International Journal of Future Generation Communication and Networking
Vol. 14, No. 1, (2021), pp. 1511-1519

On Indexed Topology

R.Geetha Jeyalakshmi[1] and K. Dass [2]

[1]Research Scholar (Regn.No. 18221072092008), [2]Assistant Professor

PG & Research Department of Mathematics, The Madurai Diraviyam Thayumanavar Hindu College, Affiliated to Manonmaniam Sundaranar University, Tirunelveli -627010,Tamilnadu, India.

Abstract

A structure on a non empty set X is a collection of subsets of X**.** Recently the authors introduced and studied hyper relations, micro relations on structures, structure union and structure intersection. This study reveals that these relations and operations do not helpful to introduce a topology of structures. Therefore quite recently the authors introduced the notions of indexed structures, indexed relations and indexed operations. The purpose of this paper is to introduce a topological structure known as indexed topology on indexed structures and to extend the recent concepts in general topology to this indexed topology.

Key words: Index set, indexed structure, indexed operations, indexed topology.

A structure on a non empty set X is a collection of subsets of X**.** The authors [2, 3] introduced and studied hyper relations, micro relations on structures, structure union and structure intersection operators. From the investigations of these operators on structures we infer that even though the operators have several interesting properties of structures they can not be used to define a topology on structures. Recently the authors[4] introduced the concept of indexed structure and studied the indexed relations and operations so as to define a topology on indexed structure. The purpose of this paper is to introduce a topological structure known as indexed topology on indexed structures and to extend the recent concepts in general topology to this indexed topology.

1.Preliminaries , Notations and Terminologies

Throughout this paper X is a non empty set and κ is an index set. By a structure on X we mean a collection of subsets of X. The letter P, Q, R, S, Ω denote the structures on X.

Notations 1.1:

(i) $P_\kappa = [A_j: j\in\kappa]$ is an indexed structure on X where A_j is a subset of X.
(ii) $Q_\kappa = [B_j: j\in \kappa]$.
(iii) $[\varnothing]_\kappa = [\varnothing_j: j\in\kappa]$ where $\varnothing_j =\varnothing$.
(iv) $[X]_\kappa = [X_j: j\in\kappa]$ where $X_j = X$.
(v) $[A]_\kappa = [A_j: j\in\kappa]$ where $A_j = A$.

Whenever we say that P is a structure over (X, κ) we mean that P is an indexed structure on X with index set κ.

Remark 1.2:

In an indexed structure the repetitions are allowed where as the repetitions are not allowed in an ordinary structure of sets.

Definition 1.3: [4]

ISSN: 2233-7857IJFGCN

(i) P is an indexed hyper substructure of Q, $P ⧀ Q$ if for all $j\in\kappa$, $A_j\subseteq B_j$.
(ii) P is an indexed hyper superstructure of Q, $P ⧁ Q$ if for all $j\in\kappa$, $A_j\supseteq B_j$.
(iii) $P \boxed{\wedge} Q=[A_j\cap B_j:\ j\in\kappa]$
(iv)$P \boxed{\vee} Q =[A_j\cup B_j:\ j\in\kappa]$.
(v)$P \boxed{\setminus} Q = [A_j\setminus B_j: j\in\kappa]$ and $P \triangle Q = [A_j\Delta B_j: j\in\kappa]$.

The basic properties of the indexed relations, indexed operations and indexed difference operators are found in [4].

2. Indexed topology

Definition 2.1:
Let $\mathcal{G}$ be a collection of indexed structures on X with index set κ. $\mathcal{G}$ is said to be a $(\kappa,\mathcal{G})$-topology of indexed structures over (X, κ) if the following three conditions hold.

(i) $[\varnothing]_\kappa \in \mathcal{G}$ and $[X]_\kappa \in \mathcal{G}$.
(ii) If Ω_1 and Ω_2 are in $\mathcal{G}$ then $\Omega_1 \boxed{\wedge} \Omega_2$ lies in $\mathcal{G}$.
(iii) $\{\Omega_\alpha: \alpha\in\Delta\}\subseteq \mathcal{G}$ then $\boxed{\vee}\{\Omega_\alpha: \alpha\in\Delta\}$ lies in $\mathcal{G}$.

If $\mathcal{G}$ is a $(\kappa, \mathcal{G})$-topology on X then the ordered triplet $(X, \kappa, \mathcal{G})$ is a $(\kappa, \mathcal{G})$-topological space and the members of $\mathcal{G}$ are called $(\kappa, \mathcal{G})$-open structures.

Example 2.2:
$X = \{a,b,c\}$ and $\kappa = \{x,y\}$. Let $P = [A_x, A_y]$ where $A_x = \{a\}$, $A_y= \{b\}$ and $Q =[B_x, B_y]$ where $B_x = \{a\}$, $B_y= \{b,c\}$. If $\mathcal{G} = \{[\varnothing]_\kappa , P, Q, [X]_\kappa \}$ then $\mathcal{G}$ is a $(\kappa,\mathcal{G})$-topology on X and $(X, \kappa, \mathcal{G})$ is a $(\kappa, \mathcal{G})$-topological space. The indexed structures $[\varnothing]_\kappa$, P, Q, $[X]_\kappa$ are $(\kappa, \mathcal{G})$-open structures.

Proposition 2.3:
Let $(X, \kappa, \mathcal{G})$ be a $(\kappa, \mathcal{G})$-topological space. For each $j\in\kappa$, let $\mathcal{G}_j = \{A_j: A_j\in P$ for some $P\in \mathcal{G} \}$. Then $\mathcal{G}_j$ is a topology on X.
Proof:
Since $[\varnothing]_\kappa \in \mathcal{G}$ and $[X]_\kappa \in \mathcal{G}$, it follows that $\varnothing = \varnothing_j\in \mathcal{G}_j$ and $X = X_j\in \mathcal{G}_j$.
Let $A_j\in \mathcal{G}_j$ and $B_j\in \mathcal{G}_j$. Then there are Ω_1 and Ω_2 in $\mathcal{G}$ with $A_j\in\Omega_1$ and $B_j\in\Omega_2$. This implies $A_j\cap B_j\in\Omega_1\boxed{\wedge}\Omega_2\in\mathcal{G}$ so that $A_j\cap B_j\in \mathcal{G}_j$.
Suppose $\{A_{\alpha j}: \alpha\in\Delta\}\subseteq \mathcal{G}_j$. Then for each $\alpha\in\Delta$, there is an indexed structure $\Omega_\alpha\in \mathcal{G}$ with $A_{\alpha j}\in \mathcal{G}_j$. Now $\cup\{A_{\alpha j}: \alpha\in\Delta\}\in \boxed{\vee}\{\Omega_\alpha: \alpha\in\Delta\} \in \mathcal{G}$. Therefore $\mathcal{G}_j$ is a topology on X.

Remark 2.4:
Every $(\kappa,\mathcal{G})$-topology $\mathcal{G}$ over (X, κ) determines a collection of topologies on X. This collection $\{ \mathcal{G}_j : j \in\kappa\}$ of topologies on X is said to be the indexed family of topologies on X induced by a $(\kappa,\mathcal{G})$-topology $\mathcal{G}$ over (X, κ).

Example 2.5:
$X = \{a,b,c\}$ and $\kappa = \{1,2\}$. Let $P = [A_1, A_2]$ where $A_1 = \{a\}$, $A_2= \{b\}$ and $Q =[B_1, B_2]$ where $B_1 =\{a\}$, $B_2= \{b,c\}$. Then $\mathcal{G} =\{[\varnothing]_\kappa , P, Q, [X]_\kappa \}$is a $(\kappa,\mathcal{G})$-topology on X and $(X, \kappa,\mathcal{G})$ is a $(\kappa, \mathcal{G})$-topological space.

ISSN: 2233-7857IJFGCN

Clearly $\mathcal{G}_1$ = {∅ , {a}, X} and $\mathcal{G}_1$ = {∅ , {b}, {b,c}, X} are topologies on X and { $\mathcal{G}_1$, $\mathcal{G}_2$ } is the indexed family of topologies induced by the (κ,$\mathcal{G}$)-topology $\mathcal{G}$ = {$[\varnothing]_\kappa$, P, Q, $[X]_\kappa$ }.

Definition 2.6:

Ω is (κ, $\mathcal{G}$)-closed in (X, κ, $\mathcal{G}$) if [X] $\boxed{\setminus}$Ω is (κ, $\mathcal{G}$)-open in (X, κ, $\mathcal{G}$) .

Proposition 2.7:

(i) $[\varnothing]_\kappa$ and $[X]_\kappa$ are (κ, $\mathcal{G}$)- closed in (X, κ, $\mathcal{G}$) .

(ii) The family of (X, κ, $\mathcal{G}$)-closed structures in (X, κ, $\mathcal{G}$) is closed under finite indexed union and arbitrary indexed intersection.

Proof:

Since $[X]_\kappa \boxed{\setminus} [\varnothing]_\kappa = [X]_\kappa$ and since $[X]_\kappa \boxed{\setminus} [X]_\kappa = [\varnothing]_\kappa$ it follows that $[\varnothing]_\kappa$ and $[X]_\kappa$ are (κ,$\mathcal{G}$)- closed in (X, κ, $\mathcal{G}$) that proves (i).

If P and Q are (κ, $\mathcal{G}$)- closed in (X, κ, $\mathcal{G}$) then $[X]_\kappa \boxed{\setminus}$P and $[X]_\kappa \boxed{\setminus}$Q are (κ, $\mathcal{G}$)- open in (X, κ, $\mathcal{G}$) that implies($[X]_\kappa \boxed{\setminus}$P) $\boxed{\wedge}$ ($[X]_\kappa \boxed{\setminus}$Q) is (κ, $\mathcal{G}$)- open that is $[X]_\kappa \boxed{\setminus}$(P $\boxed{\vee}$Q) is (κ, $\mathcal{G}$)- open so that P $\boxed{\vee}$Q is (κ, $\mathcal{G}$)- closed.

Suppose {Ω_α: α∈Δ} is a family of (κ, $\mathcal{G}$)- closed structures in (X, κ, $\mathcal{G}$). Then for each α∈Δ, $[X]_\kappa \boxed{\setminus} \Omega_\alpha$ is (κ, $\mathcal{G}$)- open.

$\boxed{\vee}${$[X]_\kappa \boxed{\setminus} \Omega_\alpha$: α∈Δ} is (κ, $\mathcal{G}$)- open that is $[X]_\kappa \boxed{\setminus}$($\boxed{\wedge}$ {Ω_α: α∈Δ}) is (κ, $\mathcal{G}$)- open so that $\boxed{\wedge}$ {Ω_α: α∈Δ} is (κ, $\mathcal{G}$)- closed in (X, κ, $\mathcal{G}$) .

Definition 2.8:

(κ, $\mathcal{G}$)-*Int*Ω = $\boxed{\vee}${P: P is (κ, $\mathcal{G}$)-open and P ⧀ Ω } and

(κ, $\mathcal{G}$)-*Cl*Ω = $\boxed{\wedge}${Q: Q is (κ, $\mathcal{G}$)-closed and Q ⧁ Ω }.

Proposition 2.9: Let Ω, Ω_1 and Ω_2 be the indexed structures over (X, κ) . Then

(i) (κ, $\mathcal{G}$)-*Int*$[\varnothing]_\kappa = [\varnothing]_\kappa$ and (κ, $\mathcal{G}$)-*Cl* $[\varnothing]_\kappa = [\varnothing]_\kappa$.

(ii) (κ, $\mathcal{G}$)-*Int*$[X]_\kappa = [X]_\kappa$ and (κ, $\mathcal{G}$)-*Cl* $[X]_\kappa = [X]_\kappa$.

(iii) (κ, $\mathcal{G}$)-*Int*Ω is (κ, $\mathcal{G}$)- open

(iv) (κ, $\mathcal{G}$)-*Cl*Ω is (κ, $\mathcal{G}$)- closed

(v) (κ, $\mathcal{G}$)-*Int*Ω ⧀ Ω ⧀ (κ, $\mathcal{G}$)-*Cl*Ω

(vi) Ω1 ⧀ Ω2 ⇒ (κ, $\mathcal{G}$)-*Int*Ω_1 ⧀(κ, $\mathcal{G}$)- *Int*Ω_2 and (κ, $\mathcal{G}$)- *Cl*Ω_1 ⧀ (κ, $\mathcal{G}$)-*Cl*Ω_2

(vii) Ω is (κ, $\mathcal{G}$)- open iff (κ, $\mathcal{G}$)-*Int*Ω = Ω.

(viii) Ω is (κ, $\mathcal{G}$)- closed iff (κ, $\mathcal{G}$)-*Cl*Ω = Ω

Proof:

The assertions (i) and (ii) are obviously true. If Ω is $[\varnothing]_\kappa$ or $[X]_\kappa$ then from (i) and (ii) it follows that (κ, $\mathcal{G}$)-*Int*Ω and (κ, $\mathcal{G}$)-*Cl*Ω are (κ, $\mathcal{G}$)-open and (κ, $\mathcal{G}$)- closed in (X, κ, $\mathcal{G}$) respectively.

ISSN: 2233-7857IJFGCN

Since the arbitrary indexed union of $(\kappa, \mathcal{G})$-open structures is $(\kappa, \mathcal{G})$-open and since the arbitrary indexed intersection of $(\kappa, \mathcal{G})$-closed structures is $(\kappa, \mathcal{G})$-closed it is clear that $(\kappa, \mathcal{G})$-$Int\Omega$ and $(\kappa, \mathcal{G})$-$Cl\Omega$ are $(\kappa, \mathcal{G})$-open and $(\kappa, \mathcal{G})$- closed in $(X, \kappa, \mathcal{G})$ respectively. This proves (iii) and (iv).

$(\kappa, \mathcal{G})$-$Int\Omega \olessthan \Omega \olessthan (\kappa, \mathcal{G})$-$Cl\Omega$ is obviously true when Ω is $[\emptyset]_\kappa$ or $[X]_\kappa$. Suppose Ω is neither $[\emptyset]_\kappa$ nor $[X]_\kappa$.

$(\kappa, \mathcal{G})$-$Int\Omega = \boxed{\vee}\{P: P$ is $(\kappa, \mathcal{G})$- open and $P \olessthan \Omega\} \olessthan \boxed{\vee}\{\Omega: P$ is $(\kappa, \mathcal{G})$- open and $P \olessthan \Omega\} = \Omega$

$(\kappa, \mathcal{G})$-$Cl\Omega = \boxed{\wedge}\{Q: Q$ is $(\kappa, \mathcal{G})$-closed and $Q \ogreaterthan \Omega\} \ogreaterthan \boxed{\wedge}\{\Omega: Q$ is $(\kappa, \mathcal{G})$-closed and $Q \ogreaterthan \Omega\} = \Omega$.

This proves (v).

Suppose $\Omega_1 \olessthan \Omega_2$.

$(\kappa, \mathcal{G})$-$Int\Omega_1 = \boxed{\vee}\{P: P$ is $(\kappa, \mathcal{G})$- open and $P \olessthan \Omega_1\} \olessthan \boxed{\vee}\{P: P$ is $(\kappa, \mathcal{G})$- open and $P \olessthan \Omega_2\} = (\kappa, \mathcal{G})$-$Int\Omega_2$.

$(\kappa, \mathcal{G})$-$Cl\Omega_2 = \boxed{\wedge}\{Q:Q$ is $(\kappa, \mathcal{G})$-closed and $Q \ogreaterthan \Omega_2\} \ogreaterthan \boxed{\wedge}\{Q:Q$ is $(\kappa, \mathcal{G})$-closed and $Q \ogreaterthan \Omega_1\} = (\kappa, \mathcal{G})$-$Cl\Omega_1$.

This proves (vi).

$(\kappa, \mathcal{G})$-$Int\Omega = \Omega. \Rightarrow \Omega$ is $(\kappa, \mathcal{G})$-open . Conversely
Ω is $(\kappa, \mathcal{G})$-open $\Rightarrow (\kappa, \mathcal{G})$-$Int\Omega = \boxed{\vee}\{P: P$ is $(\kappa, \mathcal{G})$-open and $P \olessthan \Omega\} \ogreaterthan \Omega$ that implies
$(\kappa, \mathcal{G})$-$Int\Omega = \Omega$..

$(\kappa, \mathcal{G})$-$Cl\Omega = \Omega. \Rightarrow \Omega$ is $(\kappa, \mathcal{G})$-closed . Conversely
Ω is $(\kappa, \mathcal{G})$-closed $\Rightarrow$ m-$Cl\Omega = \boxed{\wedge}\{Q: Q$ is $(\kappa, \mathcal{G})$- closed and $Q \ogreaterthan \Omega\} \olessthan \Omega$ that implies
$(\kappa, \mathcal{G})$-$Cl\Omega = \Omega$. This proves (vii) and (viii).

Proposition 2.10:

Let Ω_1 and Ω_2 be the indexed structures on X. Then

(i) $(\kappa, \mathcal{G})$-$Int\Omega_1 \boxed{\vee} (\kappa, \mathcal{G})$-$Int\Omega_2 \olessthan (\kappa, \mathcal{G})$-$Int\,(\Omega_1 \boxed{\vee} \Omega_2)$

(ii) $(\kappa, \mathcal{G})$-$Int\Omega_1 \boxed{\wedge} (\kappa, \mathcal{G})$-$Int\Omega_2 = (\kappa, \mathcal{G})$-$Int\,(\Omega_1 \boxed{\wedge} \Omega_2)$

(iii) $(\kappa, \mathcal{G})$-$Cl\,\Omega_1 \boxed{\vee} (\kappa, \mathcal{G})$-$Cl\,\Omega_2 = (\kappa, \mathcal{G})$-$Cl\,(\Omega_1 \boxed{\vee} \Omega_2)$

(iv) $(\kappa, \mathcal{G})$-$Cl\,\Omega_1 \boxed{\wedge} (\kappa, \mathcal{G})$-$Cl\,\Omega_2 \ogreaterthan (\kappa, \mathcal{G})$- $Cl\,(\Omega_1 \boxed{\wedge} \Omega_2)$

Proof:

$(\kappa, \mathcal{G})$-$Int\Omega_1 \olessthan \Omega_1$ and $(\kappa, \mathcal{G})$-$Int\Omega_2 \olessthan \Omega_2 \Rightarrow (\kappa, \mathcal{G})$-$Int\Omega_1 \boxed{\vee} (\kappa, \mathcal{G})$-$Int\Omega_2 \olessthan (\Omega_1 \boxed{\vee} \Omega_2)$
$\Rightarrow (\kappa, \mathcal{G})$-$Int\,((\kappa, \mathcal{G})$-$Int\Omega_1 \boxed{\vee} (\kappa, \mathcal{G})$-$Int\Omega_2) \olessthan (\kappa, \mathcal{G})\ Int\,(\Omega_1 \boxed{\vee} \Omega_2)$.

Since $(\kappa, \mathcal{G})$-$Int\Omega_1 \boxed{\vee} (\kappa, \mathcal{G})$- $Int\Omega_2$ is $(\kappa, \mathcal{G})$-open, from the above we have
$\Rightarrow (\kappa, \mathcal{G})$-$Int\Omega_1 \boxed{\vee} (\kappa, \mathcal{G})$-$Int\Omega_2 \olessthan (\kappa, \mathcal{G})$-$Int\,(\Omega_1 \boxed{\vee} \Omega_2)$. This proves (i).

Applying the same technique we also have $(\kappa, \mathcal{G})$-$Int\Omega_1 \boxed{\wedge} (\kappa, \mathcal{G})$-$Int\Omega_2 \olessthan (\kappa, \mathcal{G})$-$Int\,(\Omega_1 \boxed{\wedge} \Omega_2)$. $\Omega_1 \boxed{\wedge} \Omega_2 \olessthan \Omega_1$ and $\Omega_1 \boxed{\wedge} \Omega_2 \olessthan \Omega_2 \Rightarrow (\kappa,\mathcal{G})$-$Int(\Omega_1 \boxed{\wedge} \Omega_2) \olessthan (\kappa,\mathcal{G})$-$Int\Omega_1$ and
$(\kappa,\mathcal{G})$-$Int(\Omega_1 \boxed{\wedge} \Omega_2) \olessthan (\kappa,\mathcal{G})$-$Int\Omega_2$
$\Rightarrow (\kappa, \mathcal{G})$-$Int\,(\Omega_1 \boxed{\wedge} \Omega_2) \olessthan (\kappa, \mathcal{G})$-$Int\Omega_1 \boxed{\wedge} (\kappa, \mathcal{G})$-$Int\Omega_2$.

$(\kappa,\mathcal{G})$-$Int\Omega_1 \boxed{\wedge} (\kappa,\mathcal{G})$-$Int\Omega_2 \olessthan (\kappa,\mathcal{G})$-$Int(\Omega_1 \boxed{\wedge} \Omega_2)$ and
$(\kappa,\mathcal{G})$-$Int(\Omega_1 \boxed{\wedge} \Omega_2) \olessthan (\kappa,\mathcal{G})$-$Int\Omega_1 \boxed{\wedge} (\kappa,\mathcal{G})$-$Int\Omega_2$ which implies
$(\kappa, \mathcal{G})$-$Int\Omega_1 \boxed{\wedge} (\kappa, \mathcal{G})$-$Int\Omega_2 = (\kappa, \mathcal{G})$-$Int\,(\Omega_1 \boxed{\wedge} \Omega_2)$. This proves (ii).

$(\kappa, \mathcal{G})$-$Cl\Omega_1 \ogreaterthan \Omega_1$ and $(\kappa, \mathcal{G})$-$Cl\Omega_2 \ogreaterthan \Omega_2 \Rightarrow (\kappa, \mathcal{G})$- $Cl\Omega_1 \boxed{\vee} (\kappa, \mathcal{G})$-$Cl\Omega_2 \ogreaterthan (\Omega_1 \boxed{\vee} \Omega_2)$
$\Rightarrow (\kappa, \mathcal{G})$-$Cl\,(Cl\Omega_1 \boxed{\vee} (\kappa, \mathcal{G})$-$Cl\Omega_2) \ogreaterthan (\kappa, \mathcal{G})$-$Cl\,(\Omega_1 \boxed{\vee} \Omega_2)$.

Since $(\kappa, \mathcal{G})$-$Cl\Omega_1 \boxed{\vee} (\kappa, \mathcal{G})$-$Cl\Omega_2$ is $(\kappa, \mathcal{G})$-closed we have

ISSN: 2233-7857IJFGCN

$(\kappa, \mathcal{G})$-$Cl\Omega_1 \vee (\kappa, \mathcal{G})$-$Cl\Omega_2 \ogreaterthan (\kappa, \mathcal{G})$-$Cl(\Omega_1 \vee \Omega_2)$.
Since $\Omega_1 \olessthan \Omega_1 \vee \Omega_2$ and $\Omega_2 \olessthan \Omega_1 \vee \Omega_2$ we have $(\kappa, \mathcal{G})$-$Cl\Omega_1 \olessthan (\kappa, \mathcal{G})$-$Cl(\Omega_1 \vee \Omega_2)$ and $(\kappa, \mathcal{G})$-$Cl\Omega2 \olessthan (\kappa, \mathcal{G})$-$Cl(\Omega_1 \vee \Omega_2)$ so that $(\kappa, \mathcal{G})$-$Cl\Omega_1 \vee (\kappa, \mathcal{G})$-$Cl\Omega_2 \olessthan (\kappa, \mathcal{G})$-$Cl(\Omega_1 \vee \Omega_2)$. This proves (iii).

Since $(\kappa, \mathcal{G})$-$Cl\Omega_1 \ogreaterthan \Omega_1$ and $(\kappa, \mathcal{G})$-$Cl\Omega_2 \ogreaterthan \Omega_2$ we have
$(\kappa, \mathcal{G})$-$Cl\ \Omega_1 \wedge (\kappa, \mathcal{G})$-$Cl\ \Omega_2 \ogreaterthan (\Omega_1 \wedge \Omega_2)$ so that
$(\kappa, \mathcal{G})$-$Cl\ \Omega_1 \wedge (\kappa, \mathcal{G})$-$Cl\ \Omega_2 \ogreaterthan (\kappa, \mathcal{G})$-$Cl\ (\Omega_1 \wedge \Omega_2)$. This proves (iv).

Proposition 2.11:
Let Ω be an indexed structure on X.

(i) $[X]_\kappa \setminus (\kappa, \mathcal{G})$-$Int\Omega = (\kappa, \mathcal{G})$- $Cl([X]_\kappa \setminus \Omega)$
(ii) $[X]_\kappa \setminus (\kappa, \mathcal{G})$-$Cl\Omega = (\kappa, \mathcal{G})$-$Int([X]_\kappa \setminus \Omega)$

(iii) $[X]_\kappa \setminus (\kappa, \mathcal{G})$-$Int([X]_\kappa \setminus \Omega) = (\kappa, \mathcal{G})$-$Cl\ \Omega$

(iv) $[X]_\kappa \setminus (\kappa, \mathcal{G})$-$Cl([X]_\kappa \setminus \Omega) = (\kappa, \mathcal{G})$-$Int\ \Omega$

Proof:

$[X]_\kappa \setminus (\kappa, \mathcal{G})$-$Int\Omega = [X]_\kappa \setminus (\vee\{P: P$ is $(\kappa, \mathcal{G})$- open and $P \olessthan \Omega \})$

$= \wedge\{[X]_\kappa \setminus P: P$ is $(\kappa, \mathcal{G})$- open and $P \olessthan \Omega \}$

$= \wedge\{Q: Q$ is $(\kappa, \mathcal{G})$-closed and $Q \ogreaterthan ([X]\kappa \setminus \Omega) \}$

$= (\kappa, \mathcal{G})$-$Cl([X]_\kappa \setminus \Omega)$.

$[X]\kappa \setminus (\kappa, \mathcal{G})$-$Cl\Omega = [X]\kappa \setminus (\wedge\{Q: Q$ is $(\kappa, \mathcal{G})$-closed and $Q \ogreaterthan \Omega \})$

$= \vee\{[X]\kappa \setminus Q: Q$ is $(\kappa, \mathcal{G})$-closed and $Q \ogreaterthan \Omega \}$

$= \vee\{ P: P$ is $(\kappa, \mathcal{G})$-open and $P \olessthan ([X]\kappa \setminus \Omega)\}$

$= (\kappa, \mathcal{G})$- $Int([X]\kappa \setminus \Omega)$.

$[X]_\kappa \setminus (\kappa, \mathcal{G})$-$Int([X]_\kappa \setminus \Omega) = [X]_\kappa \setminus ([X]_\kappa \setminus Cl\ \Omega)$

$= (\kappa, \mathcal{G})$-$Cl\ \Omega$

$[X]_\kappa \setminus (\kappa, \mathcal{G})$-$Cl([X]_\kappa \setminus \Omega) = [X]_\kappa \setminus ([X]_\kappa \setminus (\kappa, \mathcal{G})$- $Int\Omega)$

$= (\kappa, \mathcal{G})$-$Int\ \Omega$.

3.Nearly (κ, $\mathcal{G}$)-open structures

The following expressions for an indexed structure Ω will be useful in sequel.

Expressions 3.1:

(i) $\eta(\Omega) = (\kappa, \mathcal{G})$-$Int\ ((\kappa, \mathcal{G})$-$Cl\ ((\kappa, \mathcal{G})$-$Int\ \Omega))$
(ii) $\lambda(\Omega) = (\kappa, \mathcal{G})$-$Cl\ ((\kappa, \mathcal{G})$-$Int\ \Omega)$

ISSN: 2233-7857IJFGCN

(iii) $\mu(\Omega)=(\kappa, \mathcal{G})\text{-}Int\ ((\kappa, \mathcal{G})\text{-}Cl\ \Omega)$

(iv) $\nu(\Omega)=(\kappa, \mathcal{G})\text{-}Cl\ ((\kappa, \mathcal{G})\text{-}Int\ ((\kappa, \mathcal{G})\text{-}Cl\ \Omega))$

Properties:

(i) $(\kappa, \mathcal{G})\text{-}Int\ \Omega \olessthan \eta(\Omega) \olessthan \lambda(\Omega) \olessthan \nu(\Omega) \olessthan (\kappa, \mathcal{G})\text{-}Cl\ \Omega$.

(ii) $(\kappa, \mathcal{G})\text{-}Int\ \Omega \olessthan \eta(\Omega) \olessthan \mu(\Omega) \olessthan \nu(\Omega) \olessthan (\kappa, \mathcal{G})\text{-}Cl\ \Omega$.

(iii) $\lambda(\lambda(\Omega)) = \lambda(\Omega)$.

(iv) $\mu(\mu(\Omega)) = \mu(\Omega)$.

Definition 3.2 :

Let Ω be an indexed structure of X. Then Ω is

(i) $(\kappa, \mathcal{G})$-regular open if $\Omega = \mu(\Omega)$,

(ii) $(\kappa, \mathcal{G})$-semi open if $\Omega \olessthan \lambda(\Omega)$,

(iii) $(\kappa, \mathcal{G})$-pre open if $\Omega \olessthan \mu(\Omega)$,

(iv) $(\kappa, \mathcal{G})$-α-open if $\Omega \olessthan \eta(\Omega)$,

(v) $(\kappa, \mathcal{G})$-β-open if $\Omega \olessthan \nu(\Omega)$

(vi) $(\kappa, \mathcal{G})$-b-open if $\Omega \olessthan \mu(\Omega) \boxvee \lambda(\Omega)$

(vii) $(\kappa, \mathcal{G})$-$b^{\#}$-open if $\Omega = \mu(\Omega) \boxvee \lambda(\Omega)$

(viii) $(\kappa, \mathcal{G})$-*b-open if $\Omega \olessthan \mu(\Omega) \boxwedge \lambda(\Omega)$

(ix) a $(\kappa, \mathcal{G})$-p-structure if $\lambda(\Omega) \olessthan \mu(\Omega)$

(x) a $(\kappa, \mathcal{G})$-q-structure if $\mu(\Omega) \olessthan \lambda(\Omega)$

(xi) a $(\kappa, \mathcal{G})$-Q-structure if $\mu(\Omega) = \lambda(\Omega)$

Diagram 3.3:

$(\kappa, \mathcal{G})$-regular open ↣ $(\kappa, \mathcal{G})$-open ↣ $(\kappa, \mathcal{G})$-α-open ↣ $(\kappa, \mathcal{G})$-semiopen
↧ ↧
$(\kappa, \mathcal{G})$-preopen ↣ $(\kappa, \mathcal{G})$-b-open ↣ $(\kappa, \mathcal{G})$-β-open

Diagram 3.4:

$(\kappa, \mathcal{G})$-*b-open ↣ $(\kappa, \mathcal{G})$-b-open ↢ $(\kappa, \mathcal{G})$- $b^{\#}$-open

Diagram 3.5:

$(\kappa, \mathcal{G})$-p-structure ↢ $(\kappa, \mathcal{G})$-Q-structure ↣ $(\kappa, \mathcal{G})$-q-structure

Proposition 3.6:

ISSN: 2233-7857IJFGCN

$[\varnothing]_\kappa$ and $[X]_\kappa$ are (i) $(\kappa, \mathcal{G})$-regular open, (ii) $(\kappa, \mathcal{G})$-semi open, (iii) $(\kappa, \mathcal{G})$-pre open ,

(iv) $(\kappa, \mathcal{G})$-α-open , (v) $(\kappa, \mathcal{G})$-β-open , (vi) $(\kappa, \mathcal{G})$-b-open , (vii) $(\kappa, \mathcal{G})$-$b^{\#}$-open,

(viii)$(\kappa, \mathcal{G})$-*b-open, (ix) $(\kappa, \mathcal{G})$-p-structures , (x) $(\kappa, \mathcal{G})$-q-structures , (xi) $(\kappa, \mathcal{G})$-Q-structures

Proposition 3.7:

(i) $\mu(\Omega)$ is $(\kappa, \mathcal{G})$-regular open

(ii) Ω is $(\kappa, \mathcal{G})$-semi open if and only if $(\kappa, \mathcal{G})$-$Cl\ \Omega = \lambda(\Omega)$,

(iii) Ω is $(\kappa, \mathcal{G})$-β-open if and only if $(\kappa, \mathcal{G})$-$Cl\ \Omega = \nu(\Omega)$

(iv)) Ω is a $(\kappa, \mathcal{G})$-p-structure if and only if $[X]_\kappa \boxbslash \Omega$ is a $(\kappa, \mathcal{G})$-p-structure

(v) Ω is a $(\kappa, \mathcal{G})$-q-structure if and only if $[X]_\kappa \boxbslash \Omega$ is a $(\kappa, \mathcal{G})$-q-structure

(vi) Ω is a $(\kappa, \mathcal{G})$-Q-structure if and only if $[X]_\kappa \boxbslash \Omega$ is a $(\kappa, \mathcal{G})$-Q-structure

(vii) Ω is a $(\kappa, \mathcal{G})$-Q-structure if and only if it is a $(\kappa, \mathcal{G})$-p-structure and a $(\kappa, \mathcal{G})$-q-structure

Corollary 3.8:

(i)If Ω is $(\kappa, \mathcal{G})$-pre open then $(\kappa, \mathcal{G})$-$Cl\ \Omega = \nu(\Omega)$.

(ii) If Ω is $(\kappa, \mathcal{G})$-α-open then $(\kappa, \mathcal{G})$-$Cl\ \Omega = \lambda(\Omega)$.

(iii)If Ω is $(\kappa, \mathcal{G})$-b-open then $(\kappa, \mathcal{G})$-$Cl\ \Omega = \nu(\Omega)$.

(iv)If Ω is $(\kappa, \mathcal{G})$-*b-open then $(\kappa, \mathcal{G})$-$Cl\ \Omega = \nu(\Omega)$.

(v)If Ω is $(\kappa, \mathcal{G})$- $b^{\#}$-open then $(\kappa, \mathcal{G})$-$Cl\ \Omega = \nu(\Omega)$.

4.Nearly $(\kappa, \mathcal{G})$-closed structures

Definition 4.1 :

Let Ω be an indexed structure of X. Then Ω is

(i) $(\kappa, \mathcal{G})$-regular closed if $\Omega = \lambda(\Omega)$,

(ii) $(\kappa, \mathcal{G})$-semi closed if $\mu(\Omega) \olessthan \Omega$,

(iii) $(\kappa, \mathcal{G})$-pre closed if $\lambda(\Omega) \olessthan \Omega$,

(iv) $(\kappa, \mathcal{G})$-α-closed if $\nu(\Omega) \olessthan \Omega$,

ISSN: 2233-7857IJFGCN

(v) $(\kappa, \mathcal{G})$-β-closed if $\eta(\Omega) \subseteq \Omega$

(vi) $(\kappa, \mathcal{G})$-b-closed if $\mu(\Omega) \cap \lambda(\Omega) \subseteq \Omega$

(vii) $(\kappa, \mathcal{G})$-$b^{\#}$-closed if $\mu(\Omega) \cap \lambda(\Omega) = \Omega$

(viii) $(\kappa, \mathcal{G})$-*b-closed if $\mu(\Omega) \cup \lambda(\Omega) \subseteq \Omega$

Proposition 4.2 :

Let Ω be an indexed structure of X. Ω is

(i) $(\kappa, \mathcal{G})$-regular closed if and only if $[X]_\kappa \setminus \Omega$ is $(\kappa, \mathcal{G})$-regular open.

(ii) $(\kappa, \mathcal{G})$-semi closed if and only if $[X]_\kappa \setminus \Omega$ is $(\kappa, \mathcal{G})$-semi open.,

(iii) $(\kappa, \mathcal{G})$-pre closed if and only if $[X]_\kappa \setminus \Omega$ is $(\kappa, \mathcal{G})$-pre open.

(iv) $(\kappa, \mathcal{G})$-α-closed if and only if $[X]_\kappa \setminus \Omega$ is $(\kappa, \mathcal{G})$-α-open.,

(v) $(\kappa, \mathcal{G})$-β-closed if and only if $[X]_\kappa \setminus \Omega$ is $(\kappa, \mathcal{G})$-β-open.

(vi) $(\kappa, \mathcal{G})$-b-closed if and only if $[X]\kappa \setminus \Omega$ is $(\kappa, \mathcal{G})$-b-open.

(vii) $(\kappa, \mathcal{G})$-$b^{\#}$-closed if and only if $[X]\kappa \setminus \Omega$ is $(\kappa, \mathcal{G})$-$b^{\#}$-open.

(viii) $(\kappa, \mathcal{G})$-*b-closed if and only if $[X]\kappa \setminus \Omega$ is $(\kappa, \mathcal{G})$-*b -open.

Diagram 4.3:

$(\kappa, \mathcal{G})$-regular closed ↣ $(\kappa, \mathcal{G})$-closed ↣ $(\kappa, \mathcal{G})$- α-closed ↣ $(\kappa, \mathcal{G})$-semiclosed
↧ ↧
$(\kappa, \mathcal{G})$-preclosed ↣ $(\kappa, \mathcal{G})$-b-closed ↣ $(\kappa, \mathcal{G})$- β-closed

Diagram 4.4:

$(\kappa, \mathcal{G})$-*b-closed ↣ $(\kappa, \mathcal{G})$-b-closed ↢ $(\kappa, \mathcal{G})$- $b^{\#}$-closed

Proposition 4.5:

$[\varnothing]_\kappa$ and $[X]_\kappa$ are (i)$(\kappa, \mathcal{G})$-regular closed, (ii) $(\kappa, \mathcal{G})$-semi closed ,(iii) $(\kappa, \mathcal{G})$-pre closed (iv)$(\kappa, \mathcal{G})$-α-closed , (v) $(\kappa, \mathcal{G})$-β-closed , (vi) $(\kappa, \mathcal{G})$-b-closed, (vii) $(\kappa, \mathcal{G})$-$b^{\#}$-closed (viii) $(\kappa, \mathcal{G})$-*b-closed.

Proposition 4.6:

(i) $\lambda(\Omega)$ is $(\kappa, \mathcal{G})$-regular closed

(ii) Ω is $(\kappa, \mathcal{G})$-semi closed if and only if $(\kappa, \mathcal{G})$-*Int* $\Omega = \mu(\Omega)$,

(iii) Ω is $(\kappa, \mathcal{G})$-β-closed if and only if $(\kappa, \mathcal{G})$-*Int* $\Omega = \eta(\Omega)$

ISSN: 2233-7857IJFGCN

Corollary 4.7:

(i)If Ω is (κ, $\mathcal{G}$)-pre closed then (κ, $\mathcal{G}$)-*Int* Ω =η(Ω).

(ii) If Ω is (κ, $\mathcal{G}$)-α-closed then (κ, $\mathcal{G}$)-*Int* Ω =μ(Ω).

(iii)If Ω is (κ, $\mathcal{G}$)-b-closed then (κ, $\mathcal{G}$)-*Int* Ω =η(Ω)

(iv)If Ω is (κ, $\mathcal{G}$)-*b-closed then (κ, $\mathcal{G}$)-*Int* Ω =η(Ω)

(v)If Ω is (κ, $\mathcal{G}$)-$b^{\#}$-closed then (κ, $\mathcal{G}$)-*Int* Ω =η(Ω)

Conclusion:

A topological structure has been introduced on the indexed structures. This topology is known as indexed topology. The weak and strong form of open sets in general topology have been studied in the settings of indexed topology.

References:

[1] Geetha Jeyalakshmi R, Dass K and Pasunkili Pandian S, On Structures induced by a bitopology, International Workshop and Conference on Topology and Applications(IWCTA2018), Dec. 05-11, 2018, Kerala , India.

[2] Geetha Jeyalakshmi R and Dass K, Structures and Relations, International Conference on Advanced Research in Mathematical Sciences(ICARMS), Feb. 24-25, 2021, Pondicherry , India.

[3] Geetha Jeyalakshmi R and Dass K, Structures, Operations and Their Applications to Topology(submitted).

[4] Geetha Jeyalakshmi R and Dass K, On Indexed Structures, 2nd International Conference on Mathematical Techniques and Applications, e-ICMTA - 2021(Virtual Mode), March 24-26, 2021, SRMIST, Tamilnadu, India.

ISSN: 2233-7857IJFGCN