

Hardware Distortion Correlation Has Negligible Impact On Uplink Massive MIMO Spectral Efficiency

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Abstract

This Paper Analyzes How The Distortion Created By Hardware Impairments In A Multiple-Antenna Base Station Affects The Uplink Spectral Efficiency (S_e), With Focus On Massive MIMO. This Distortion Is Correlated Across The Antennas, But Has Been Often Approximated As Uncorrelated To Facilitate (Tractable) S_e Analysis. To Determine When This Approximation Is Accurate, Basic Properties Of Distortion Correlation Are First Uncovered. Then, We Separately Analyze The Distortion Correlation Caused By Third Order Non-Linearities And By Quantization. Finally, We Study The S_e Numerically And Show That The Distortion Correlation Can Be Safely Neglected In Massive MIMO When There Are Sufficiently Many Users. Rayleigh Fading And Equal Signal-To Noise Ratios (S_nrs), This Occurs For More Than Five Transmitting Users. Other Channel Models And Snr Variations Have Only Minor Impact On The Accuracy. We Also Demonstrate The Importance Of Taking The Distortion Characteristics Into Account In The Receive Combining.

Key Words: *Uplink Massive MIMO, Hardware Distortion, Spectral Efficiency, Matlab.*

I. Introduction

The Move To Digital Modulation Provides More Information Capacity, Compatibility With Digital Data Services, Higher Data Security, Better Quality Communications, And Quicker System Availability. Developers Of Communications Systems Face These Constraints:

- Available Bandwidth
- Permissible Power
- Inherent Noise Level Of The System

The Rf Spectrum Must Be Shared, Yet Every Day There Are More Users For That Spectrum As Demand For Communications Services Increases. Digital Modulation Schemes Have Greater Capacity To Convey Large Amounts Of Information Than Analog Modulation Schemes [1].

1.1 Trading Off Simplicity And Bandwidth

There Is A Fundamental Tradeoff In Communication Systems. Simple Hardware Can Be Used In Transmitters And Receivers To Communicate Information. However, This Uses A Lot Of Spectrums Which Limits The Number Of Users. Alternatively, More Complex Transmitters And Receivers Can Be Used To Transmit The Same Information Over Less Bandwidth. The Transition To More And More Spectrally Efficient Transmission Techniques Requires More And More Complex Hardware. Complex Hardware Is Difficult To Design, Test, And Build. This Tradeoff Exists Whether Communication Is Over Air Or Wire, Analog Or Digital [2].

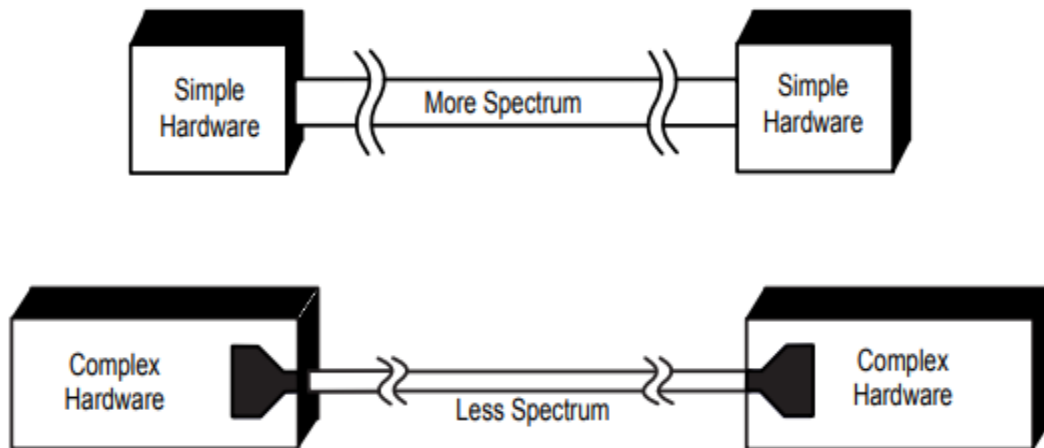


Figure 1. The Fundamental Trade-Off

In Many Wireless Systems, Massive Multiple-Input Multiple-Output (Mimo) Techniques Have Been Used To Improve The Spectral Efficiency And User Experience [3]. In Contrast To The Traditional Single User Mimo (Su-Mimo) Systems Where The Time-Frequency Resource Element Is Dedicated To A Single User, Massive Mimo System Allows Multiple Users To Use The Same Time-Frequency Resources Via A Proper Control Of The Interference Among Co-Scheduled Users At The Base Station. Control Of This Massive Interference Is Achieved By Applying A Precoding To The Symbol Vectors Of All Users Scheduled In The Same Time-Frequency Resources. Since The Precoding Matrix Is Generated Using The Downlink Channel State Information (Csi) Which Relies On The Feedback Information From The Mobile Users, Inaccurate Precoding Operation From Imperfect Csi Causes A Severe Degradation In Massive Interference Cancellation At The Transmitter Side And The Channel Estimation And Detection At The Receiver Side, Undermining The Benefits Of Massive Mimo. In Order To Mitigate The Degradation Of Channel Estimation Quality Caused By Massive Interference, Dedicated Pilots (E.G., Demodulation Reference Signal (Dm-Rs)) Has Been Introduced In The Long Term Evolution Advanced (Lte-Advanced) Standard [4].

While The Purpose Of Common Pilots Is To Serve All Users In The Cell, Dedicated Pilots Are Literally Dedicated To A Single Or A Group Of Users For Better Estimation Of Channels. The Precoding Matrix Applied To The Dedicated Pilots Is The Same As That Applied To The Information Vector. Thus, The Estimates Of The Precoded Channel (Compound Of The Precoding Matrix And The Physical Channels) Can Be Used For The Detection Purpose. As The Wireless Cellular Industry Is Moving Fast Towards The Fifth Generation (5g) Communication Systems, An Attempt To Use Large Number Of Antennas At The Base Station Has Received Much Attention In Recent Years. For Example, Lte-Advancedpro, The Recent Standard Of 3rd Generation Partnership Project (3gpp) Lte, Considers Using Up To 64 Antennas At The Base Station [5].

There Have Been Some Studies To Improve The Channel Estimation Quality Of Massive Mimo Systems [6]–[13]. In [6], Massive Interferences Are Suppressed Using The Channel Estimates Predicted From Adjacent Symbols. Using The Interference Suppressed Signals, Tentative Decisions Are Performed On Data Symbols. In [7], A Joint Detection Algorithm For Massive Environment Has Been Suggested. In This Work, Effect Of The Residual Interference Was Considered To Improve The Performance Of Joint Demodulation And Decoding. Also, Hannel Estimation (Ce) Techniques Accounting For The Effect Of Massive Interferences Have Been Proposed. Notable Examples Include The Maximum A Posteriori

(Map)-Based Ce [8], [9], Joint Maximum Likelihood Ce And Detection [10], Kalman Filter-Based Soft Decision Ce [11], And Ce Combined With Interference Suppression [12], [13].

An Aim Of This Paper Is To Propose An Improved Channel Estimation Technique For The Massive Mimo Systems. The Proposed Method Exploits The Channel Information At The Data Tones To Improve The Channel Estimation And Subsequent Detection Quality. Towards This End, We Pick A Small Number Of Reliable Data Tones And Then Use Them For Virtual Pilots To Generate The Refined Channel Estimates. Our Framework Is Based On The Expectation Maximization (Em) Algorithm [14], Where The Channel Estimation And Data Decoding Are Performed Iteratively To Generate The Joint Estimate Of The Channel And Data Symbols.

The Distortion Correlation Is Studied Both Analytically And Experimentally In [20], In The Related Scenario Of Multi-Antenna Transmission With Non-Linear Hardware. The Authors Observed Distortion Correlation For Single-Stream Transmission In The Sense That Each Element Of The Complex-Valued Distortion Vector Has The Same Angle As The Corresponding Element In The Signal Vector, Leading To A Similar Directivity Of The Radiated Signal And Distortion. However, The Authors Of [20] Conjectured That The Hardware Distortion Is “Practically Uncorrelated” When Transmitting Multiple Precoded Data Streams. In That Case, Each Signal Vector Is A Linear Combination Of The Precoding Vectors Where The Coefficients Are The Random Data. Hence, The Directivity Changes Rapidly With The Data And Does Not Match Any Of The Individual Precoding Vectors. The Conjecture From [20] Was Proved Analytically In [21] For The Case When Many Data Streams Are Transmitted With Similar Power. The Error-Vector Magnitude Analysis In [22] Shows That An Approximate Model Where The Bs Distortion Is Uncorrelated Across The Antennas Gives Accurate Results, While [23] Provides A Similar Results When Considering A Model That Also Includes Mutual Coupling And Noise At The Transmitter Side. Nevertheless, [24] Recently Claimed That The Model With Uncorrelated Bs Distortion “Do Not Necessarily Reflect The True Behavior Of The Transmitted Distorted Signals”, [25] Stated That The Model Is “Physically Inaccurate”, And [26] Says That It “Can Yield Incorrect Conclusions In Some Important Cases Of Interest When The Number Of Antennas Is Large”. These Papers Focus On The Power Spectral Density And Out-Of-Band Interference, Assuming Setups With Non-Linearities At The Bs, While The Ues Are Equipped With Ideal Hardware. In Contrast, The Works [8], [18] Promoting The Uncorrelated Bs Distortion Model Also Consider Hardware Distortion At The Ues And Claim That It Is The Dominant Factor In Massive Mimo. Hence, The Question Is If The Statements From [24]–[26] Are Applicable When Quantifying The Uplink Se In Typical Massive Mimo Scenarios With Hardware Impairments At Both The Ues And Bss.

II. Methodology

We Consider The Uplink Of A Single-Cell System Where K Single-Antenna Ues Communicate With A Bs Equipped With M Antennas. We Consider A Symbol-Sampled Complex Baseband System Model [15]. The Channel From Ue K Is Denoted By $\mathbf{h}_k = [\mathbf{h}_{k1} \dots \mathbf{h}_{km}]^T \in \mathbb{C}^m$. A Block-Fading Model Is Considered Where The Channels Are Fixed Within A Time Frequency Coherence Block And Take Independent Realizations In Each Block, According To An Ergodic Stationary Random Process. In An Arbitrary Coherence Block, The Noise-Free Signal $\mathbf{u} = [u_1 \dots u_m]^T \in \mathbb{C}^m$ Received At The Bs Antennas Is

$$\mathbf{u} = \sum_{k=1}^K \mathbf{h}_k s_k = \mathbf{H}\mathbf{s} \quad (1)$$

Where $\mathbf{H} = [\mathbf{h}_1 \dots \mathbf{h}_K] \in \mathbb{C}^{m \times K}$ Is The Channel Matrix And $s_k \in \mathbb{C}$ Is The Information-Bearing Signal Transmitted By Ue K. Since We Want To Quantify The Se And Gaussian Codebooks Maximizes

The Differential Entropy, We Assume $S = [S_1 \dots S_m]^T \sim N_c(0, P_{ik})$ Where P Is The Variance. The U_{es} Can Have Unequal Pathlosses And/Or Transmit Powers, Which Are Both Absorbed Into How The Channel Realizations H_k Are Generated. Several Different Channel Distributions Are Considered In Section V. We Will Now Analyze How U Is Affected By Non-Ideal Receiver Hardware And Additive Noise. To Focus On The Distortion Characteristics Only, H Is Assumed To Be Known In The Analytical Part Of The Paper. Imperfect Channel Knowledge Is Considered.

A. Basic Modeling Of Bs Receiver Hardware Impairments

We Focus Now On An Arbitrary Coherence Block With A Fixed Channel Realization H And Use $E\{H\{\cdot\}}\}$ To Denote The Conditional Expectation Given H . Hence, The Conditional Distribution Of U Is $N_c(0, C_{uu})$ Where $C_{uu} = E\{H\{U_{uh}\}\} = P_{hhh} \in C^{m \times m}$ Describes The Correlation Between Signals Received At Different Antennas. It Is Only When C_{uu} Is A Scaled Identity Matrix That U Has Uncorrelated Elements. This Can Only Happen When $K \geq M$. However, We Stress That $K < M$ Is Of Main Interest In Massive MIMO [8] (Or More Generally In Any Multiuser MIMO System). The BS Hardware Is Assumed To Be Non-Ideal But Quasi Memoryless, Which Means That Both The Amplitude And Phase Are Affected [16].

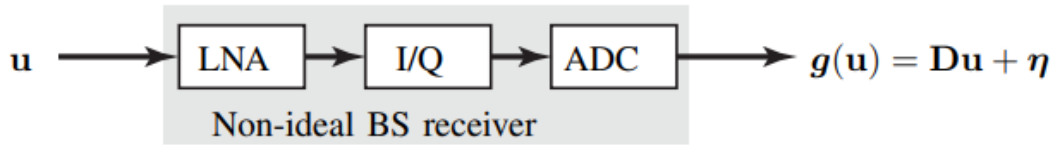


Fig. 2: The Signal Vector U That Reaches The Antennas Is Processed By The Non-Ideal BS Receiver Hardware, Which Includes Lna, I/Q Demodulator, And Adc.

The Impairments At Antenna M Are Modeled By An Arbitrary Deterministic Function $G_m(\cdot) : C \rightarrow C$, For $M = 1, \dots, M$, Which Can Describe Both Continuous Non-Linearities And Discontinuous Quantization Errors. These Functions Distort Each Of The Components In U , Such That The Resulting Signal Is

$$z = \begin{bmatrix} g_1(u_1) \\ \vdots \\ g_M(u_M) \end{bmatrix} \triangleq g(u).$$

By Defining $C_{zu} = E\{H\{Z_{uh}\}\}$, We Can Express Z As A Linear Function Of The Input U By Exploiting That $C_{zu}C_{uu}^\dagger U_{uu}$ Is The Linear Minimum Mean-Squared Error (Lmmse) Estimate Of Z Given U [28, Sect. 15.8]. Notice That We Used The Moorepenrose Inverse Since C_{uu} Is Generally Rank-Deficient. We Can Further Define The “Estimation Error” $H, G(U) - C_{zu}C_{uu}^\dagger U_{uu}$, Which We Will Call The Additive Distortion Term. This Leads To The Input-Output Relation

$$z = g(u) = C_{zu}C_{uu}^\dagger u + \eta.$$

By Construction, The Signal U Is Uncorrelated With H ; That Is,2

However, U And H Are Clearly Not Independent. This Derivation Has Not Utilized The Fact That U Is Gaussian Distributed (For A Given H), But Only Its First And Second Order Moments. Hence, The Model In (3) Holds Also For Finite-Sized Constellations. Since The SE Will Be Our Metric, We Utilize The Full Distribution To Simplify $C_{zu}C_{uu}^\dagger U_{uu}$ In (3) Using A Discrete Complex-Valued Counterpart To The Busgang Theorem [3].

B. Spectral Efficiency With Bs Hardware Impairments

Using The Signal And Distortion Characteristics Derived Above, We Can Compute The SE . The Signal Detection Is Based On The Received Signal $Y \in C^m$ That Is Available In The Digital Baseband And Is Given By

$$\mathbf{y} = \mathbf{z} + \mathbf{n} = \mathbf{D}\mathbf{u} + \boldsymbol{\eta} + \mathbf{n} = \sum_{k=1}^K \mathbf{D}\mathbf{h}_k s_k + \boldsymbol{\eta} + \mathbf{n}$$

Where $\mathbf{N} \sim \mathcal{N}_c(0, \Sigma_2 \mathbf{I}_M)$ Accounts For Thermal Noise That Is (Conditionally) Uncorrelated³ With \mathbf{U} And \mathbf{H} . The Combining Vector \mathbf{v}_k Is Used To Detect The Signal Of u_k As

$$\mathbf{v}_k^H \mathbf{y} = \mathbf{v}_k^H \mathbf{D}\mathbf{h}_k s_k + \sum_{i=1, i \neq k}^K \mathbf{v}_k^H \mathbf{D}\mathbf{h}_i s_i + \mathbf{v}_k^H \boldsymbol{\eta} + \mathbf{v}_k^H \mathbf{n}.$$

In The Given Coherence Block, \mathbf{H} Is Uncorrelated With \mathbf{U} , Thus The Distortion Is Also Uncorrelated With The Informationbearing Signals s_1, \dots, s_k (Which Are Mutually Independent By Assumption). Hence, We Can Use The Worst-Case Uncorrelated Additive Noise Theorem [5] To Lower Bound The Mutual Information Between The Input s_k And Output $\mathbf{v}_k^H \mathbf{y}$ In (10) As

$$\mathcal{I}(s_k; \mathbf{v}_k^H \mathbf{y}) \geq \log_2(1 + \gamma_k)$$

For The Given Deterministic Channel Realization \mathbf{H} , Where Γ_k Represents The Instantaneous Signal-To-Interference-And-Noise Ratio (Sinr) And Is Given By

$$\gamma_k = \frac{p \mathbf{v}_k^H \mathbf{D}\mathbf{h}_k \mathbf{h}_k^H \mathbf{D}^H \mathbf{v}_k}{\mathbf{v}_k^H \left(\sum_{i \neq k} p \mathbf{D}\mathbf{h}_i \mathbf{h}_i^H \mathbf{D}^H + \mathbf{C}_{\eta\eta} + \sigma^2 \mathbf{I}_M \right) \mathbf{v}_k}.$$

The Lower Bound Represents The Worst-Case Situation When The Uncorrelated Distortion Plus Noise Term $\mathbf{H} + \mathbf{N}$ Is Colored Gaussian Noise That Is Independent Of The Desired Signal And Distributed As $\mathbf{H} + \mathbf{N} \sim \mathcal{N}_c(0, \mathbf{C}_{\eta\eta} + \Sigma_2 \mathbf{I}_M)$. Note That This Is Only The Worst-Case Conditional Distribution In A Coherence Block (For A Given \mathbf{H}), While The Marginal Distribution Of $\mathbf{H} + \mathbf{N}$ Is The Product Of Gaussian Random Variables .

C. Spectral Efficiency With u_k Hardware Impairments

In Practice, There Are Hardware Impairments At Both The B_s And U_s . To Quantify The Relative Impact Of Both Impairments, We Next Assume That $s_k = \Sigma_k + \Omega_k$ For $k = 1, \dots, K$, Where $\Sigma_k \sim \mathcal{N}_c(0, K_p)$ Is The Actual Desired Signal From u_k And $\Omega_k \sim \mathcal{N}_c(0, (1 - K)P)$ Is A Distortion Term. The Parameter $K \in [0, 1]$ Determines The Level Of Hardware Impairments At The U_s Side, Potentially After Some Predistortion Algorithm Has Been Applied. For Analytical Tractability, We Assume That Σ_k And Ω_k Are Independent, Thus The Transmit Power Is $E\{|s_k|^2\} = K_p + (1 - K)P = P$ Irrespective Of k . The Independence Can Be Viewed As A Worst-Case Assumption [5], [8], But Is Mainly Made To Apply The Same Methodology As Above To Obtain The Achievable S_e

$$\mathcal{I}(s_k; \mathbf{v}_k^H \mathbf{y}, \mathbf{H}) = \mathbb{E}_{\mathbf{H}}\{\mathcal{I}(s_k; \mathbf{v}_k^H \mathbf{y})\} \geq \mathbb{E}_{\mathbf{H}}\{\log_2(1 + \gamma'_k)\} \quad (17)$$

with

$$\gamma'_k = \frac{\kappa p \mathbf{v}_k^H \mathbf{D}\mathbf{h}_k \mathbf{h}_k^H \mathbf{D}^H \mathbf{v}_k}{\mathbf{v}_k^H \left(\sum_{i \neq k} p \mathbf{D}\mathbf{h}_i \mathbf{h}_i^H \mathbf{D}^H + (1 - \kappa) p \mathbf{D}\mathbf{h}_k \mathbf{h}_k^H \mathbf{D}^H + \mathbf{C}_{\eta\eta} + \sigma^2 \mathbf{I}_M \right) \mathbf{v}_k}.$$

This Sinr Is Also Maximized By Da-Mmse In (13), As It Can Be Proved Using [8, Lemma B.4]. The Reason Is That The Desired Signal And u_k Distortion Are Received Over The Same Channel $\mathbf{D}\mathbf{h}_k$, Thus Such Distortion Cannot Be Canceled By Receive Combining Without Also Canceling The Desired Signal.

III. Simulation Results

The Proposed System Implemented In Matlab, Figure 3 Shows Correlation Data And Figure 4 Represents Linear Amplifier Data. Figure Shows Desired Signal And Figure 6 Shows User Set Data.

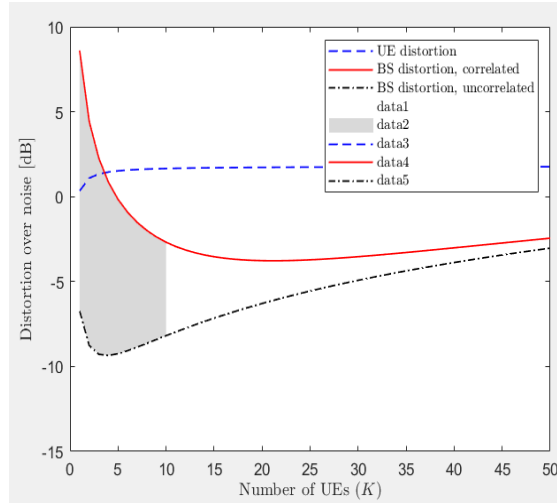


Figure 3. Distortion Correlation Data

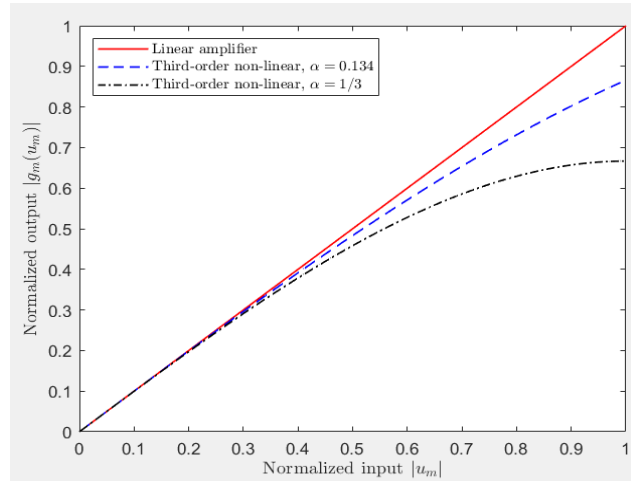


Figure 4 Linear Amplifier Data

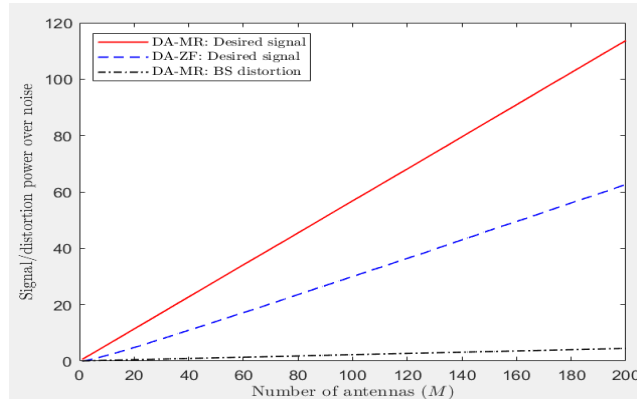


Figure 5 Desired With Signal With Distortion

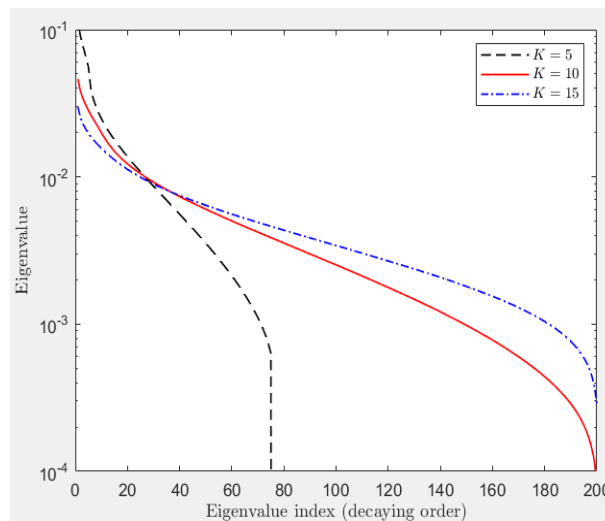


Figure 6 User Set Data

IV. Conclusion

The Hardware Distortion In A Multiple-Antenna Bs Is Generally Correlated Across Antennas. The Correlation Reduces The Sinr, But We Have Shown That Its Impact On The Se Is Negligible, Particularly When Using Da-Mmse Combining Which Can Utilize The Correlation To Effectively Suppress The Distortion. Even In Massive Mimo Systems With 100-200 Antennas, Approximating The Bs Distortion As Uncorrelated When Computing The Se Only Leads To Overestimating The Se By A Few Percent And The Bias Reduces As More Ues Are Added. Since Massive Mimo Is Typically Designed To Serve Tens Of Ues, The Error Is Negligible In The Typical Use Cases. We Demonstrated This By First Deriving Se Expressions With Arbitrary Quasi-Memoryless Distortion Functions And Then Quantifying The Impact Of Third-Order Am-Am Non-Linearities And Quantization Errors. Analytical Results Were Used To Establish The Basic Phenomena Qualitatively And Then Numerical Results Were Used To Quantify Their Impact On The Se. The Worst-Case Scenario For Hardware Distortion Seems To Be When Serving One Single-Antenna Ue At High-Snr In Freespace Propagation [15]. In That Special Case, Which Is Not Mimo Since The Ue Has Only One Antenna, Approximating The Bs Distortion As Uncorrelated Leads To An Se Overestimation Of Around 1 Bit/S/Hz, Assuming That The Bs And Ue Hardware Are Of Similar Quality.

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