4-Total difference cordial graphs obtained from path and cycle

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Abstract

Let G be a graph. Let $f : V(G) \to \{0, 1, 2, \ldots, k-1\}$ be a map where $k \in N$ and $k > 1$. For each edge uv, assign the label $|f(u)-f(v)|$. f is called k-total difference cordial labeling of G if $|t_d(i)-t_d(j)| \le 1$, $i, j \in \{0, 1, 2, \ldots, k-1\}$ where $t_d(x)$ denotes the total number of vertices and the edges labeled with x. A graph with admits a k-total difference cordial labeling is called k-total difference cordial graphs. In this paper we investigate the 4-total difference cordial labeling behaviour of SC_n , $P_n(G_6)$, $P_n(G_8)$.

Definition : 1

Consider the eight copies of the cycles C_{4n}. Let the i^{th} copy of the cycle be $v_{i1}v_{i2}$,......v_{i4n} v_{i1}. The graph SC_n obtained from eight copies of C_{4n} by identify $u_{1,4n}$ with $u_{2,1}$, $u_{2,4n}$ with $u_{3,1}$, $u_{3,4n}$ with $u_{4,1}$, $u_{4,4n}$ with $u_{5,1}$, $u_{5,4n}$ with $u_{6,1}$, $u_{6,4n}$ with $u_{7,1}$, $u_{7,4n}$ with $u_{8,1}$, $u_{8,4n}$ with $v_{1,1}$. This is called Eight Sprocket graph.

Definition : 2

Let P_n be the path v_1v_2.......v_n. Consider the graph G_6^i, (i= 1,2,.....n) with $V(G_6^i)$ = { $u_1^i, u_2^i, u_3^i, u_4^i, u_5^i, u_6^i$} and E($u_6^i$) = {$u_1^i\, u_2^i, u_2^iu_3^i, u_3^iu_4^i, u_4^iu_5^i, u_5^iu_6^i, u_6^iu_1^i, u_2^iu_4^i, u_4^iu_6^i$}. The graph $P_n(G_6)$ is obtained from P_n and G_6^i by indentify by v_i with $u_1^i (i = 1,2, \ldots . n)$.

Definition: 3

Let P_n be the path$v_1v_2 \ldots \ldots \ldots . v_n$. Consider the graph $G_8^i(i = 1,2, \ldots n)$ with $V(G_8^i) = \{u_1^i, u_2^i, u_3^i, \ldots \ldots \ldots .. u_8^i\}$ and $E(G_8^i) = \{u_1^iu_2^i, u_2^iu_3^i, \ldots \ldots u_7^iu_8^i, u_8^iu_1^i, u_2^iu_4^i, u_4^iu_6^i, u_6^iu_8^i\}$. The graph $P_n(G_8)$ is obtained from G_8^i and P_n by identify by v_i with u_1^i $(i = 1,2, \ldots . n)$.

Theorem: 1

Eight Sprocket graph SC_n is 4-Total difference cordial

Proof:

Take the vertex set and edge set as in definition 1. Clearly $|V(SC_n)| + |E(SC_n)| = 248$.

We fine labelling function $f : V(G) \to \{0,1,2,3\}$ as follows

$$f(u_{1,j}) = f(u_{2,j}) = f(u_{3,j}) = f(u_{4,j}) = \begin{cases} 1 \text{ if } j \neq 3,4,7,10 \\ 2 \text{ if } j = 8,9 \\ 3 \text{ if } j = 1,2,5,6,11,12,15,16 \end{cases}$$

$$f(u_{5,j}) = f(u_{6,j}) = \begin{cases} 1 \text{ if } j = 3,4,7,10,13,14 \\ 2 \text{ if } j = 9 \\ 3 \text{ if } j = 1, 2, 5, 6, 8, 11, 12, 15,16 \end{cases}$$

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$$f\left(u_{7,j}\right) = f\left(u_{8,j}\right) = \begin{cases} o \ if \quad j = 9 \\ 1 \ if \quad j = 3, 4, 7, 10, 13, 14 \\ 3 \ if \quad j = 1, 2, 5, 6, 8, 11, 12, 15, 16 \end{cases}$$

Clearly t_{df}(o) = $t_{df}(1) = t_{df}(2) = t_{df}(3) = 62$.

Theorem: 2

$P_n\ (G_6)$ is 4-total difference cordial.

Proof:

Take the vertex set and edge set as in definition 2

Clearly $|V(P_n(G_6)| + E(P_n(G_6)| = 16n - 1$

Case (i) n is even

Assign the label 3 to the vertices $v_1, v_2, \dots \dots \dots v_n$. Next assign the label 3 to the vertices $u_3^1, u_4^1, u_5^1, u_3^2, u_4^2, u_5^2 \dots \dots \dots, u_3^n, u_4^n, u_5^n$. We now assign the label 1 to the vertices $u_2^1, u_6^1, u_2^2, u_6^2 \dots \dots .. u_2{}^{n/2}, u_6{}^{n/2}$. Finally assign the label 2 to the vertices $u_2{}^{n+2/2}, u_6{}^{n+2/2}, u_2{}^{n+4/2}, \dots \dots \dots\ u_2^n, u_6^n$.

Case(ii) n is odd

We now assign the label 3 to the vertices. $v_1, v_2, \dots \dots \dots \dots . v_n$ Next we assign the label 3 to the vertices. $u_3^1, u_4^1, u_5^1, u_3^2, u_4^2, u_5^2, \dots \dots \dots \dots .. u_3^{n-1}, u_4^{n-1}, u_5^{n-1}$. Now we assign the label 1 to the vertices$u_2^1, u_6^1, u_2^2, u_6^2 \dots \dots .. u_2{}^{n-1/2}, u_6{}^{n-1/2}$. We now assign the label 2 to the vertices $u_2{}^{n+1/2}, u_6{}^{n+1/2}, u_2{}^{n+3/2}, u_6{}^{n+3/2}, \dots ..\ u_2^{n-1},\ u_6^{n-1}$. At last we assign the label 3,3,1,2 and 3 respectively 3 to the vertices $u_2^n, u_3^n, u_4^n, u_5^n$ and u_6^n.

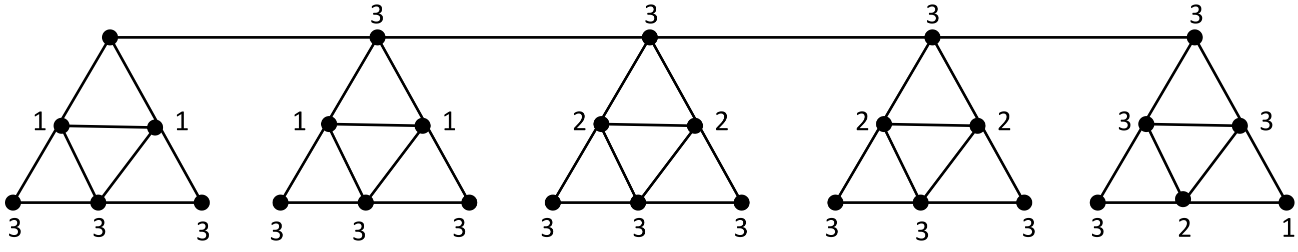
The table 1 given below establish that this vertex labelling is a 4-total difference cordial

Value of n	$tdf(0)$	$tdf(1)$	$tdf(2)$	$tdf(3)$
n is odd	$4n$	$4n - 1$	$4n$	$4n$
n is even	$4n - 1$	$4n$	$4n$	$4n$

Table:1

Example: 1

$P_5(G_6)$ is 4-total difference cordial.



Theorem:3

$P_n(G_8)$ is 4-total difference cordial.

Proof:

Take the vertex set and edge set as in definition 3

Obviously $|V(P_n(G8)| + |E(P_n(G8)| = 16n - 1$

Case(i) $n \leq 4$

A 4- total difference cordial labeling is given in table 2

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Value of n	u_2^1	u_2^2	u_2^3	u_2^4	u_3^1	u_3^2	u_3^3	u_3^4	u_4^1	u_4^2	u_4^3	u_4^4	u_5^1	u_5^2	u_5^3	u_5^4	u_6^1	u_6^2	u_6^3	u_6^4	u_7^1	u_7^2	u_7^3	u_7^4	u_8^1	u_8^2	u_8^3	u_8^4
n=1	3				3				1				3				2				2				3			
n=2		3				3				1				3				2				1				3		
n=3			3				3				2				3				1				1				3	
n=4				3				2				0				3				2				1				3

Table 2

Case(ii) $n > 4$

Assign the label 0,0,3,3 to the vertices $u_2^5, u_2^6, u_2^7, u_2^8$. Next assign the label 0,0,3,3 Respectively to the vertices $u_2^9, u_2^{10}, u_2^{11}, u_2^{12}$. Proceeding like this until reach the vertex u_2^n. Assign the label 3 to the vertices, $u_8^5, u_8^6, u_8^7, u_8^8 \dots \dots .. u_8^n$.

First assign the labels 2,2 to the vertices u_3^5, u_3^6. Next assign the labels 3,2,2,2 to the vertices $u_3^7, u_3^8, u_3^9, u_3^{10}$. We now assign the labels 3,2,2,3 to the next four vertices $u_3^{11}, u_3^{12}, u_3^{13}, u_3^{14}$ and so on. Continue in this way until reach the vertex u_3^n.

Next we assign the labels 2 to the vertices u_4^5, u_4^6, u_4^7. Now we assign the labels 2,2,1 to the vertices u_6^5, u_6^6, u_6^7. Assign the labels 0,2,2,2 to the vertices $u_4^8, u_4^9, u_4^{10}, u_4^{11}$. We now assign the labels 0,2,2,2 to the vertices $u_4^{12}, u_4^{13}, u_4^{14}, u_4^{15}$. Proceeding like this until we reach the vertex u_4^n. Assign the labels 2,2,2,1 to the vertices to the vertices $u_6^8, u_6^9, u_6^{10}, u_6^{11}$. Now we assign the labels 2,2,2,1 to the vertices $u_6^{12}, u_6^{13}, u_6^{14}, u_6^{15}$. Continue in this way until we reach the vertex u_6^n.

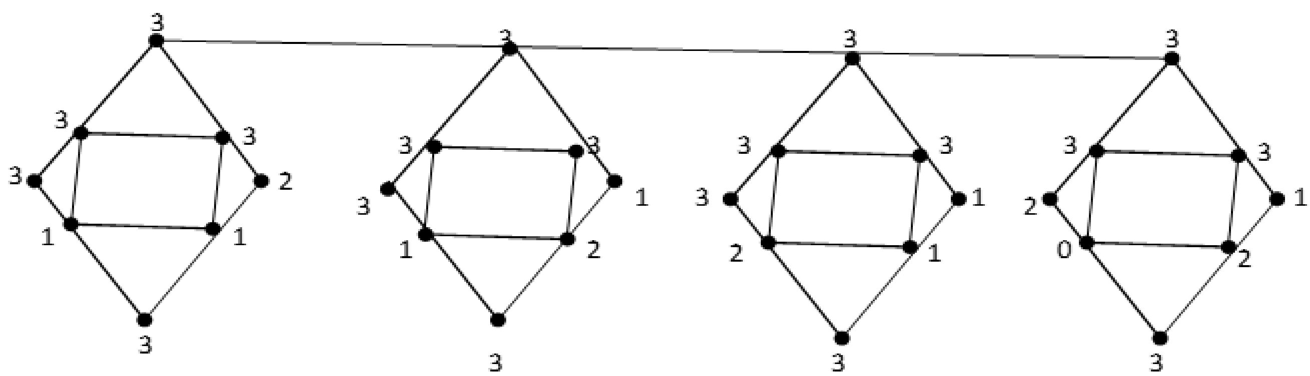
The table 3 given below shows that this labeling pattern is 4-total difference cordial labelling.

Value of n	tdf(0)	tdf(1)	tdf(2)	tdf(3)
n= 0 (mod 4)	$21\,n/_4$	$21\,n/_4$	$\frac{21\,n-4}{4}$	$\frac{21n}{4}$
n=1(mod 4)	$\frac{21\,n-1}{4}$	$\frac{21\,n-1}{4}$	$\frac{21\,n-1}{4}$	$\frac{21\,n-1}{4}$
n=2(mod 4)	$\frac{21\,n-2}{4}$	$\frac{21\,n-2}{4}$	$\frac{21\,n-2}{4}$	$\frac{21\,n-2}{4}$
n=3(mod 4)	$\frac{21\,n+1}{4}$	$\frac{21\,n+1}{4}$	$\frac{21\,n+1}{4}$	$\frac{21\,n+1}{4}$

Table 3

Example: 2

$P_4(G_8)$ is 4-total difference cordial



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