \to (Z, H)$ Be The Identity Function. Clearly F Is $B\delta g$-Closed Map And G Is Closed Map. But
$G \circ F : (X, T) \to (Z, H)$ Is Not An $B\delta g$ Closed Map Because $(G \circ F)(\{A, C\}) = \{B, C\}$ Is Not An $B\delta g$-Closed Set Of (Z, H) Where $\{A, C\}$ Is A Closed Set Of (X, T).

Proposition 2.1.8. *If A Map $F : (X, T) \to (Y, \Sigma)$ Is $B\delta g$-Closed, Then $B\delta g\text{-}Cl(F(A)) \subset F(Cl(A))$ For Every Subset A Of (X, T).*

Proof. Suppose F Is $B\delta g$-Closed Map And Let $A \subset X$. Then $F(Cl(A))$ Is $B\delta g$-Closed Set In (Y, Σ). But $F(A) \subset F(Cl(A))$ And $B\delta g\text{-}Cl(F(A)) \subset B\delta g\text{-}Cl(F(Cl(A))) = F(Cl(A))$.

Remark 2.1.9. The Converse Of Proposition 2.1.8 Need Not Be True As Seen From The Following Example.

Example 2.1.10. *Let $X = \{A, B, C\}$ And $Y = \{P, Q, R\}$ With The Topologies $T = \{\Phi, \{A\}, X\}$ And $\Sigma = \{\Phi, \{Q\}, \{P, Q\}, Y\}$. Define A Map $F : (X, T) \to (Y, \Sigma)$ By $F(A) = Q$, $F(B) = P$ And $F(C) = R$. Let $A = \{A\}$. Here $B\delta g\text{-}Cl(F(A)) \subset F(Cl(A))$. But F Is Not A $B\delta g$-Closed Map, Since The Set $\{B, C\}$ Is Closed In (X, T), But $F(\{B, C\}) = \{P, R\}$ Is Not $B\delta g$-Closed Set In (Y, Σ).*

Theorem 2.1.11. *A Map $F : (X, T) \to (Y, \Sigma)$ Is $B\delta g$-Closed If And Only If For Each Subset G Of (Y, Σ) And For Each Open Set U Of (X, T) Containing $F^{-1}(G)$, There Exists An $B\delta g$-Open Set V Of (Y, Σ) Such That $G \subset V$ And $F^{-1}(V) \subset U$.*

Proof. Let F Be A $B\delta g$-Closed Map And Let G Be An Subset Of (Y, Σ) And U Be An Open Set Of (X, T) Containing $F^{-1}(G)$. Then $X - U$ Is Closed In (X, T). Since F Is $B\delta g$-Closed Map, $F(X - U)$ Is $B\delta g$-Closed Set In (Y, Σ). Hence $Y - F(X-U)$ Is $B\delta g$-Open Set In (Y, Σ). Take $V = Y - F(X - U)$.

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Then V Is $B\delta g$-Open Set In (Y, Σ) Containing G Such That $F^{-1}(V) \subset U$. Conversely, Let F Be An Closed Subset Of (X, T). Then $F^{-1}(Y - F(F)) \subset X - F$ And $X - F$ Is Open. By Hypothesis There Is An $B\delta g$-Open Set V Of (Y, Σ) Such That $Y - F(F) \subset V$ And $F^{-1}(V) \subset X - F$. Therefore $F \subset X - F^{-1}(V)$. Hence $Y - V \subset F(F) \subset F(X - F^{-1}(V)) \subset Y - V$ Which Implies $F(F) = Y - V$ And Hence $F(F)$ Is $B\delta g$-Closed In (Y, Σ).
Thus F Is An $B\delta g$-Closed Map.

Theorem 2.1.12. *Let* $F : (X, T) \to (Y, \Sigma)$ *And* $G : (Y, \Sigma) \to (Z, H)$ *Be Any Two Maps.*

(I) If $G \circ F : (X, T) \to (Z, H)$ *Is* $B\delta g$*-Closed Map And G Is* $B\delta g$ *Irresolute, Injective Map Then F Is* $B\delta g$ *Closed.*

(Ii) If $G \circ F : (X, T) \to (Z, H)$ *Is* $B\delta g$*-Irresolute And G Is* $B\delta g$*-Closed, Injective Map Then F Is* $B\delta g$*-Continuous.*

Proof. (I) Let U Be Any Closed Set In (X, T). Since $G \circ F$ Is $B\delta g$ Closed, $(G \circ F)(U)$ Is $B\delta g$-Closed Set In (Z, H). Therefore $G(F(U))$Is $B\delta g$-Closed In (Z, H). Since G Is $B\delta g$-Irresolute, $G^{-1}(G(F(U)))$ Is $B\delta g$-Closed In (Y, Σ). That Is $F(U)$ Is $B\delta g$-Closed In (Y, Σ). Hence F Is $B\delta g$-Closed Map. (Ii) Let U Be Any Closed Set In (Y, Σ). Since G Is $B\delta g$-Closed Map, $G(U)$ Is $B\delta g$-Closed Set In (Z, H). Since $G \circ F$ Is $B\delta g$-Irresolute, $(G \circ F)^{-1}(G(U))$ Is $B\delta g$-Closed In (X, T). Therefore $(F^{-1}(G^{-1}(G(U))))$ Is $B\delta g$-Closed In (X, T). Hence $F^{-1}(U)$ Is $B\delta g$-Closed In (X, T). This
Shown That F Is $B\delta g$-Continuous Function.

Theorem .2.1.13. *Let* $F : (X, T) \to (Y, \Sigma)$ *And* $G : (Y, \Sigma) \to (Z, H)$ *Be Any Two Maps And* $G \circ F$: $(X, T) \to (Z, H)$ *Be An* $B\delta g$*-Closed Map. If F Is Continuous Then G Is* $B\delta g$*-Closed.*
Proof. Let V Be Any Closed In (Y, Σ). Since F Is Continuous, $F^{-1}(V)$ Is Closed In (X, T). Since $G \circ F$ Is $B\delta g$-Closed Map, $(G \circ F)(F^{-1}(V))$ Is $B\delta g$-Closed In (Z, H). That Is $G(V)$ Is $B\delta g$-Closed In (Z, H).
Hence G Is $B\delta g$-Closed Map.

Theorem 2.1.14. *A Bijection* $F : (X, T) \to (Y, \Sigma)$ *Is* $B\delta g$*-Closed Map If And Only If* $F(U)$ *Is* $B\delta g$*-Open In* (Y, Σ) *For Every Open Set U In* (X, T).
Proof. *Necessity:* Suppose $F : (X, T) \to (Y, \Sigma)$ Is $B\delta g$-Closed Map. Let U Be An Open Set In (X, T). Then $U^c$is Closed In (X, T). Since F Is $B\delta g$-Closed Map, $F(U^c)$ Is $B\delta g$-Closed Set In (Y, Σ). But $F(U^c) = [F(U)]^C$ And Hence $[F(U)]^C$is $B\delta g$-Closed In (Y, Σ). Hence $F(U)$ Is $B\delta g$-Open In (Y, Σ). *Sufficiency:* Let $F(U)$ Be $B\delta g$-Open In (Y, Σ) For Every Open Set U Of (X, T). Then $U^c$is Closed Set In (X, T) And $[F(U)]^C$is $B\delta g$ Closed Set In (Y, Σ). But $[F(U)]^C = F(U^c)$ And Hence $F(U^c)$ Is $B\delta g$ Closed Set In (Y, Σ). Therefore F Is $B\delta g$-Closed Map.

Definition 2.1.15. A Map $F : (X.T) \to (Y, \Sigma)$ Is Said To Be $B\delta g$ Open If The Image Of Every Open Set In (X, T) Is $B\delta g$-Open In (Y, Σ).

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Remark .2.1.16. $B\delta g$-Openness And $B\delta g$-Continuity Are Indepen Dent As Shown In The Following Example.

Example 2.1.17. Let $X = \{A, B, C\}$ And $Y = \{P, Q, R\}$ With The Topologies $T = \{\Phi, \{A\}, \{B\}, \{A, B\}, X\}$ And $\Sigma = \{\Phi, \{Q\}, \{P, Q\}, Y\}$. Define $F : (X, T) \to (Y, \Sigma)$ By $F(A) = P$, $F(B) = Q$ And $F(C) = R$. Clearly F Is $B\delta g$-Continuous But Not $B\delta g$-Open Map Because $\{B\}$ Is Open In (X, T) But $F(\{B\}) = \{Q\}$ Is Not $B\delta g$-Open In (Y, Σ).

Example 2.1.18. Let $X = \{A, B, C\}$ And $Y = \{P, Q, R\}$ With The Topologies $T = \{\Phi, \{A\}, \{A, C\}, X\}$ And

$\Sigma = \{\Phi, \{P\}, \{Q\}, \{P, Q\}, Y\}$. Define The Function $F : (X, T) \to (Y, \Sigma)$ By $F(A) = Q$, $F(B) = R$ And $F(C) = P$. Then F Is $B\delta g$-Open But Not $B\delta g$-Continuous Map Because $\{R\}$ Is Closed In (Y, Σ) But $F^{-1}(\{R\}) = \{B\}$ Is Not $B\delta g$-Closed In (X, T).

Theorem 2.1.19. *A Map $F : (X, T) \to (Y, \Sigma)$ Is $B\delta g$-Open If And Only If For Each Subset G Of (Y, Σ) And For Each Closed Set U Of (X, T)Containing $F^{-1}(G)$, There Exists An $B\delta g$-Closed Set V Of (Y, Σ) Such That $G \subset V$ And $F^{-1}(V) \subset U$.*

Proof. Suppose F Is $B\delta g$-Open Map. Let G Be Any Subset Of (Y, Σ) And U Be An Closed Set (X, T) Containing $F^{-1}(G)$. Then $U^c$is Open In (X, T). Since F Is $B\delta g$-Open Map, $F(U^c)$ Is $B\delta g$-Open Set In (Y, Σ). Hence $(F(U^c))^C$is $B\delta g$-Closed Set In (Y, Σ). Put $V = (F(U^c))^C$. Then V Is $B\delta g$-Closed Set In (Y, Σ) Containing G Such That $F^{-1}(V) \subset U$. Conversely, Let F Be An Open Subset Of (X, T). Then $F^{-1}(F(F^c)) \subset F^c$ And $F^c$is Closed. By Hypothesis There Is An $B\delta g$-Closed Set V Of (Y, Σ) Such That $(F(F))^C \subset V$ And $F^{-1}(V) \subset F^c$. Therefore $F \subset (F^{-1}(V))^C$. Hence $V^c \subset F(F) \subset F(F^{-1}(V))^C \subset V^c$, Which Implies $F(F) = V^c$ And Hence $F(F)$ Is $B\delta g$-Open In (Y, Σ). Thus F Is A $B\delta g$ Open Map.

Corollary 2.1.20. *A Map $F : (X, T) \to (Y, \Sigma)$ Is $B\delta g$-Open If And Only If $F^{-1}(B\delta g\text{-}Cl(B)) \subset Cl(F^{-1}(B))$ For Every Subset B Of (Y, Σ).*

Proof. Suppose That F Is $B\delta g$-Open Map. For Any Subset B Of (Y, Σ), $F^{-1}(B) \subset Cl(F^{-1}(B))$. Hence By Theorem 4.2.19 There Ex Ists A $B\delta g$-Closed Set A Of (Y, Σ) Such That $B \subset A$ And $F^{-1}(A) \subset Cl(F^{-1}(B))$. Hence We Obtain $F^{-1}(B\delta g\text{-}Cl(B)) \subset F^{-1}(A) \subset Cl(F^{-1}(B))$,

Since A Is $B\delta g$-Closed Set In (Y, Σ). Conversely, Let B Be Any Sub Set Of (Y, Σ) And Let U Be Any Closed Set Containing $F^{-1}(B)$. Take $A = B\delta g\text{-}Cl(B)$. Then A Is $B\delta g$-Closed And $B \subset A$. By Assump Tion $F^{-1}(A) = F^{-1}(B\delta g\text{-}Cl(B)) \subset Cl(F^{-1}(B)) \subset U$ And Therefore By Theorem 2.1.19 F Is $B\delta g$-Open.

2.2 Weakly *Bδg*-Closed Maps

We Introduce The Following Definition.

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Definition 2.2.1. A Map $F : (X, T) \rightarrow (Y, \Sigma)$ Is Called *Weakly Bδg Closed* (Resp. Weakly *Bδg*-Open) If The Image Of Every *Δ*-Closed (Resp. *Δ*-Open) Set In (X, T) Is *Bδg*-Closed (Resp. *Bδg*-Open) Set In(Y, Σ).

Theorem 2.2.2. *Every Bδg-Closed Map Is Weakly Bδg-Closed.*

Proof. Let $F : (X, T) \rightarrow (Y, \Sigma)$ Be An *Bδg*-Closed Map And G Be A *Δ*-Closed Set In (X, T). Every *Δ*-Closed Set Is Closed, G Is Closed Set In (X, T). Since F Is *Bδg*-Closed Map, $F(G)$ Is *Bδg*-Closed In (Y, Σ).
Hence F Is Weakly *Bδg*-Closed Map.

Remark 2.2.3. The Converse Of The Above Theorem Need Not Be True As Shown In The Following Example.

Example 2.2.4. Let $X = \{A, B, C\}$ And $Y = \{P, Q, R\}$ With The Topologies $T = \{\Phi, \{A\}, X\}\Sigma = \{\Phi, \{Q\}, \{P, Q\}, Y\}$. Define A Map $F : (X, T) \rightarrow (Y, \Sigma)$ By $F(A) = Q$, $F(B) = P$ And $F(C) = R$. Then F Is Weakly *Bδg*-Closed But Not *Bδg*-Closed Map. Because $F(\{B, C\}) = \{P, R\}$ Is Not *Bδg*-Closed In (Y, Σ) Where $\{B, C\}$ Is Closed In (X, T).

Theorem 2.2.5. *Every Δ-Closed Map Is Weakly Bδg-Closed Map.*

Proof. It Is True That Every *Δ*-Closed Set Is *Bδg*-Closed.

Remark 2.2.6. The Converse Of The Above Theorem Need Not Be True As Shown In The Following Example.

Example2.2.7. Let $X = \{A, B, C\}$ And $Y = \{P, Q, R\}$ With The Topologies $T = \{\Phi, \{A\}, \{B, C\}, X\}$ And $\Sigma = \{\Phi, \{R\}, \{P, Q\}, Y\}$. De Fine $F : (X, T) \rightarrow (Y, \Sigma)$ By $F(A) = P$, $F(B) = R$ And $F(C) = Q$. Then F Is Weakly *Bδg*-Closed Map But F Is Not *Δ*-Closed Because $F(\{B, C\}) = \{Q, R\}$ Is Not *Δ*-Closed In (Y, Σ) Where $\{B, C\}$ Is *Δ*-Closed In (X, T).

Remark 2.2.8. The Composition Of Two Weakly *Bδg*-Closed Maps Need Not Be Weakly *Bδg*-Closed As Shown In The Following Example.

Example 2.2.9. Let $X = \{A, B, C\} = Y = Z$ With The Topologies $T = \{\Phi, \{A\}, \{B, C\}, X\}$; $\Sigma = \{\Phi, \{C\}, \{A, B\}, Y\}$ And $H = \{\Phi, \{A\}, \{C\}, \{A, B\}, \{A, C\}, Z\}$. Define $F : (X, T) \rightarrow (Y, \Sigma)$ By $F(A) = B$, $F(B) = A$ And $F(C) = C$ And Let $G : (Y, \Sigma) \rightarrow (Z, H)$ Be The Identity Function. Clearly F And G Are Weakly *Bδg*-Closed Maps But $G \circ F : (X, T) \rightarrow (Z, H)$ Is Not An Weakly *Bδg*-Closed Map. Because $\{B, C\}$ Is *Δ*-Closed In (X, T) But $(G \circ F)(\{B, C\}) = G(F(\{B, C\})) = \{A, C\}$ Is Not *Bδg*-Closed Set In (Z, H).

Theorem 2.2.10. *Let* $F : (X, T) \rightarrow (Y, \Sigma)$ *And* $G : (Y, \Sigma) \rightarrow (Z, H)$ *Be Any Two Maps. Then* *(I)* $G \circ F : (X, T) \rightarrow (Z, H)$ *Is Weakly Bδg-Closed Map, If F Is Δ Closed Map And G Is Weakly Bδg-*

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Closed Map. (Ii) If $G \circ F : (X, T) \to (Z, H)$ *Is Weakly* $B\delta g$*-Closed And G Is* $B\delta g$ *Irresolute, Injective Map Then F Is Weakly* $B\delta g$*-Closed.*

Proof. (I) Let V Be Δ-Closed In (X, T). Since F Is Δ-Closed Map, $F(V)$ Is Δ-Closed In (Y, Σ). Since G Is Weakly $B\delta g$-Closed Map, $G(F(V))$ Is $B\delta g$-Closed In (Z, H). That Is $(G \circ F)(V)$ Is $B\delta g$-Closed In (Z, H). Hence $(G \circ F)$ Is Weaklly $B\delta g$-Closed Map. (Ii) Let U Be Δ-Closed In (X, T). Since $G \circ F$ Is Weakly $B\delta g$ Closed Map, $(G \circ F)(U)$ Is $B\delta g$-Closed In (Z, H). Therefore $G(F(U))$ Is $B\delta g$-Closed In (Z, H). Since G Is $B\delta g$-Irresolute, $G^{-1}(G(F(U)))$ Is $B\delta g$-Closed In (Y, Σ). That Is $F(U)$ Is $B\delta g$-Closed In (Y, Σ). Hence F Is Weakly $B\delta g$-Closed Map.

Theorem 2.2.11. *A Bijection* $F : (X, T) \to (Y, \Sigma)$ *Is Weakly* $B\delta g$ *Closed Map If And Only If* $F(U)$ *Is* $B\delta g$*-Open In* (Y, Σ) *For Every* Δ*-Open Set U In* (X, T).

Proof. Let $F : (X, T) \to (Y, \Sigma)$ Be Weakly $B\delta g$-Closed Map And U Be Any Δ-Open Set In (X, T). Then $U^c$is Δ-Closed Set In (X, T). Since F Is Weakly $B\delta g$-Closed Map, $F(U^c)$ Is $B\delta g$-Closed Set In (Y, Σ). But $F(U^c) = [F(U)]^C$ And Hence $[F(U)]^C$is $B\delta g$-Closed Set In (Y, Σ). There Fore $F(U)$ Is $B\delta g$-Open In (Y, Σ). Conversely, Assume That $F(U)$ Is $B\delta g$-Open In (Y, Σ) For Every Δ-Open Set U Of (X, T). Then $U^c$is Δ Closed Set In (X, T) And $[F(U)]^C$is $B\delta g$-Closed In (Y, Σ). Hence $F(U^c)$ Is $B\delta g$-Closed In (Y, Σ). Thus F Is Weakly $B\delta g$-Closed Map.

Theorem 2.2.12. *A Map* $F : (X, T) \to (Y, \Sigma)$ *Is Weakly* $B\delta g$*-Closed Map If And Only If For Each Subset B Of* (Y, Σ) *And For Each* Δ*-Open Set U Of* (X, T) *Containing* $F^{-1}(B)$, *There Exists An* $B\delta g$*-Open Set V Of* (Y, Σ) *Such That* $B \subset V$ *And* $F^{-1}(V) \subset U$.

Proof. *Necessity:* Suppose F Is Weakly $B\delta g$-Closed Map. Let B Be Any Subset Of (Y, Σ) And U Be An Δ-Open Set Of (X, T) Containing $F^{-1}(B)$. Then $X - U$ Is Δ-Closed Subset Of (X, T). Since F Is Weakly $B\delta g$-Closed Map, $F(X - U)$ Is $B\delta g$-Closed Set In (Y, Σ). That Is $Y - F(X - U)$ Is $B\delta g$-Open In (Y, Σ). Put $V = Y - F(X - U)$. Then V Is An $B\delta g$-Open Set In (Y, Σ) Containing B Such That $F^{-1}(V) \subset U$.

Sufficiency: Let F Be Any Δ-Closed Subset Of (X, T). Then $F^{-1}(Y - F(F)) \subset X - F$ And $X - F$ Is Δ-Open In (X, T). Put $B = Y - F(F)$ Then $F^{-1}(B) \subset X - F$. There Exists An $B\delta g$-Open Set V Of (Y, Σ) Such That $B = Y - F(F) \subset V$ And $F^{-1}(V) \subset X - F$. Therefore We Obtain $F(F) = Y - V$ And Hence $F(F)$ Is $B\delta g$-Closed In (Y, Σ). Thus F Is Weakly $B\delta g$-Closed Map.

2.3 Contra-*Bδg*-Closed Maps

This Section Deals With Contra-$B\delta g$-Closed Maps And Contra-$B\delta g$ Open Maps, Their Weaker Forms Namely Contra-Weakly-$B\delta g$-Closed Maps And Contra-Weakly $B\delta g$-Open Maps Respectively.

Definition 2.3.1. A Map $F : (X, T) \to (Y, \Sigma)$ Is Called *Contra-*$B\delta g$*-Closed* (Resp. Contra-$B\delta g$-Open) If The Image Of Each Closed (Resp. Open) Set In (X, T) Is $B\delta g$-Open (Resp. $B\delta g$-Closed) Set In (Y, Σ).

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Definition 2.3.2. A Map $F : (X, T) \to (Y, \Sigma)$ Is Said To Be Contra Weakly $B\delta g$-Closed (Resp. Contra-Weakly $B\delta g$-Open) If The Image Of Each Δ-Closed (Resp. Δ-Open) Set In (X, T) Is $B\delta g$-Open (Resp. $B\delta g$ Closed) Set In (Y, Σ).

Theorem 2.3.3. *Every Contra-Bδg-Closed Map Is Contra-Weakly Bδg Closed.*

Proof. Let $F : (X, T) \to (Y, \Sigma)$ Be Contra-$B\delta g$-Closed Map And G Be A Δ-Closed Set In (X, T). Every Δ-Closed Set Is Closed, G Is Closed In (X, T). Since F Is Contra-$B\delta g$-Closed Map, $F(G)$ Is $B\delta g$-Open In

(Y, Σ). Hence F Is Contra-Weakly $B\delta g$-Closed Map.

Remark 2.3.4. The Converse Of The Above Theorem Need Not Be True As Shown In The Following Example.

Example 2.3.5. Let $X = \{A, B, C\}$; $Y = \{P, Q, R\}$ With The Topolo Gies $T = \{\Phi, \{A\}, X\}$ And $\Sigma = \{\Phi, \{P\}, \{P, R\}, Y\}$. Define A Map $F : (X, T) \to (Y, \Sigma)$ By $F(A) = Q$, $F(B) = R$ And $F(C) = P$. Then F Is Contra-Weakly $B\delta g$-Closed But F Is Not Contra $B\delta g$-Closed Map, Because $F(\{B, C\}) = \{P, R\}$ Is Not $B\delta g$-Open In (Y, Σ) Where $\{B, C\}$ Is Closed In (X, T).

Theorem 2.3.6. *Every Contra-Bδg-Open Map Is Contra-Weakly Bδg Open.*

Proof. Follows From The Definitions.

Remark 2.3.7. The Converse Of The Theorem 4.4.6 Need Not Be True As Shwon In The Following Example.

Example 2.3.8. Let $X = \{A, B, C\}$; $Y = \{P, Q, R\}$ With The Topologies $T = \{\Phi, \{A\}, \{A, C\}, X\}$ And $\Sigma = \{\Phi, \{R\}, \{P, R\}, Y\}$. Define A Map $F : (X, T) \to (Y, \Sigma)$ By $F(A) = R$, $F(B) = Q$ And $F(C) = P$. Then Clearly F Is Contra-Weakly $B\delta g$-Open But F Is Not Contra $B\delta g$-Open Map, Because $F(\{A\}) = \{R\}$ Is Not $B\delta g$-Closed In (Y, Σ) Where $\{A\}$ Is Open In (X, T).

Remark 2.3.9. Contra-$B\delta g$-Closedness And $B\delta g$-Closedness Are In Dependent Notions As Shown In The Following Examples. Example 4.4.10. Let $X = \{A, B, C\}$; $Y = \{P, Q, R\}$ With The Topolo Gies $T = \{\Phi, \{A\}, X\}$ And $\Sigma = \{\Sigma, \{Q\}, Y\}$. Define A Map $F : (X, T) \to (Y, \Sigma)$ By $F(A) = P$, $F(B) = Q$ And $F(C) = R$. Then F Is $B\delta g$-Closed Map But F Is Not Contra-$B\delta g$-Closed, Because $F(\{B, C\}) = \{Q, R\}$ Is Not $B\delta g$-Open In (Y, Σ) Where $\{B, C\}$ Is Closed In (X, T).

Example 2.3.11. Let $X = \{A, B, C\}$; $Y = \{P, Q, R\}$ With The Topolgies $T = \{\Phi, \{B\}, \{A, B\}, X\}$ And $\Sigma = \{\Phi, \{P\}, \{R\}, \{P, R\}, Y\}$. Define A Map $F : (X, T) \to (Y, \Sigma)$ By $F(A) = P$, $F(B) = Q$ And $F(C) = R$. Then F Is Contra-$B\delta g$-Closed. It Is Not $B\delta g$-Closed Map, Because $\{A, C\}$ Is Closed In (X, T) But $F(\{A, C\}) = \{P, R\}$ Is Not $B\delta g$-Closed In (Y, Σ).

Theorem 2.3.12. *A Bijection* $F : (X, T) \to (Y, \Sigma)$ *Is Contra-Bδg Closed Map If And Only If* $F(U)$ *Is Bδg-Closed In* (Y, Σ) *For Every Open Set U In* (X, T).

Proof. Let $F : (X, T) \to (Y, \Sigma)$ Be A Contra-$B\delta g$-Closed Map And U Be A Open Set In (X, T). Then U^c is Closed In (X, T). Since F Is Contra-$B\delta g$-Closed Map, $F(U^c)$ Is $B\delta g$-Open Set In (Y, Σ). But $F(U^c) = [F(U)]^C$

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And Hence $[F(U)]^C$is $B\delta g$-Open Set In (Y, Σ). Hence $F(U)$ Is $B\delta g$-Closed In (Y, Σ). Conversely, $F(U)$ Is $B\delta g$-Closed In (Y, Σ) For Every Open Set U In (X, T). Then $U^c$is Closed Set In (X, T) And $[F(U)]^C$is $B\delta g$-Open In (Y, Σ). But $[F(U)]^C = F(U^c)$ And Hence $F(U^c)$ Is $B\delta g$-Open In (Y, Σ). Therefore F Is Contra-$B\delta g$-Closed Map.

Remark 2.3.13. Contra-$B\delta g$-Opendness And $B\delta g$-Opendness Are Independent Notions As Shown In The Following Examples.

Example 2.3.14. Let $X = \{A, B, C\}$; $Y = \{P, Q, R\}$ With The Topolo Gies $T = \{\Phi, \{C\}, X\}$ And $\Sigma = \{\Phi, \{R\}, \{Q, R\}, Y\}$. Define A Function $F : (X, T) \rightarrow (Y, \Sigma)$ By $F(A) = R$, $F(B) = P$ And $F(C) = Q$. Then Clearly F Is $B\delta g$-Open Map But Not Contra-$B\delta g$-Open Because $\{C\}$ Is Open In (X, T) But $F(\{C\}) = \{Q\}$ Is Not $B\delta g$-Closed In (Y, Σ).

Example 2.3.15. Let $X = \{A, B, C\}$; $Y = \{P, Q, R\}$ With The Topolo Gies $T = \{\Phi, \{A, B\}, X\}$, And $\Sigma = \{\Phi, \{Q\}, \{Q, R\}, Y\}$. Define A Map $F : (X, T) \rightarrow (Y, \Sigma)$ By $F(A) = P$, $F(B) = Q$ And $F(C) = R$. Then Clearly F Is Contra-$B\delta g$-Open But Not $B\delta g$-Open Map Because $F(\{A, B\}) = \{P, Q\}$ Is Not $B\delta g$-Open (Y, Σ) Where $\{A, B\}$ Is Open In (X, T).

Theorem 2.3.16. *A Bijection $F : (X, T) \rightarrow (Y, \Sigma)$ Is Contra-Weakly $B\delta g$-Closed Map, If And Only If $F(U)$ Is $B\delta g$-Closed In (Y, Σ) For Every Δ-Open Set U In (X, T).*

Proof. Let $F : (X, T) \rightarrow (Y, \Sigma)$ Be An Contra-Weakly $B\delta g$-Closed Map And U Be Any Δ-Open Set In (X, T). Then $U^c$is Δ-Closed Set In (X, T). Since F Is Contra-Weakly $B\delta g$-Closed Map, $F(U^c)$ Is $B\delta g$-Open Set In (Y, Σ). But $F(U^c) = [F(U)]^C$ And Hence $[F(U)]^C$is $B\delta g$-Open Set In (Y, Σ). Hence $F(U)$ Is $B\delta g$-Closed In (Y, Σ). Conversely, $F(U)$ Is $B\delta g$-Closed In (Y, Σ) For Every Δ-Open Set U Of (X, T). Then $U^c$is Δ-Closed Set In (X, T) And $[F(U)]^C$is $B\delta g$-Open In (Y, Σ). Hence $F(U^c)$
Is $B\delta g$-Open In (Y, Σ). Thus F Is Contra-Weakly $B\delta g$-Closed Map.

Theorem 2.3.17. *For A Map $F : (X, T) \rightarrow (Y, \Sigma)$ The Followings Are Equivalent.*

- *(i)* *F Is Contra-Weakly $B\delta g$-Open.*
- (ii) *(Ii) For Every Subset B Of (Y, Σ) And For Every Δ-Closed Subset F Of (X, T) With $F^{-1}(B) \subseteq F$, There Exists An $B\delta g$-Open Subset U Of (Y, Σ) With $B \subseteq U$ And $F^{-1}(U) \subseteq F$.*
- (iii) *For Every Subset $Y \in Y$ And For Every Δ-Closed Subset F Of (X, T) With $F^{-1}(Y) \subseteq F$, There Exists An $B\delta g$-Open Subset U Of (Y, Σ) With $Y \in U$ And $F^{-1}(U) \subseteq F$.*

Proof. (I) $\Rightarrow$ (Ii). Let B Be A Subset Of (Y, Σ) And F Be A Δ-Closed Subset Of (X, T) With $F^{-1}(B) \subseteq F$. Since F Is Contra-Weakly $B\delta g$ Open And $F^c$is Δ-Open Subset Of (X, T), $F(F^c)$ Is $B\delta g$-Closed Subset Of (Y, Σ). Then $[F(F^c)]^C$is $B\delta g$-Open Subset Of (Y, Σ). Put $U = [F(F^c)]^C$. Then U Is $B\delta g$-Open Subset Of (Y, Σ) And Since $F^{-1}(B) \subseteq F$. We Get $F(F^c) \subseteq B^c$ And Hence $B \subseteq U$. Moreover $F^{-1}(U) = F^{-1}(F(F^c))^C \subseteq F$.

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(Ii) $\Rightarrow$ (Iii). It Is Sufficient But $B = \{Y\}$. (Iii) $\Rightarrow$ (I). Let A Be A Δ-Open Subset Of (X, T) And $Y \in (F(A)^C)^C$ And Let $F = A^c$. Then F Is A Δ-Closed Subset Of (X, T). BY (Iii), There Exist An $B\delta g$-Open Subset U_y Of (Y, Σ) With $Y \in U_y$ And $F^{-1}(U_y) \subseteq F$. Then We See That $Y \in U_y \subseteq (F(A))^C$. Hence $(F(A))^C = \cup\{U_y/Y \in (F(A))^C\}$ Is $B\delta g$-Open. Therefore $F(A)$ Is $B\delta g$ Closed Subset Of (Y, Σ). Hence F Is Contra-Weakly $B\delta g$-Open.

Remark 2.3.18. The Composition Of Two Contra-$B\delta g$-Closed Maps Need Not Be Contra-$B\delta g$-Closed As Shown In The Following Example.

Example 2.3..19. Let $X = \{A, B, C\} = Y = Z$; With The Topologies $T = \{\Phi, \{A, B\}, X\}$, $\Sigma = \{\Phi, \{A\}, Y\}$ And

$H = \{\Phi, \{A\}, \{B\}, \{A, B\}, \{B, C\}, Z\}$. Let $F : (X, T) \to (Y, \Sigma)$ And $G : (Y, \Sigma) \to (Z, H)$ Be The Identity Functions. Then F And G Are Contra-$B\delta g$-Closed Maps But $G \circ F : (X, T) \to (Z, H)$ Is Not Contra-$B\delta g$-Closed Map Because $(G \circ F)(\{A\}) = \{B\}$ Is Not $B\delta g$-Open In (Z, H) Where $\{C\}$ Is Closed In (X, T).

Remark 2.3.20. The Composition Of Two Contra-Weakly $B\delta g$-Closed Maps Need Not Be Contra-Weakly $B\delta g$-Closed Map.

Theorem 2.3.21. *Let $F : (X, T) \to (Y, \Sigma)$ And $G : (Y, \Sigma) \to (Z, H)$ Be Any Two Maps.*

(I) If $G \circ F : (X, T) \to (Z, H)$ Is Contra-$B\delta g$-Closed Map And G Is $B\delta g$-Irresolute, Injective Then F Is Contra-$B\delta g$-Closed Map.
(Ii) If $G \circ F : (X, T) \to (Z, H)$ Is Contra-Weakly $B\delta g$-Closed And G Is $B\delta g$-Irresolute, Injective Then F Is Contra-Weakly $B\delta g$-Closed.

Proof. (I) Let U Be Closed In (X, T). Since $G \circ F$ Is Contra-$B\delta g$-Closed Map, $(G \circ F)(U)$ Is $B\delta g$-Open In (Z, H). Since G Is $B\delta g$-Irresolute Injective $G^{-1}(G \circ F)(U) = F(U)$ Is $B\delta g$-Open In (Y, Σ). Hence F Is Contra-$B\delta g$-Closed Map.

(Ii) Let U Be The Δ-Closed In (X, T). Since $G \circ F$ Is Contra Weakly $B\delta g$-Closed Map, $(G \circ F)(U)$ Is $B\delta g$-Open In (Z, H). Since G Is $B\delta g$-Irresolute, $G^{-1}(G \circ F)(U) = F(U)$ Is $B\delta g$-Open In (Y, Σ). Hence

F Is Contra-Weakly $B\delta g$-Closed Map.

Theorem 2.3.22. *If $G \circ F : (X, T) \to (Z, H)$ Is Contra-Weakly $B\delta g$ Closed Map And $G : (Y, \Sigma) \to (Z, H)$ Is Contra-$B\delta g$-Irresolute, Injec Tion Then $F : (X, T) \to (Y, \Sigma)$ Is Weakly-$B\delta g$-Closed.*

Proof. Let V Be Δ-Closed In (X, T). Since $G \circ F$ Is Contra-Weakly $B\delta g$-Closed Map, $(G \circ F)(V)$ Is $B\delta g$-Open In (Z, H). Since G Is Contra $B\delta g$-Irresolute, $G^{-1}(G \circ F)(V)$ Is $B\delta g$-Closed In (Y, Σ). That Is $F(V)$

Is $B\delta g$-Closed In (Y, Σ). Hence F Is Weakly-$B\delta g$-Closed Map.

2.4 Applications

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Theorem 2.4.1. *Let $F : (X, T) \to (Y, \Sigma)$ And $G : (Y, \Sigma) \to (Z, H)$ Be Two Functions. Let (Y, Σ) Be $_Bt_{\delta g}$-Space. Then*

(I) $G \circ F : (X, T) \to (Z, H)$ Is $B\delta g$-Closed Map If G Is $B\delta g$-Closed And F Is $B\delta g$-Closed Map.

(Ii) $G \circ F : (X, T) \to (Z, H)$ Is Weakly $B\delta g$-Closed Map If G Is Weakly $B\delta g$-Closed And F Is Weakly $B\delta g$-Closed.

Proof. (I) Let V Be Closed In (X, T). Since F Is $B\delta g$-Closed Map, $F(V)$ Is $B\delta g$-Closed Set In (Y, Σ). Also Since (Y, Σ) $_Bt_{\delta g}$-Space, $F(V)$ Is Δ-Closed In Y. Since G Is $B\delta g$-Closed Map, $G(F(V))$ Is $B\delta g$-Closedin (Z, H). That Is $(G \circ F)(V)$ Is $B\delta g$-Closed In (Z, H). Hence $(G \circ F)$ Is $B\delta g$-Closed Map.

(Ii) Let U Be Δ-Closed In (X, T). Since F Is Weakly $B\delta g$ Closed Map, $F(U)$ Is $B\delta g$-Closed In $(Y\ \Sigma)$. Since (Y, Σ) Is $_Bt_{\delta g}$-Space, $F(U)$ Is Δ-Closed In Y. Since G Is Weakly $B\delta g$-Closed $G(F(U))$ Is $B\delta g$ Closed In (Z, H). That Is $(G \circ F)(U)$ Is $B\delta g$-Closed In (Z, H). Hence $G \circ F$ Is Weakly $B\delta g$-Closed Map.

We Introduce The Following Definition.

Definition 2.4.2. A Space (X, T) Is Said To Be $B\delta g$-Regular If For Each Closed Set F Of X And Each Point $X \in F$ There Exists Disjoint $B\delta g$-Open Sets U And V Such That $F \subset U$ And $X \in V$.

Theorem 2.4.3. *In A Topological Space (X, T) The Following State Ments Are Equivalent.*

(i) (X, T) *Is $B\delta g$-Regular.*

(ii) *(Ii) For Every Point Of (X, T) And Every Open Set V Containing X There Exists An $B\delta g$-Open Set A Such That $X \in A \subset Cl_\delta(A) \subset V$.*

Proof. (I) $\Rightarrow$ (Ii) Let $X \in X$ And V Be An Open Set Containing X. Then $X - V$ Is Closed And $X \notin X - V$. By (I) There Exists An $B\delta g$-Open Sets A And B Such That $X \in A$ And $X - V \subset B$. That Is $X - B \subset V$. Since Every Open Set Is B-Set, V Is B-Set, $X - B$ Is $B\delta g$-Closed. Therefore $Cl_\delta(X - B) \subset V$. Since $A \cap B = \Phi$, $A \subset X - B$. Hence $X \in A \subset Cl_\delta(A) \subset Cl_d(X - B) \subset V$. Thus $X \in A \subset Cl_\delta(A) \subset V$.

(Ii) $\Rightarrow$ (I) Let F Be A Closed Set And $X \notin F$. This Implies That $X - F$ Is Open Set Containing X. By (Ii), There Exists An $B\delta g$-Open Set A Such That $X \in A \subset Cl_\delta(A) \subset X - F$. That Is $F \subset X - Cl_\delta(A)$. Since Every Δ-Closed Set Is $B\delta g$-Closed, $Cl_\delta(A)$ Is $B\delta g$-Closed And $X - Cl_\delta(A)$ Is $B\delta g$-Open. Therefore A And $X - Cl_\delta(A)$ Are The Required $B\delta g$-Open Sets.

Theorem 2.4.4. *Let $F : (X, T) \to (Y, \Sigma)$ Is Continuous And $B\delta g$ Closed, Bijective Map And (X, T) Is A Regular Space Then (Y, Σ) Is $B\delta g$-Regular.*

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Proof. Let $Y \in Y$ And V Be An Open Set Containing Y Of (Y, Σ). Let X Be A Point Of (X, T) Such That $Y = F(X)$. Since F Is Continuous, $F^{-1}(V)$ Is Open In (X, T). Since (X, T) Is Regular, There Exists An Open Set U Such That $X \in U \subset Cl(U) \subset F^{-1}(V)$. Hence $Y = F(X) \in F(U) \subset F(Cl(U)) \subset V$. Since F Is A $B\delta g$-Closed Map, $F(Cl(U))$ Is An $B\delta g$-Closed Set Contained In The Open Set V, Which Is B-Set. Hence We Have $Cl_\delta(F(Cl(U))) \subset V$. Therefore $Y \in F(U) \subset Cl_\delta(F(U)) \subset Cl_\delta(F(Cl(U))) \subset V$. This Implies $Y \in F(U) \subset Cl_\delta(F(U)) \subset V$. Since F Is $B\delta g$-Closed Map And $U^c$is Closed In X, $F(U^c)$ Is $B\delta g$-Closed In (Y, Σ). But $F(U^c) = [F(U)]^C$is $B\delta g$-Closed In (Y, Σ). Hence $F(U)$ Is $B\delta g$-Open In (Y, Σ). Thus For Every Point Y Of (Y, Σ) And Every Open Set V Containing Y There Exists An $B\delta g$-Open Set $F(U)$ Such That $Y \in F(U) \subset Cl_\delta(F(U)) \subset V$. Hence By The Theorem 4.5.3 (Y, Σ) Is $B\delta g$-Regular.

Theorem 2.4.5. *If $F : (X, T) \to (Y, \Sigma)$ Is A Continuous And Weakly $B\delta g$-Closed Bijective Map And If (X, T) Is ${}_Bt_{\delta g}$-Space And Regular Space Then (Y, Σ) Is $B\delta g$-Regular.*

Proof. Let $Y \in (Y, \Sigma)$ And V Be An Open Set Containing Y. Let X Be A Point Of (X, T), Such That $Y = F(X)$. Since F Is Continuous, $F^{-1}(V)$ Is Open In (X, T). BY Assumptions And Theorem 4.5.3, There Exists An $B\delta g$-Open Set U Such That $X \in U \subset Cl_\delta(U) \subset F^{-1}(V)$. Then $Y \in F(U) \subset F(Cl_\delta(U)) \subset V$. We Know That $Cl_\delta(U)$ Is Δ-Closed. Since F Is Weakly $B\delta g$-Closed, $F(Cl_\delta(U))$ Is $B\delta g$-Closed Set In (Y, Σ). Every Open Set Is B-Set And Hence V Is B-Set. Therefore We Get $Cl_\delta(F(Cl_\delta(U))) \subset V$. This Implies $Y \in F(U) \subset Cl_\delta(F(U)) \subset Cl_\delta(F(Cl_\delta(U))) \subset V$. That Is $Y \in F(U) \subset Cl_\delta(F(U)) \subset V$. Now U Is $B\delta g$-Open Which Implies That $U^c$is $B\delta g$-Closed In (X, T). Since (X, T) Is ${}_Bt_{\delta g}$-Space And F Is Weakly $B\delta g$-Closed Map, $F(U^c)$ Is $B\delta g$-Closed In (Y, Σ). But $F(U^c) = [F(U)]^C$. That Is $[F(U)]^C$is $B\delta g$-Closed In (Y, Σ). That Implies $F(U)$ Is $B\delta g$ Open In (Y, Σ). Thus For Every Point Y Of (Y, Σ) And Every Open Set V Containing Y, There Exists An $B\delta g$-Open Set $F(U)$ Such That $Y \in F(U) \subset Cl_\delta(F(U)) \subset V$. Hence By Theorem 4.5.3 (Y, Σ) Is $B\delta g$ Regular.

Theorem 2.4.6. *Let $F : (X, T) \to (Y, \Sigma)$ And $G : (Y, \Sigma) \to (Z, H)$ Be Any Two Maps Such That $G \circ F : (X, T) \to (Z, H)$ Where (Y, Σ) Is ${}_Bt_{\delta g}$-Space.*

- *(i)* *If F Is Weakly-$B\delta g$-Closed And G Is Contra-Weakly $B\delta g$-Closed Then $G \circ F$ Is Contra-Weakly $B\delta g$-Closed Map.*
- ***(ii)*** *If F Is Contra-Weakly-$B\delta g$-Closed And G Is Weakly-$B\delta g$-Open Then $G \circ F$ Is Contra-Weakly $B\delta g$-Closed Map.*

Proof. (I) Let U Be Δ-Closed In (X, T). Since F Weakly-$B\delta g$-Closed, $F(U)$ Is $B\delta g$-Closed In (Y, Σ). Since (Y, Σ) Is ${}_Bt_{\delta g}$-Space, $F(U)$ Is Δ-Closed In (Y, Σ). Since G Is Contra-Weakly-$B\delta g$-Closed, $G(F(U))$ Is $B\delta g$-Open In (Z, H). Hence $G \circ F$ Is Contra-Weakly $B\delta g$-Closed Map. (Ii) Let U Be A Δ-Closed In (X, T). Since F Is Contra-Weakly $B\delta g$-Closed Map, $F(U)$ Is $B\delta g$-Open In (Y, Σ). Since (Y, Σ) Is ${}_Bt_{\delta g}$ Space And G Is Weakly-$B\delta g$-Open, $G(F(U))$ Is $B\delta g$-Open In (Z, H). Hence $G \circ F$is Contra-Weakly $B\delta g$-Closed Map.

Theorem 2.4.7. *Let $F : (X, T) \to (Y, \Sigma)$ And $G : (Y, \Sigma) \to (Z, H)$ Be Any Two Maps Such That*

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$G \circ F : (X, T) \to (Z, H)$ Where (Y, Σ) Is ${}_B t_{\delta g}$-Space.

(I) If F Is Weakly-Bδg-Open And G Is Contra-Weakly Bδg-Open Then $G \circ F$ Is Contra-Weakly Bδg-Open.

(Ii) If F Is Contra-Weakly Bδg-Open And G Is Weakly-Bδg-Closed Then $G \circ F$ Is Contra-Weakly Bδg-Open.

Proof. (I) Let U Be Δ-Open In (X, T). Since F Weakly-$B\delta g$-Open, $F(U)$ Is $B\delta g$-Open In (Y, Σ). Since (Y, Σ) Is ${}_B t_{\delta g}$-Space, $F(U)$ Is Δ-Open In (Y, Σ). Since G Is Contra-Weakly $B\delta g$-Open, $G(F(U))$ Is $B\delta g$-Closed In (Z, H). Hence $G \circ F$ Is Contra-Weakly $B\delta g$-Open Map.

(Ii) Let U Be A Δ-Open In (X, T). Since F Is Contra-Weakly $B\delta g$-Open Map, $F(U)$ Is $B\delta g$-Closed In (Y, Σ). Since (Y, Σ) Is ${}_B t_{\delta g}$ Space And G Is Weakly-$B\delta g$-Closed, $G(F(U))$ Is $B\delta g$-Closed In (Z, H). Hence $G \circ F$ Is Contra-Weakly $B\delta g$-Open Map.

Theorem 2.4.8. *If $G \circ F : (X, T) \to (Z, H)$ Is Contra-Bδg-Irresolute And $G : (Y, \Sigma) \to (Z, H)$ Is Contra-Weakly Bδg-Closed, Injective Where (Y, Σ) Is ${}_B t_{\delta g}$-Space Then F Is Bδg-Irresolute.*

Proof. Let V Be A $B\delta g$-Closed In (Y, Σ). Since (Y, Σ) Is ${}_B t_{\delta g}$-Space And G Is Contra-Weakly $B\delta g$-Closed, $G(V)$ Is $B\delta g$-Open In (Z, H). Since $G \circ F$ Is Contra-$B\delta g$-Irresolute, $(G \circ F)^{-1}(G(V))$ Is $B\delta g$-Closed In (X, H).That Is $F^{-1}(V)$ Is $B\delta g$-Closed In (X, H). Hence F Is $B\delta g$-Irresolute Map.

REFERENCES:

1.Veera Kumar M.K.R.S, Bg-Closed Sets In Topological Spaces, Bull.Allah. Math. Soc, 18 (2003), 99{112.
2. Malghan S.R., Generalized Closed Maps, J.Karnatak Univ, Sci.,27 (1982), 82{88.
3. Noiri T., On _-Continuous Functions, J. Korean Math.Soc., 18(1980), 161{166.
4. Devi R., Balachandran K. And Maki H., On Generalized _- Continuous And _-Generalized Continuous Functions, East J. Math Sci., Special Volume, Part I (1997), 1{15.

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