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Multiple Attribute Group Decision Making Methods Using Intuitionistic Trapezoidal Fuzzy Numbers

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Abstract

Solving Multiple Attribute Group Decision Making (MAGDM) problems has become one of the most important researches in the recent trends. The information or data is in the form of Intuitionistic Trapezoidal Fuzzy Number (ITzFN). The methods are applied in decision making problems. The Intuitionistic Trapezoidal Fuzzy Ordered Weighted Geometric (ITzFOWG) operator and the Intuitionistic Trapezoidal Fuzzy Hybrid Geometric (ITzFHG) operator are used to combine the decision matrix. The correlation coefficient is used for ranking the best alternatives. Numerical illustration is proposed to show the effectiveness of the method.

Key words: MAGDM, Intuitionistic Trapezoidal Fuzzy Number, ITzFOWG, ITzFHG.

1.1. Introduction

Multiple Attribute Group Decision Making problems are wide spread in real life condition. A MAGDM problem is to find a desirable solution from a finite number of feasible alternatives assessed on multiple attribute quantitative and qualitative. In the process of MAGDM problems with intuitionistic fuzzy information and the attribute values taken in the form of intuitionistic trapezoidal fuzzy number. Atanassov [1,2] introduced the concept of intuitionistic fuzzy set (IFS) and characterized by a membership and a non-membership function, which is generalization of the concept of fuzzy set. Atanassov & Gargov [3] are investigated the concept of interval valued intuitionistic fuzzy sets. Atanassov [4,5] provided some operstors over interval valued intuitionistic fuzzy sets and new operations defined over the intuitionistic fuzzy sets. Burillo [6] proposed the definition of intuitionistic fuzzy number. John Robinson & Amirtharaj [7,8] are presented a short primer on the correlation co-efficient of vague sets and a search for the correlation co-efficient of triangular and trapezoidal intuitionistic fuzzy sets for multiple attribute group decision making.

Aggregation operators with MAGDM problems are used in the field of mathematics, physics, economics, natural sciences, computer engineering and so on. Jian Wu & Qing-wei cao [9] are provided by same families of geometric aggregation operators with intuitionistic trapezoidal fuzzy numbers. Wei Wu et al. [13,14] are solved by some arithmetic aggregation with intuitionistic trapezoidal fuzzy numbers and their application to group decision making and an approach to multiple attribute group decision making with interval intuitionistic trapezoidal fuzzy information. In this paper basic concepts of IFS, ITzFN and Intuitionistic trapezoidal fuzzy sets are presented. Some aggregation operators are discussed and the procedure for decision making using the correlation co-efficient for ITzFNs is also presented. A numerical example is proposed to explain the developed decision making model.

Intuitionistic Trapezoidal Fuzzy Number

Definition 1.2.1: (Intuitionistic Fuzzy Set) An IFS A in X is given by $A=\left\{\left\langle x,\ \mu_A(x),\ \gamma_A(x)\right\rangle / x\in X\right\}$, where $\mu_A: X\rightarrow[0,1]$, $\gamma_A: X\rightarrow[0,1]$, with the condition $0\le\mu_A(x)+\gamma_A(x)\le 1,\ \ \forall x\in X$ .The numbers $\mu_A(x)$

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and $\gamma_A(x)$ represent, the membership degree and non-membership degree of the element x to the set A respectively.

Definition 1.2.2: (Intuitionistic Trapezoidal Fuzzy Number ITzFN) A ITzFN is an IFS in R with the following membership function $\mu_A(x)$ and non-membership function $\nu_A(x)$:

$$\mu_A(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & for \quad a_1\le x<a_2\\ 1 & for \quad a_2\le x<a_3\\ \dfrac{a_3-x}{a_4-a_3} & for \quad a_3\le x<a_4\\ 0 & otherwise\end{cases}\qquad \gamma_A(x)=\begin{cases}\dfrac{a_2-x}{a_2-a_1'} & for \quad a_1\le x<a_2\\ 0 & for \quad a_2\le x<a_3\\ \dfrac{x-a_3}{a_4'-a_3} & for \quad a_3\le x<a_4'\\ 1 & otherwise\end{cases} \quad (1)$$

where $a_1'\le a_1\le a_2\le a_3\le a_4\le a_4'$ and $\mu_A(x),\gamma_A(x)\le 0.5$ for $\mu_A(x)=\gamma_A(x)$. This ITzFN is denoted by $\left(a_1,a_2,a_3,a_4;a',a_2,a_3,a_4'\right)$. This ITzFN is also denoted as:

$$A=\left\langle\left(\left[a_1,a_2,a_3,a_4\right];\mu_A\right),\left(\left[a',a_2,a_3,a'\right];\gamma_A\right)\right\rangle$$

1.3. Correlation Co-efficient of Intuitionistic Trapezoidal Fuzzy Sets

Let $A=\left(\left[a_1,b_1,c_1,d_1\right];\mu_A,\gamma_A\right),B=\left(\left[a_2,b_2,c_2,d_2\right];\mu_B,\gamma_B\right)$ be two intuitionistic trapezoidal fuzzy sets. Then for each $A,B\in ITzFS\left(X\right)$. We define the informational intuitionistic trapezoidal energy of A as follows:

$$E_{ITzFS}\left(A\right)=\frac{1}{n}\sum_{i=1}^{n}\left[\frac{a_1+2b_1+2c_1+d_1}{6}\right]^2\left(\mu_A{}^2\left(x_i\right)+\gamma_A{}^2\left(x_i\right)+\pi_A{}^2\left(x_i\right)\right) \quad (2)$$

and $$E_{ITzFS}\left(B\right)=\frac{1}{n}\sum_{i=1}^{n}\left[\frac{a_2+2b_2+2c_2+d_2}{6}\right]^2\left(\mu_B{}^2\left(x_i\right)+\gamma_B{}^2\left(x_i\right)+\pi_B{}^2\left(x_i\right)\right) \quad (3)$$

Now we define the correlation of A and B as:

$$C_{ITzFS}\left(A,B\right)=\frac{1}{n}\sum_{i=1}^{n}\left[\frac{a_1+2b_1+2c_1+d_1}{6}\right]\left[\frac{a_2+2b_2+2c_2+d_2}{6}\right]\left(\mu_A\left(x_i\right)\mu_B\left(x_i\right)+\gamma_A\left(x_i\right)\gamma_B\left(x_i\right)+\pi_A\left(x_i\right)\pi_B\left(x_i\right)\right) \quad (4)$$

Then we define the correlation co-efficient between A and B as:

$$K_{ITzFS}\left(A,B\right)=\frac{C_{ITzFS}\left(A,B\right)}{\sqrt{E_{ITzFS}\left(A\right).E_{ITzFS}\left(B\right)}}. \quad (5)$$

1.4. Aggregation Operators for Decision Making

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For a normalized intuitionistic trapezoidal fuzzy decision matrix, $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}=\left(\left[a_{ij},b_{ij},c_{ij},d_{ij}\right];\mu_{ij},\gamma_{ij}\right)_{m\times n}$, where $0\le a_{ij}\le b_{ij}\le c_{ij}\le d_{ij}\le 1$, $0\le \mu_{ij}+\gamma_{ij}\le 1$. The intuitionistic trapezoidal fuzzy positive ideal solution and intuitionistic trapezoidal fuzzy negative ideal solution are defined as follows:

$$\tilde{r}^{+}=\left(\left[a^{+},b^{+},c^{+},d^{+}\right];\mu^{+},\gamma^{+}\right)=\left(\left[1,1,1,1\right];1,0\right),\tilde{r}^{-}=\left(\left[a^{-},b^{-},c^{-},d^{-}\right];\mu^{-},\gamma^{-}\right)=\left(\left[0,0,0,0\right];0,1\right)$$

Definition 1.4.1: Let $\tilde{\alpha}_i\left(i=1,2,...,n\right)$ be a collection of intuitionistic trapezoidal fuzzy number. A intuitionistic trapezoidal fuzzy ordered weighted geometric (ITzFOWG) operator of dimension n is a mapping $ITzFOWG:\Omega^n\rightarrow\Omega$, if

$$ITzFOWG\left(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n\right)=\prod_{i=1}^{n}\tilde{\alpha}_{\sigma(i)}^{\,w_i}$$
$$=\left(\left[\prod_{i=1}^{n}\left(a_{\sigma(i)}\right)^{w_i},\prod_{i=1}^{n}\left(b_{\sigma(i)}\right)^{w_i},\prod_{i=1}^{n}\left(c_{\sigma(i)}\right)^{w_i},\prod_{i=1}^{n}\left(d_{\sigma(i)}\right)^{w_i}\right];\prod_{i=1}^{n}\left(\mu_{\tilde{\alpha}_{\sigma(i)}}\right)^{w_i},1-\prod_{i=1}^{n}\left(1-\gamma_{\tilde{\alpha}_{\sigma(i)}}\right)^{w_i}\right)$$

Where, $\left(\sigma(1),\sigma(2),...,\sigma(n)\right)$ is a permutation of $(1,2,...,n)$ such that $\tilde{\alpha}_{\sigma(i-1)}\ge\tilde{\alpha}_{\sigma(i)}$ for all i, and $w=\left(w_1,w_2,...,w_n\right)^T$ is the weighted vector of $\tilde{\alpha}_i\left(i=1,2,...,n\right)$, $\sum_{i=1}^{n}w_i=1$, $w_i\in\left[0,1\right]$.

Definition 1.4.2: Let $\tilde{\alpha}_i\left(i=1,2,...,n\right)$ be a collection of intuitionistic trapezoidal fuzzy number. A intuitionistic trapezoidal fuzzy hybrid geometric (ITzFHG) operator of dimension n is a mapping $ITzFHG:\Omega^n\rightarrow\Omega$, which has an associated vector $\omega=\left(\omega_1,\omega_2,...,\omega_n\right)^T$ with $\sum_{i=1}^{n}\omega_i=1$, $\omega_i\in\left[0,1\right]$ such that

$$ITrFHG_{\omega,w}\left(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n\right)=\tilde{\beta}_{\sigma(1)}^{\,\omega_1}\otimes\tilde{\beta}_{\sigma(2)}^{\,\omega_2}\otimes...\otimes\tilde{\beta}_{\sigma(n)}^{\,\omega_n}$$
$$=\left(\left[\prod_{i=1}^{n}\left(\ddot{a}_{\sigma(i)}\right)^{\omega_i},\prod_{i=1}^{n}\left(\ddot{b}_{\sigma(i)}\right)^{\omega_i},\prod_{i=1}^{n}\left(\ddot{c}_{\sigma(i)}\right)^{\omega_i},\prod_{i=1}^{n}\left(\ddot{d}_{\sigma(i)}\right)^{\omega_i}\right];\prod_{i=1}^{n}\left(\ddot{\mu}_{\tilde{\beta}_{\sigma(i)}}\right)^{\omega_i},1-\prod_{i=1}^{n}\left(1-\ddot{\gamma}_{\tilde{\beta}_{\sigma(i)}}\right)^{\omega_i}\right)$$

Where, $\tilde{\beta}_{\sigma(i)}$ is the i[th] largest of the weighted intuitionistic trapezoidal fuzzy number $\tilde{\beta}_i\left(\tilde{\beta}_i=\tilde{\alpha}_i^{nw_i},i=1,2,...,n\right)$, $w=\left(w_1,w_2,...,w_n\right)^T$ is weight vector of $\tilde{\alpha}_i\left(i=1,2,...,n\right)$ with $\sum_{i=1}^{n}w_i=1,w_i\in\left[0,1\right]$.

1.5. An Approach to Group Decision Making with Intuitionistic Trapezoidal Fuzzy Information

Let $A=\left\{A_1,A_2,...,A_n\right\}$ be a discrete set of alternatives, and $G=\left\{G_1,G_2,...,G_n\right\}$ be the set of attributes, $w=\left\{w_1,w_2,...,w_n\right\}$ is the weighting vector of the attribute, $G_j\left(j=1,2,...,n\right)$, where $w_j\in\left[0,1\right]\sum_{j=1}^{n}w_j=1$. Let $D=\left\{D_1,D_2,...,D_t\right\}$ be the set of decision makers, $\omega=\left\{\omega_1,\omega_2,...,\omega_n\right\}$ be the weighting vector of decision makers, with $\omega_j \in [0,1]$, $\sum_{j=1}^{n} \omega_j = 1$; Suppose that, $R_k = \left(\tilde{r}_{ij}^{(k)}\right)_{m\times n} = \left(\left[a_{ij}^{(k)}, b_{ij}^{(k)}, c_{ij}^{(k)}, d_{ij}^{(k)}\right]; \mu_{ij}^{(k)}, \gamma_{ij}^{(k)}\right)$, $0 \le a_{ij}^{(k)} \le b_{ij}^{(k)} \le c_{ij}^{(k)} \le d_{ij}^{(k)} \le 1$, $0 \le \mu_{ij}^{(k)} + \gamma_{ij}^{(k)} \le 1$, is the normalized intuitionistic trapezoidal fuzzy decision matrix, where $i = 1, 2, ..., n$; $j = 1, 2, ..., n$; $k = 1, 2, ..., t$. Then the intuitionistic trapezoidal fuzzy positive ideal solution and intuitionistic trapezoidal fuzzy negative ideal solution are defined as follows: $\tilde{r}^+ = \left(\left[a^+, b^+, c^+, d^+\right]; \mu^+, \gamma^+\right) = \left([1,1,1,1]; 1, 0\right), \tilde{r}^- = \left(\left[a^-, b^-, c^-, d^-\right]; \mu^-, \gamma^-\right) = \left([0,0,0,0]; 0, 1\right)$ In the following, the ITzFOWG and ITzFHG operator are applied to MAGDM problem based on intuitionistic trapezoidal fuzzy information.

Step 1: Utilize the decision information given in the intuitionistic trapezoidal fuzzy decision matrix R_k , and the ITzFOWG operator to derive the individual overall preference intuitionistic trapezoidal fuzzy values $\tilde{r}_i^{(k)}$ of the alternatives A_i .

Step 2: Utilize the ITzFHG operator to derive the collective preference intuitionistic trapezoial fuzzy values $\tilde{r}_i, i = 1, 2, ..., m$ of the alternative A_i .

Step 3: To calculate the correlation coefficient using (5) between collective overall values $\tilde{r}_i = \left(\left[a_i, b_i, c_i, d_i\right]; \mu_i, \gamma_i\right)$ and the intuitionistic trapezoidal fuzzy positive ideal solution $\tilde{r}^+$.

$$K_{ITzFS}(A, B) = \frac{C_{ITzFS}(A, B)}{\sqrt{E_{ITzFS}(A).E_{ITzFS}(B)}} .$$

Step 4: Rank all the alternatives $A_i\left(i = 1, 2, ..., m\right)$, and select one in accordance with $K_{ITzFS}(A, B), i = 1, 2, ..., m$. The greater values of $K_{ITzFS}(A, B)$ will be the better alternatives A_i .

1.6. Numerical Illustration

In the following, going to develop an illustrative example of the new approach in a decision making problem. Let us consider an investor wants to invest some money in a game company in order to get high profits. Initially an investor considers five possible alternatives: A_1 is Tencent Company, A_2 is Sony Computer Entertainment Company, A_3 is Activision Blizzard Company, A_4 is Microsoft Company, and A_5 is Apple Company. In order to evaluate these investments, the investor uses a group of experts. This group of experts considers that the key factor is the economic environments of the economy. After careful analysis, they consider four possible attributes; S_1 : Contribution of Organization Performance, S_2 : The Effort to Transform from Current System, S_3 : The Costs of Hardware/Software investment, S_4 : The out-Sourcing Software developer Reliability. The five possible alternatives $(A_1, A_2, A_3, A_4, A_5)$ are to be evaluated using the intuitionistic fuzzy numbers by weighting vector, $\omega = \left(0.3346, 0.2457, 0.3461, 0.0731\right)^T$ under the above four attributes. And the weighting vector, $W = \left(0.3395, 0.3287, 0.2315, 0.1002\right)^T$ under the above five alternatives of the decision makers and construct respectively, the decision matrices (5×4) are

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$$\tilde{R}_1=\begin{bmatrix}([0.1,0.2,0.3,0.4];0.4,0.3) & ([0.2,0.4,0.6,0.8];0.8,0.1) & ([0.1,0.3,0.4,0.5];0.1,0.2) & ([0.3,0.4,0.5,0.7];0.3,0.6)\\ ([0.3,0.4,0.5,0.6];0.1,0.7) & ([0.5,0.6,0.7,0.8];0.3,0.5) & ([0.2,0.4,0.6,0.7];0.2,0.3) & ([0.2,0.4,0.6,0.7];0.2,0.7)\\ ([0.1,0.5,0.7,0.9];0.3,0.6) & ([0.3,0.4,0.8,0.9];0.5,0.2) & ([0.3,0.5,0.8,0.9];0.3,0.4) & ([0.1,0.3,0.7,0.8];0.1,0.4)\\ ([0.2,0.5,0.7,0.8];0.2,0.5) & ([0.1,0.2,0.5,0.7];0.7,0.2) & ([0.4,0.5,0.6,0.7];0.4,0.5) & ([0.2,0.4,0.5,0.9];0.5,0.1)\\ ([0.4,0.5,0.6,0.9];0.1,0.8) & ([0.2,0.5,0.7,0.9];0.3,0.6) & ([0.2,0.3,0.4,0.9];0.5,0.1) & ([0.1,0.3,0.8,0.9];0.1,0.8)\end{bmatrix}$$

$$\tilde{R}_2=\begin{bmatrix}([0.2,0.5,0.8,0.9];0.3,0.5) & ([0.3,0.5,0.8,0.9];0.3,0.6) & ([0.5,0.6,0.7,0.8];0.1,0.2) & ([0.2,0.4,0.6,0.7];0.1,0.7)\\ ([0.4,0.6,0.7,0.8];0.1,0.4) & ([0.2,0.5,0.7,0.8];0.3,0.4) & ([0.4,0.5,0.6,0.7];0.2,0.3) & ([0.3,0.5,0.7,0.9];0.2,0.6)\\ ([0.3,0.4,0.6,0.9];0.2,0.6) & ([0.5,0.6,0.7,0.8];0.2,0.5) & ([0.3,0.4,0.5,0.6];0.3,0.4) & ([0.1,0.4,0.6,0.8];0.3,0.2)\\ ([0.1,0.2,0.5,0.7];0.4,0.2) & ([0.1,0.3,0.4,0.6];0.7,0.2) & ([0.2,0.3,0.4,0.5];0.4,0.5) & ([0.2,0.7,0.8,0.9];0.4,0.3)\\ ([0.5,0.6,0.7,0.9];0.8,0.1) & ([0.2,0.3,0.4,0.9];0.1,0.2) & ([0.1,0.2,0.3,0.4];0.7,0.2) & ([0.3,0.5,0.6,0.7];0.8,0.1)\end{bmatrix}$$

$$\tilde{R}_3=\begin{bmatrix}([0.1,0.3,0.4,0.6];0.1,0.3) & ([0.2,0.3,0.4,0.6];0.7,0.2) & ([0.3,0.4,0.5,0.7];0.5,0.4) & ([0.1,0.4,0.8,0.9];0.2,0.7)\\ ([0.4,0.5,0.7,0.9];0.3,0.5) & ([0.3,0.4,0.8,0.9];0.3,0.6) & ([0.1,0.2,0.3,0.4];0.2,0.3) & ([0.2,0.3,0.6,0.7];0.1,0.2)\\ ([0.3,0.7,0.8,0.9];0.2,0.4) & ([0.1,0.5,0.7,0.8];0.5,0.4) & ([0.3,0.4,0.5,0.6];0.5,0.3) & ([0.3,0.4,0.6,0.7];0.4,0.3)\\ ([0.1,0.2,0.3,0.4];0.4,0.5) & ([0.3,0.5,0.6,0.8];0.1,0.5) & ([0.4,0.5,0.6,0.7];0.4,0.2) & ([0.6,0.7,0.8,0.9];0.5,0.2)\\ ([0.2,0.4,0.6,0.8];0.7,0.2) & ([0.4,0.5,0.6,0.7];0.4,0.3) & ([0.1,0.2,0.3,0.4];0.3,0.4) & ([0.4,0.5,0.6,0.8];0.1,0.4)\end{bmatrix}$$

$$\tilde{R}_4=\begin{bmatrix}([0.1,0.3,0.7,0.8];0.1,0.2) & ([0.3,0.5,0.7,0.9];0.4,0.3) & ([0.6,0.7,0.8,0.9];0.4,0.3) & ([0.1,0.5,0.7,0.9];0.3,0.6)\\ ([0.2,0.4,0.5,0.9];0.1,0.6) & ([0.4,0.5,0.6,0.7];0.3,0.4) & ([0.1,0.2,0.3,0.4];0.1,0.5) & ([0.4,0.6,0.7,0.9];0.6,0.3)\\ ([0.6,0.7,0.8,0.9];0.3,0.6) & ([0.1,0.2,0.3,0.4];0.1,0.5) & ([0.2,0.3,0.5,0.6];0.3,0.4) & ([0.5,0.6,0.7,0.9];0.4,0.5)\\ ([0.1,0.2,0.3,0.4];0.7,0.1) & ([0.3,0.4,0.5,0.6];0.3,0.6) & ([0.1,0.3,0.7,0.9];0.3,0.6) & ([0.1,0.2,0.3,0.5];0.1,0.2)\\ ([0.3,0.4,0.5,0.6];0.6,0.3) & ([0.2,0.3,0.4,0.5];0.4,0.3) & ([0.5,0.6,0.7,0.8];0.1,0.5) & ([0.2,0.3,0.5,0.6];0.2,0.3)\end{bmatrix}$$

Step 1:

$$ITzFOWG(\tilde{\alpha}_1,\tilde{\alpha}_2,...,\tilde{\alpha}_n)=\prod_{i=1}^{n}\tilde{\alpha}_{\sigma(i)}{}^{w_i}$$

$$=\left(\left[\prod_{i=1}^{n}\left(a_{\sigma(i)}\right)^{w_i},\prod_{i=1}^{n}\left(b_{\sigma(i)}\right)^{w_i},\prod_{i=1}^{n}\left(c_{\sigma(i)}\right)^{w_i},\prod_{i=1}^{n}\left(d_{\sigma(i)}\right)^{w_i}\right];\prod_{i=1}^{n}\left(\mu_{\tilde{\alpha}_{\sigma(i)}}\right)^{w_i},1-\prod_{i=1}^{n}\left(1-\gamma_{\tilde{\alpha}_{\sigma(i)}}\right)^{w_i}\right)$$

$$r_1^{(1)}=\left(\begin{array}{l}\left[\begin{array}{l}(0.1)^{0.4}\times(0.2)^{0.3}\times(0.1)^{0.2}\times(0.3)^{0.1},(0.2)^{0.4}\times(0.4)^{0.3}\times(0.3)^{0.2}\times(0.4)^{0.1},\\ (0.3)^{0.4}\times(0.6)^{0.3}\times(0.4)^{0.2}\times(0.5)^{0.1},(0.4)^{0.4}\times(0.8)^{0.3}\times(0.5)^{0.2}\times(0.7)^{0.1}\end{array}\right];\\ (0.4)^{0.4}\times(0.8)^{0.3}\times(0.1)^{0.2}\times(0.3)^{0.1},1-\left[(1-0.3)^{0.4}\times(1-0.1)^{0.3}\times(1-0.2)^{0.2}\times(1-0.6)^{0.1}\right]\end{array}\right)$$

$$r_1^{(1)}=([0.1374,0.2862,0.4117,0.5446];0.3626,0.2669)$$

Similarly,

$r_2^{(1)} = ([0.3096, 0.4517, 0.5842, 0.6850]; 0.1712, 0.5857)$

$r_3^{(1)} = ([0.1732, 0.4443, 0.7483, 0.8895]; 0.3133, 0.4438)$

$r_4^{(1)} = ([0.1866, 0.3714, 0.5933, 0.7572]; 0.3666, 0.3894)$

$r_3^{(1)} = ([0.1732, 0.4443, 0.7483, 0.8895]; 0.3133, 0.4438)$

$r_4^{(1)} = ([0.1866, 0.3714, 0.5933, 0.7572]; 0.3666, 0.3894)$

$r_5^{(1)} = ([0.2462, 0.4289, 0.5964, 0.9000]; 0.2164, 0.6674)$

$r_1^{(2)} = ([0.2713, 0.5071, 0.7568, 0.8572]; 0.2158, 0.5119)$

$r_2^{(2)} = ([0.3157, 0.5378, 0.6787, 0.7881]; 0.1712, 0.4058)$

$r_3^{(2)} = ([0.3133, 0.4517, 0.6059, 0.7917]; 0.2259, 0.5029)$

$r_4^{(2)} = ([0.1231, 0.2449, 0.4687, 0.6408]; 0.4731, 0.2814)$

$r_5^{(2)} = ([0.2616, 0.3841, 0.4919, 0.7463]; 0.4174, 0.1515)$

$r_1^{(3)} = ([0.1534, 0.3270, 0.4483, 0.6444]; 0.2651, 0.3509)$

$r_2^{(3)} = ([0.2595, 0.3699, 0.6056, 0.7463]; 0.2479, 0.4758)$

$r_3^{(3)} = ([0.1913, 0.5350, 0.6528, 0.7812]; 0.3389, 0.3716)$

$r_4^{(3)} = ([0.2195, 0.3584, 0.4680, 0.5973]; 0.2699, 0.4243)$

$r_5^{(3)} = ([0.2297, 0.3807, 0.5223, 0.6691]; 0.4112, 0.2950)$

$r_1^{(4)} = ([0.1990, 0.4360, 0.7189, 0.8586]; 0.2232, 0.3018)$

$r_2^{(4)} = ([0.2297, 0.3877, 0.4931, 0.7097]; 0.1663, 0.5005)$

$r_3^{(4)} = ([0.2763, 0.3996, 0.5354, 0.6507]; 0.2221, 0.5257)$

$r_4^{(4)} = ([0.1390, 0.2670, 0.4143, 0.5433]; 0.3772, 0.4070)$

$r_5^{(4)} = ([0.2825, 0.3866, 0.5002, 0.6017]; 0.3326, 0.3456)$

Step 2: Utilize the ITzFHG operator to derive the collective overall preference intuitionistic trapezoidal fuzzy values. Consider

$W = (0.3395, 0.3287, 0.2315, 0.1002)^T$

$r_1^{(1)} = ([0.1374, 0.2862, 0.4117, 0.5446]; 0.3626, 0.2669)$

$r_1^{(2)} = ([0.2713, 0.5071, 0.7568, 0.8572]; 0.2158, 0.5119)$

$r_1^{(3)} = ([0.1534, 0.3270, 0.4483, 0.6444]; 0.2651, 0.3509)$

$r_1^{(4)} = ([0.1990, 0.4360, 0.7189, 0.8586]; 0.2232, 0.3018)$

$a_1^{(1)} = a_1^{n\times w_1} = (0.1374)^{4\times 0.3395} = 0.0675, a_2^{(1)} = a_2^{n\times w_2} = 0.1799, a_3^{(1)} = a_3^{n\times w_3} = 0.1762,$

$$a_4^{(1)} = a_4^{n\times w_4} = 0.5236, b_1^{(1)} = b_1^{n\times w_1} = 0.1829, b_2^{(1)} = b_2^{n\times w_2} = 0.4095,$$
$$b_3^{(1)} = b_3^{n\times w_3} = 0.3552, b_4^{(1)} = b_4^{n\times w_4} = 0.7170, c_1^{(1)} = c_1^{n\times w_1} = 0.2996,$$
$$c_2^{(1)} = c_2^{n\times w_2} = 0.6932, c_3^{(1)} = c_3^{n\times w_3} = 0.4757, c_4^{(1)} = c_4^{n\times w_4} = 0.8761,$$
$$d_1^{(1)} = d_1^{n\times w_1} = 0.4381, d_2^{(1)} = d_2^{n\times w_2} = 0.8166, d_3^{(1)} = d_3^{n\times w_3} = 0.6657,$$
$$d_4^{(1)} = d_4^{n\times w_4} = 0.9407, \mu_1^{(1)} = \mu_1^{n\times w_1} = 0.2522, \mu_2^{(1)} = \mu_2^{n\times w_2} = 0.1332,$$
$$\mu_3^{(1)} = \mu_3^{n\times w_3} = 0.2925, \mu_4^{(1)} = \mu_4^{n\times w_4} = 0.5482, \gamma_1^{(1)} = \gamma_1^{n\times w_1} = 0.1663,$$
$$\gamma_2^{(1)} = \gamma_2^{n\times w_2} = 0.4146, \gamma_3^{(1)} = \gamma_3^{n\times w_3} = 0.3792, \gamma_4^{(1)} = \gamma_4^{n\times w_4} = 0.6187$$

Using ITzFHG operator we get,

$$\omega = \left(0.3346, 0.2457, 0.3461, 0.0731\right)$$

$$ITzFHG\left(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n\right) = \left(\begin{array}{l}\left[\prod_{i=1}^{n}\left(\ddot{a}_{\sigma(i)}\right)^{\omega_i}, \prod_{i=1}^{n}\left(\dddot{b}_{\sigma(i)}\right)^{\omega_i}, \prod_{i=1}^{n}\left(\ddot{c}_{\sigma(i)}\right)^{\omega_i}, \prod_{i=1}^{n}\left(\dddot{d}_{\sigma(i)}\right)^{\omega_i}\right]; \\ \prod_{i=1}^{n}\left(\dddot{\mu}_{\tilde{\beta}_{\sigma(i)}}\right)^{\omega_i}, 1-\prod_{i=1}^{n}\left(1-\dddot{\gamma}_{\tilde{\beta}_{\sigma(i)}}\right)^{\omega_i}\end{array}\right)$$

$$\tilde{r}_1 = \left(\begin{array}{l}\left[\begin{array}{l}(0.0675)^{0.3346}\times(0.1799)^{0.2459}\times(0.1762)^{0.3461}\times(0.5236)^{0.0731}, \\ (0.1829)^{0.3346}\times(0.4095)^{0.2459}\times(0.3552)^{0.3461}\times(0.7170)^{0.0731}, \\ (0.2996)^{0.3346}\times(0.6932)^{0.2459}\times(0.4757)^{0.3461}\times(0.8761)^{0.0731}, \\ (0.4381)^{0.3346}\times(0.8166)^{0.2459}\times(0.6657)^{0.3461}\times(0.9407)^{0.0731}\end{array}\right]; (0.2522)^{0.3346}\times(0.1332)^{0.2459} \\ \times(0.2925)^{0.3461}\times(0.5482)^{0.0731}, 1-\left[\begin{array}{l}(1-0.1663)^{0.3346}\times(1-0.4146)^{0.2459} \\ \times(1-0.3792)^{0.3461}\times(1-0.6187)^{0.0731}\end{array}\right]\end{array}\right)$$

$$\tilde{r}_1 = \left(\left[0.1392, 0.3102, 0.4676, 0.6242\right]; 0.2403, 0.3482\right)$$

Similarly,

$$\tilde{r}_2 = \left(\left[0.2514, 0.4032, 0.5764, 0.7028\right]; 0.2889, 0.4811\right)$$
$$\tilde{r}_3 = \left(\left[0.1757, 0.4262, 0.6385, 0.7916\right]; 0.2469, 0.4216\right)$$
$$\tilde{r}_4 = \left(\left[0.1376, 0.2749, 0.4658, 0.6356\right]; 0.3179, 0.3674\right)$$
$$\tilde{r}_5 = \left(\left[0.2063, 0.3566, 0.5003, 0.7513\right]; 0.2740, 0.4567\right)$$

Step 3: To calculate the correlation coefficient values,

$$C_{ITzFS}\left(\tilde{r}_1, \tilde{r}^+\right) = 0.0929, C_{ITzFS}\left(\tilde{r}_2, \tilde{r}^+\right) = 0.1403, C_{ITzFS}\left(\tilde{r}_3, \tilde{r}^+\right) = 0.1274,$$
$$C_{ITzFS}\left(\tilde{r}_4, \tilde{r}^+\right) = 0.1195, C_{ITzFS}\left(\tilde{r}_5, \tilde{r}^+\right) = 0.1220$$

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$K_{ITzFS}\left(\tilde{r}_1,\tilde{r}^+\right)=0.4075, K_{ITzFS}\left(\tilde{r}_2,\tilde{r}^+\right)=0.4766, K_{ITzFS}\left(\tilde{r}_3,\tilde{r}^+\right)=0.4180,$

$K_{ITzFS}\left(\tilde{r}_4,\tilde{r}^+\right)=0.5494, K_{ITzFS}\left(\tilde{r}_5,\tilde{r}^+\right)=0.4592$.

Step 4: Rank the alternatives $A_i\left(i=1,2,3,4,5\right)$ based on the values of $K_{ITzFS}\left(A,B\right)$. Then we have: $A_4 > A_2 > A_5 > A_3 > A_1$.Hence, the best alternative is A_4.

1.7. Conclusion

In this paper, the decision making is proposed with respect to MAGDM problems, where both the attribute weights and the expert weights in the form of real numbers and the attribute values take the form of a intuitionistic trapezoidal fuzzy numbers (ITzFNs). Some arithmetic operations of ITzFNs were presented and different operators, namely ITzFWG operator, ITzFOWG operator and ITzFHG operator were also utilized. Aggregates all the weighted arguments into a collective data. The correlation co-efficient and distance method for ITzFNs was introduced to choose the best alternative out of many. Finally, illustrative example is given to developed and validate the effectiveness of our proposed methods.

1.8. References

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