

Total Domination Number Of Rooted Products On Some Special Families Of Trees In Graph Theory

¹dr. Vilas Balgaonkar, ²dr. A.Krishnaraju,
³dr. Maninder Kaur, ⁴dr. Mohd Naved,
⁵dr. Dinesh Sheshrao Kharate, ⁶n. Subashini

¹asst. Professor, School Of Sustainable Rural Development
Faculty Of Sustainability Studies, Mit World Peace University, Pune.
Vilas26@Gmail.Com

²professor, Department Of Mechanical And Automation Engineering
Psn College Of Engineering And Technology, Tirunelveli
Pin Code- 627152, Tamilnadu India
Mail. Akrgen@Gmail.Com

³professor, Gnim, New Delhi-110026, India
Drmanindergnim@Gmail.Com

⁴assistant Professor, Department Of Business Analytics,
Jagannath University, Delhi-Ncr, India. Email: Mohdnaved@Gmail.Com

⁵assistant Professor, Animal Research Laboratory,
Pg Department Of Zoology, Sant Ramdas Art's,
Commerce And Science College, Ghansawngi, Jalna-431209,
Maharashtra, India. Dineshkharate2@Gmail.Com

⁶department Of Mathematics, College Of Engineering And Technology,
Srm Institute Of Science And Technology, Kattankulathur,
Chennai – 603203, India.

Abstract

Let G Be A Simple And Undirected Graph. If “T Is Called A Tree It Is Acyclic And Connected. In This Article We Find The Rooted Product Of Some Trees Like Star And Double Star Graphs And Its Total Domination Number Of Those Graphs”.

Subject Classification: Primary 93a30, Secondary 49k15

Keywords: Domination Number, Total Domination Number, Star Graph, Double Star Graph.

1. Introduction

Let $G = (V, E)$ Be A Graph With Vertex Set V And Edge Set E . “A Dominating Set Denoted D_s Of G Is A Set S Of Vertices Of G Such That Every Vertex In $V-S$ Is Adjacent To A Vertex In S ”. “The Domination Number Of G Is The Minimum Cardinality Of D_s . It Is Denoted By $\Gamma(G)$ ”. “A Total Dominating Set Tds Of G With No Isolated Vertex Is A Set S Of Vertices Of G Such That Every Vertex Is Adjacent To A Vertex In S . If No Proper Subset Of S Is A Tds Of G , Then S Is A Minimum Tds Of G ”.

Theorem 1:

Let $S_{a,B}$ Be A Double Star Graph With N Vertices. The Total Domination Number Of $\gamma_t((S_{a,b}) \square (S_{a,b})) = 2n$.

Proof:

Let G Be A Double Star Graph With N Vertices (u_1, u_2, \dots, u_n) Be One Partition Of Vertices In Double Star Graph It Is Dominated By Center Vertex U , And (v_1, v_2, \dots, v_n) Be Another Partition Of Vertices.

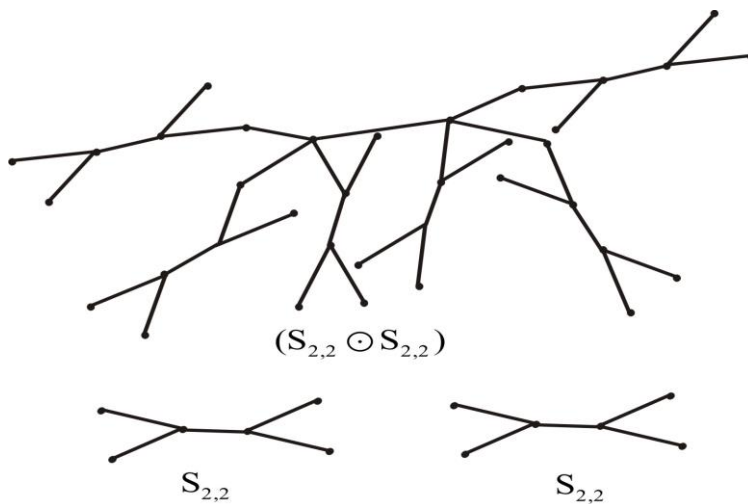


Figure 1

Dominated By Vertex U By The Definition S_0 By The Observation At Least 2 Vertices Need To Dominate The Double Star Graph By The Definition Of Rooted Product N Copies Of Double Star Graph Dominated By Each Two Vertices. So The Total Domination Number Of $\gamma_t((S_{a,b}) \square (S_{a,b}))$ Is $2n$.

Hence Proved.

Observation 1:

Let $S_{a,b}$ Be A Double Star Graph. Then Total Domination Number Of $(S_{a,b})$ Is 2.

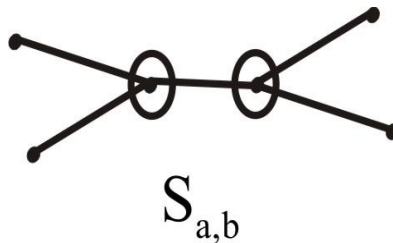


Figure 2

Observation 2:

“Let $K_{1,N}$ Be Star Graph With N Vertices. Then Total Domination Number Of Star Graph Is 2”.

Observation 3:

“Let G Be Comb Graph With N Vertices. Then Total Domination Number Of G Is $\frac{n}{2}$.”

Theorem2:

Let $K_{1,N}$ Be Star Graph Total Domination Number Product Of Star Graph Is $(K_{1,n}) \square (K_{1,n}) = 2n$.

Proof:

Let (u_1, u_2, \dots, u_n) Be Set Of Vertices Dominated By Center Vertex Of Star Graph It Is Enough To Dominate All The Vertices But By The Definition Of Total Domination One More Vertex To Need For Total Domination.

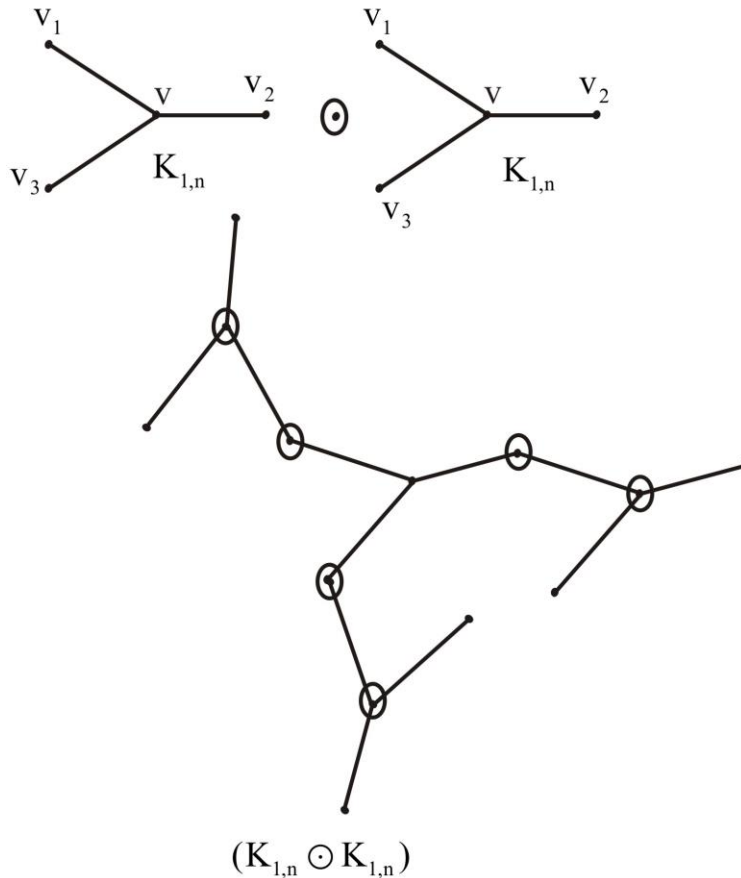


Figure 3

Then We Have Rooted Product Of Star Graph Is Need To n Copies Of Star Graph. So We Need $2n$ Vertices Of Total Dominating Set Which Is Required Number Of Total Domination Number.

Hence Proved.

Theorem3:

If Any Two Comb Graph Order N And Its Total Domination Number Of Rooted Product Is $\left\lceil \frac{n}{2} \right\rceil^2$

Proof

Let G_1, G_2 Be Two Comb Graph Each Graph Having Exactly N Vertices.

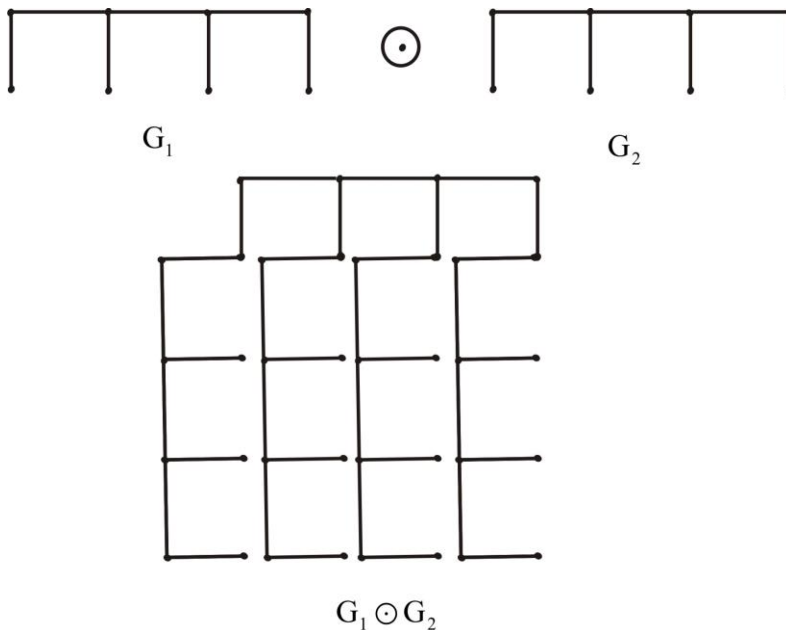


Figure 4

By The Observation3 For Comb Graph Dominated By $\left\lceil \frac{n}{2} \right\rceil$ Vertices. Then The Rooted Product Of Comb Graphs We Take N Copies Of Comb Graph. By The Definition Of

Total Domination N Copies Of $\left\lceil \frac{n}{2} \right\rceil$ Vertices Enough To Dominated By Whole Graphs.

Hence Proved.

References

1. T.W. Haynes, Stephen T. Hedetniemi And Peter J. Slater., Domination In Graphs-Advanced Topics, Marcel Dekker, Inc., New York, (1998).
2. T.W. Haynes, Stephen T. Hedetniemi And Peter J. Slater., Fundamentals Of Domination In Graphs, Marcel Dekker, Inc., New York, (1998).
3. R. Balakrishnan And K. Ranganathan, A Textbook Of Graph Theory, Second Edition , Springer, New York, (2012).
4. V.R. Kulli, B. Janakiram, The Split Domination Number Of A Graph. Graph Theory Notes, New York, (1997).
5. V. R. Kulli, Theory Of Domination In Graphs. Vishwa International Publications, (2010).
6. S.K.Vaidya And Lekha Bijukumar, Some New Families Of Mean Graphs, *Journal Of Mathematics Research*, 2.3, (2010).
7. Guze, Sambor, An Application Of The Selected Graph Theory Domination Concepts To Transportation Networks Modeling (2017).
8. M. Priyadarshini And N. Parvathi. Split Domination Number Of K -Duplication Of Outer Planar Graphs, *Aip Conference Proceedings*, 2112.1, (2019).
9. C. Berge. Theory Of Graphs And Its Applications. Methuen, London, (1962).
10. E.J. Cockayne And C.M. Mynhardt. The Sequence Of Upper And Lower
11. Domination, Independence And Irredundance Numbers Of A Graph, *Discrete*
12. *Math.*,122:89, 102, (1993).
13. [Wayne Goddard](#), [Michael A.Henning](#) Independent Domination In Graphs:
14. A Survey And Recent Results, *Discrete Mathematics*, 313.7, Pp. 839-854, (6
15. April 2013).
16. R.W. Irving. On Approximating The Minimum Independent Dominating Set.
17. *Inform. Process. Lett.*, 37:197, 200, (1991).
18. F. Harary And M. Livingston. Characterization Of Trees With Equal

19. Domination And Independent Domination Numbers, *Congr. Numer.*,
20. 55:121, 150, (1986).
21. D. Anandhababu, And N. Parvathi . On Independent Domination Number Of
22. Indubala Product Of Some Families Of Graphs, *Aip Conference Proceedings*,
23. 2112.020139, (2019).
24. M. Priyadharshini, D. Anandha Babu, A. Anuradha, To Appear In The
25. Journal Of Combinatorial Mathematics And Combinatorial Computing,
Vol.112(2020).
26. O. Favaron. A Bound On The Independent Domination Number Of A Tree,
27. *Vishwa Internat. J. Graph Theory*, 1:19, 27, 1992).
28. Amarnath Pathak, Partha Pakray & Ranjita Das, Context Guided Retrieval Of
Math Formulae From Scientific Documents, *Journal Of Information And
Optimization Sciences*, Volume 40, Issue 8 (2019), Pp.1559-1574.
29. Savio Jay Sengupta, Bikram Biswas, Dipanjan Sen, Sudhabindu
Ray, Subhashis Roy & Subir "Kumar Sarkar, A Novel Approach For Rfid
Based Distributed Security Against Physical Access Of University Data,"
Journal Of Information And Optimization Sciences, Volume 40, Issue 8
(2019),Pp. 1575-1582.
30. Dipika Singh & Rakhi Garg "Comparative Analysis Of Sequential
Community Detection Algorithms Based On Internal And External Quality
Measure," *Journal Of Statistics And Management Systems*, Volume 23, Issue
7 (2020), Pp. 1129-1146.
31. Anil Sharma & Suresh Kumar, "Bayesian Rough Set Based Information
Retrieval," *Journal Of Statistics And Management Systems*, Volume 23, Issue
7 (2020), Pp.1147-1158.