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Nonlinear wave propagation along microtubulin system using analytical method

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Abstract
In the present paper, we consider microtubules as lattice arrays of coupled local dipolestates that interact with their immediate neighbours. It is demonstrated that solitary wave likeexcitations arise as a result of the guanosine 5-triphosphate (GTP) hydrolysis and that the springconstant may cause them to propagate by admitting the nonlinear wave along a microtubule usinganalytical method.
Keywords: Solitary wave, microtubules, analytical method

1. Introduction

Microtubules was first observed by Ledbetter and Poster [1], in the cortical cytoplasm ofplant cells, which is involved in important cellular process during growth and development [2,3].Microtubules from cortical arrays during the interphase, that are generally transverse to the direction of cell elongation. On completing the cell elongation phase [3, 4], they become oblique orlongitudinal. The orientation of microtubule arrays is not always uniform across a cell and theyfrequently [5] occur with mixed array. Cortical microtubules are involved in directional cellulosedeposition, although the details remain to be established [6]. The interphase array of microtubulesgives way to a dense preprophase band (PPB) during the cell division [7, 8]. This predicts theplane of future cell division. Then the mitotic spindle is formed by microtubules, which involvedin the separation of paired chromosomes and finally between the daughter nuclei a new cell wall islaid, it is the phragmoplast. The regeneration of interphase array in the cell cortex is occurred andit is possible from microtubule-nucleating sites along the plasmalemma or the nuclear envelope[9].

A Microtubules, which forms a long hallow cylinder with its diameter about 25 nm. Theinterior part of this hallow cylinder (MTS) is filled with ordered water molecules and this impliesthe existence of electric dipoles and electric fields [10]. The oriented molecules of cytoplasm waterand enzymes [11] are surrounded in the outer surface of MT. Protofilaments with alternating andsubunits are formed by tubulin heterodimers, which are along the microtubules axis and joinedend to end. In vitro, the MTs composed of 12 to 17 protofilaments wen it is self-assembled and 13protofilaments in vivo. The internal bound of these protofilaments are strong and they are connected by weaker lateral bounds to form a sheet which is wrapped up into a tube in the nucleationprocess [12]. About 13 identical protofilaments in the wall of MT each consist of several subunitswhich are known as tubulin dimers. These dimers associated in a chain like manner resulting in socalled protofilaments. Alpha and beta tubulin are three tubulins at least associated with microtubules.Each subunit of tubulin is about 8nm peanut shaped dimer, which comprises alpha, beta tubulin. Thisconsists of single polypeptide chain folded over on itself and approximately 4nm in diameter. Both alpha and beta tubulin monomers have (GTP) Guanosine Triphosphate which is binding sites that playsa critical role in dynamics of microtubule formation. Monomers, the alpha and beta tubulin assemble toform alpha beta-heterodimers each consisting of single alpha-monomer and a single beta-monomer. The bodyof microtubule is formed by heterodimers as discussed below in detail. At the end of

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(MTOCs)Microtubule Organizing Centers, monomers of -tubule are bounded but not within microtubulesthemselves. We use the following nonlinear (PDE) partial differential equation of motion [13]:

$$m\frac{\partial^2 z}{\partial t^2} - kl^2\frac{\partial^2 z}{\partial x^2} - qE - Az + Bz^3 + \gamma\frac{\partial z}{\partial t} = 0, \tag{1}$$

where, *m* denotes the mass of the dimer, *l* - is the length of the dimer, *k* - is the harmonicconstant between the dimers belonging to the same protofilament, *E* - is the magnitude of theintrinsic electric field while *q* represents the charge within the dipole, □-viscosity coefficient, *A* and*B* represents the positive parameters of double well potential. The organization of the paper isas follows. In sec. II, we get, the soliton solutions by using analytical method, namely, extendedrational sin-cos method. It is very useful technique to obtain the periodic solitary solution. Finally,conclude this work in sec. III.

2. Solitary wave solutions using extended rational sin-cosmethod

In the recent years, various analytical methods have been used to solve the nonlinear partial differential equations (PDEs). Hirota'sbilenearization method [14], modified extended tanhmethod [15,16], Jacobi-elliptic function method [13], double exponential function method [14]. Wedescribe the first step of the new extended rational methods for finding exact solutions of partialdifferential equations (PDEs)

$$F\left[z, \frac{\partial z}{\partial t}, \frac{\partial z}{\partial x}, \frac{\partial^2 z}{\partial x}, \frac{\partial^2 z}{\partial x^2}, ...\right] = 0, \tag{2}$$

where, $z = z(x,t)$, is an unknown function and *F* is a polynomial in *z* and its various partialderivatives. According to extended rational sin-cos method, [17,18], we assume that the exactsolution can be expressed in the following forms

$$z(\xi) = \frac{a_0 sin(\mu\xi)}{a_2 + a_1 cos(\mu\xi)}, \quad cos(\mu\xi) \neq -\frac{a_2}{a_1} \tag{3}$$

where, a_0,a_1 and a_2 are parameters to be found in terms of the other parameters. The nonzero constant □ is the wave number. The derivatives of the predicted solutions are

$$z_\xi(\xi) = -\frac{a_0\mu\Big[sin(\mu\xi)a_2 + a_1\Big]}{\Big[a_2 + a_1 sin(\mu\xi)\Big]^2},$$
$$z_{\xi\xi}(\xi) = \frac{a_0 cos(\mu\xi)\mu^2\Big[-a_2^2 + a_2 a_1 sin(\mu\xi) + 2a_1^2\Big]}{\Big[a_2 + a_1 sin(\mu\xi)\Big]^3}, \tag{4}$$

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we seek its traveling wave solution of the form

$$z(x,t) = z(\xi), \quad \xi = x - ct, \tag{5}$$

where *c* is the soliton speed. Substituting Eq. (5) into Eq. (1), we get the ordinary differentialequation (ODE),

$$z_{\xi\xi}(\xi) - T_1 z_\xi(\xi) - T_2 z(\xi) + T_3 z^3(\xi) - T_4 = 0, \tag{6}$$

where,

$$T_1 = \left[\frac{c\gamma}{mc^2 - kl^2}\right],\ T_2 = \left[\frac{A}{mc^2 - kl^2}\right],\ T_3 = \left[\frac{B}{mc^2 - kl^2}\right] \text{ and } T_4 = \left[\frac{qE}{mc^2 - kl^2}\right].$$

Substituting theEq. (4) into Eq. (6), we get,

$$\begin{aligned}
&\frac{-a_0 sin(\mu\xi)\mu^2}{(a_2 + a_1 cos(\mu\xi))} + \frac{3a_0 cos(\mu\xi)\mu^2}{(a_2 + a_1 cos(\mu\xi))^2 a_1 sin(\mu\xi)} + \frac{2a_0 sin(\mu\xi)^3}{(a_2 + a_1 cos(\mu\xi))^3 a_1^2 \mu^2} \\
&- \frac{T_1(a_0 cos(\mu\xi)\mu}{(a_2 + a_1 cos(\mu\xi))} + \frac{a_0 sin(\mu\xi)^2}{(a_2 + a_1 cos(\mu\xi))^2 a_1 \mu)} - \frac{T_2 a_0 sin(\mu\xi)}{(a_2 + a_1 cos(\mu\xi))} \\
&+ \frac{T_3 a_0^3 sin(\mu\xi)^3}{(a_2 + a_1 cos(\mu\xi))^3} - T_4 = 0,
\end{aligned} \tag{7}$$

after simplifying the Eq. (7) we get,

$$\begin{aligned}
&-a_0 sin(\mu\xi)\mu^2 a_2^2 + a_0 sin(\mu\xi)\mu^2 a_2 a_1 cos(\mu\xi) + 2a_0 sin(\mu\xi)\mu^2 a_1^2 \\
&-T_1 a_0 \mu cos(\mu\xi)^2 a_2 a_1 - T_1 a_0 \mu a_2 a_1 - T_1 a_0 \mu a_1^2 cos(\mu\xi) - T_2 a_0 sin(\mu\xi) a_2^2 \\
&-2T_2 a_0 sin(\mu\xi) a_2 a_1 cos(\mu\xi) - T_2 a_0 sin(\mu\xi) a_1^2 cos(\mu\xi)^2 + T_3 a_0^3 sin(\mu\xi) \\
&-T_3 a_0^3 sin(\mu\xi) cos(\mu\xi)^2 - T_4 a_2^3 - 3T_4 a_2^2 a_1 cos(\mu\xi) - 3T_4 a_2 a_1^2 cos(\mu\xi)^2 \\
&-T_4 a_1^3 cos(\mu\xi)^3 - T_1 a_0 \mu cos(\mu\xi) a_2^2 = 0,
\end{aligned} \tag{8}$$

collecting the coefficients of cos(□□) and sin(□□), we obtain

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$$-a_0\mu^2 a_2^2 + 2a_0\mu^2 a_1^2 - T_2 a_0 a_2^2 + T_3 a_0^3 = 0,$$
$$a_0 a_1 a_2 \mu^2 - 2T_2 a_0 a_1 a_2 = 0,$$
$$-T_1 a_0 \mu a_2^2 - T_1 a_0 \mu a_1^2 - 3T_4 a_2^2 a_1 = 0,$$
$$-T_1 a_0 \mu a_2 a_1 - 3T_4 a_1^2 a_2 = 0,$$
$$-T_4 a_1^3 = 0,$$
$$-T_2 a_0 a_1^2 - T_3 a_0^3 = 0,$$
$$T_1 a_0 a_2 a_1 \mu c - T_4 a_2^3 = 0, \tag{9}$$

solving the above system of equations, we obtain a_1,□ and a_2, as follows

$$a_1 = \sqrt{\frac{-T_3}{T_2}}\ a_0,\ \mu = \sqrt{2T_2},\ a_2 = \sqrt{\frac{-T_1 a_0\left(\frac{-T_3}{T_2}\ a_0\right)(2T_2)^{\frac{1}{2}}}{T_1 a_0 (2T_2)^{\frac{1}{2}} + 3T_4\left(\sqrt{\frac{-T_3}{T_2}}\ a_0\right)}}, \tag{10}$$

substituting the above equation into Eq. (5), we obtain the solitary wave solution,

$$z(x,t) = \frac{a_0 sin(\sqrt{2T_2}\ \xi)}{\sqrt{\frac{-T_1 a_0\left(\frac{-T_3}{T_2}\ a_0\right)(2T_2)^{\frac{1}{2}}}{T_1 a_0 (2T_2)^{\frac{1}{2}} + 3T_4\left(\sqrt{\frac{-T_3}{T_2}}\ a_0\right)}} + \left(\sqrt{\frac{-T_3}{T_2}}\ a_0\right) cos(\sqrt{2T_2}\xi)}. \tag{11}$$

The above equation represents the solitary wave solutions of Eq. (1). We have plotted theEq. (11), for microtubulin system and obtained the periodic solitary wave profile. We obtain theperiodic nonlinear wave profile by choosing the value of velocity c = 0.9 *m*/*s* which is shown in theFig. (1a). By increasing the value of c = 1.0 *m*/*s*, we get the periodic wave profile with differentdirection which is depicted in the Fig. (1b). Finally, we get the periodic structure with constantamplitude for c = 1.1 *m*/*s* which shown in the Fig. (1c).

We fix the spring constant value k = 0.01x$10^{-12}$$Nm^{-1}$ and by keeping the parameter values$m$ = 10^{-26}*kg*, E = $10^5$$Vm^{-1}$, q = 6x$10^{-18}$ C,l = 8x10^{-8}*m*,c = 2.1 *m*/*s*,a_0 = 1,A = 1 and$B$ = 10^{23}, we obtain the cusp-like multi-nonperiodic solitary wave forms which is shown in the Fig.(2a). By increasing the value of k = 0.1x10^{-12} Nm^{-1}, we obtain the nonperiodic solitary waveprofile with constant amplitude which is depicted in the Fig. (2b). Further, increasing the valueof spring constant k = 0.1x10^{-12} Nm^{-1}, we received nonperiodic solitary waves are generatedand traveling on the tubulin dimers with change of direction of propagation which is shown in theFig. (2c).

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3. Conclusions

We investigate the microtubule dynamics and look for the governed nonlinear excitations inthe form of solitons. We employ symbolic computation to solve the associated dynamical equationand exhibit the periodic wave property of the solitons in microtubules under the effect of velocityand spring constant. These nonlinear excitations may be utilized as signalling and switchingmechanisms.

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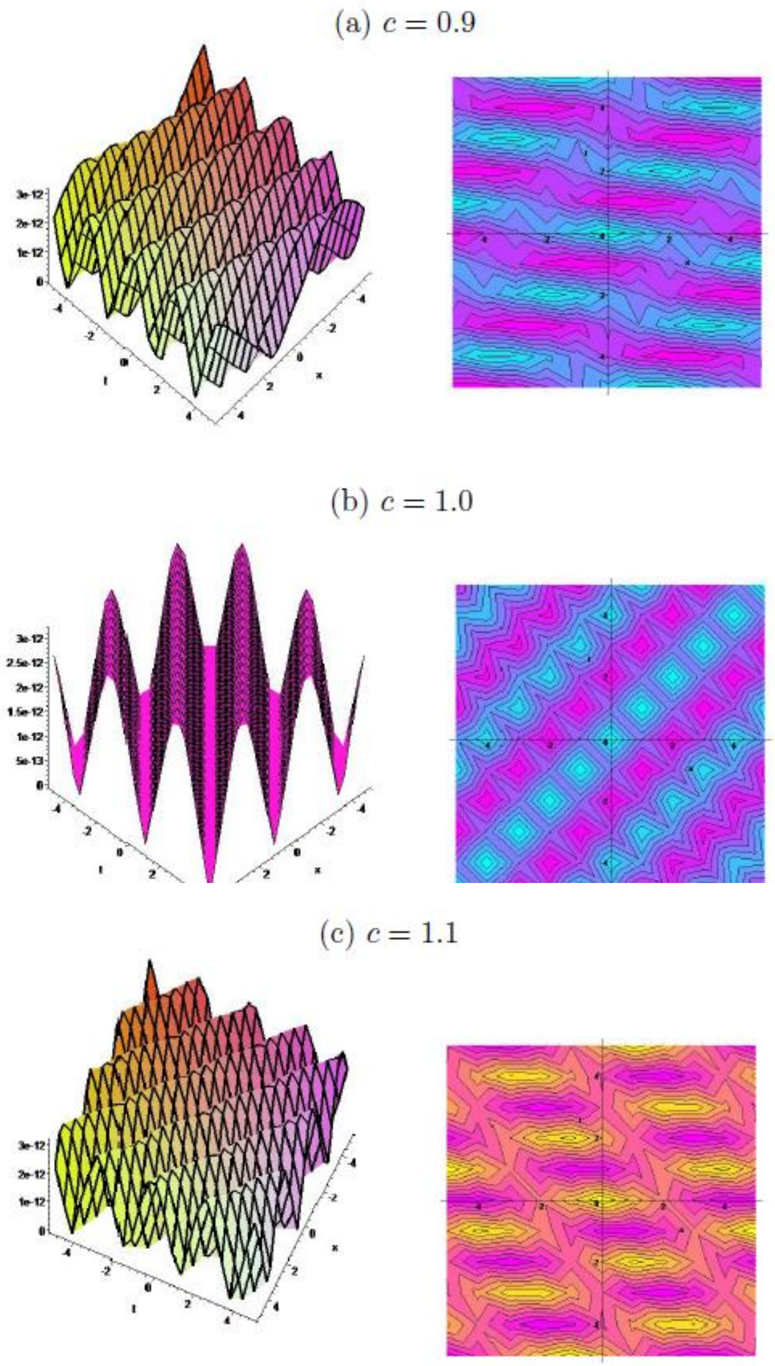

Figure 1: Solitary wave structure for Eq. (11) by varying the value velocity (c).

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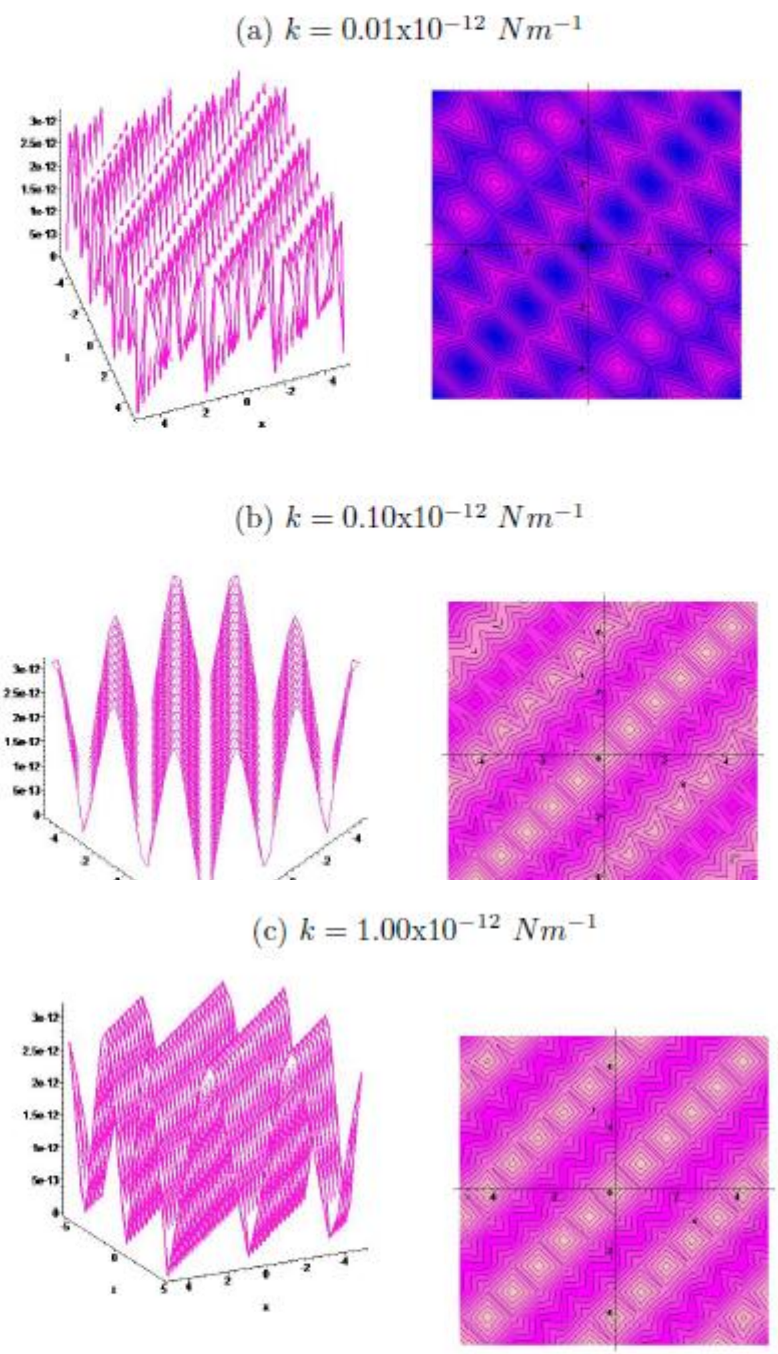

Figure 2: Solitary wave structure for Eq. (11) by varying the value spring constant (k).

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