

## Propagation of dromions in microtubulin system under the influence of viscosity

A. Muniyappan<sup>a\*</sup>, L. Sahasraari<sup>a</sup>, S. Anitha<sup>a</sup>, S. Surya<sup>a</sup>, S. Kondala Rao<sup>b</sup>  
and M. Brintha<sup>c</sup>

<sup>a</sup>Department of Physics, Theivanai Ammal College for Women (A), Villupuram-605 401, Tamilnadu, India

<sup>b</sup>Department of Science and Humanities, QIS Institute of Technology, Ongole-523 272, Andhra Pradesh, India

<sup>c</sup>Principal, Theivanai Ammal College for Women (A), Villupuram-605 401, Tamilnadu, India

*E-mail address: muniyappancnp@gmail.com*

### Abstract

*Microtubules (MTs) are the very important for cellular organization and information processing. MTs serve as structural components within cells and are involved in many cellular process including mitosis, cytokinesis and vesicular transport. The energy transfer along the protofilaments of microtubules can be understood by the underlying nonlinear excitations which explain their dynamics. We employ symbolic computation to solve the associated dynamical equation and exhibit the dromion-like solution in microtubules.*

**Keywords:** Dromions, analytical method, microtubules

### 1. Introduction

Living cells are profoundly and actively categorized by the networks of protein polymers termed cytoskeleton [1]. The cytoskeleton shows intense structure. As the cell, change their shape, it overhauls continuously, by dividing the cells which respond to the environment. The cytoskeleton comprises of intermediate filaments, actin filaments (microfilaments), and microtubules. Microtubules and filaments are co-operatively connected and manifest to form a three-dimensional network in the cell [2-5]. The primordial structure of the cytoskeleton manifested by the microtubules which gratify the vital requirements for the stimulation of vibrations and prompting of narcissistic oscillating electric field [6]. Microtubules play a crucial role for the far- reaching cellular organisation and information processing. MTs assist as structural components within cells and are take part in many cellular processes which encompass mitosis, cytokinesis and vesicular transport. MTs are nucleated and sort out by the microtubule organizing centres (MTOCs), such as centrosomes and basal bodies. The MTOCs is typically track down beside the nucleus midst interphase. MTOC which are grown out by microtubules, forming a hub and spoke array, even meanwhile interphase. MTs are numerous which is more than twice the width of an intermediate filament and three times the width of a microfilament.

Altering length from a fraction of micrometre to hundreds of micrometres, MTs are much rigid than either microfilaments or transitional filaments of their tube-like construction. A significance of this tubular design is the dexterity of MTs to trigger pushing forces without buckling, a property that is critical to the gesture of chromosomes and the mitotic spindle in mitosis. The discrimination of the replicated chromosome is brought about by a tangled cytoskeletal machine with many moving parts-the mitotic spindle. It is hammered from MTs and their federated proteins, which both pull the daughter chromosomes toward the poles apart. (MAPs (microtubule associated proteins) are of remarkable pertinent of stability of MT assembly. The presence of MAPs sustains the growth of microtubules. The mitotic spindle is superintendent for the segregation of sister chromatids in the course of cell division. Chromosomes are sequestered to the spindle with their kinetochores [7] attached to the plus ends of microtubules. Chromosome movement is tentative on kinetochore-microtubule dynamics: only

when the chromosomes kinetochore is adhering to the microtubules, a chromosome will proceed with regard to pole. In the course of mitosis, MTs similarly stretch outward from replicated centrosomes to manifest the mitotic spindle, which is sensible for the separation and allocation of chromosomes to daughter cells.

Around 20 years ago, the researcher are very much interested to study the dromion like excitations in different kinds of systems [8-13]. For constructing dromion-like forms, we consider the following nonlinear partial differential equation (nPDE) of motion for microtubulin systems [14]:

$$m \frac{\partial^2 z}{\partial t^2} - kl^2 \frac{\partial^2 z}{\partial x^2} - qE - Az + Bz^3 + \gamma \frac{\partial z}{\partial t} = 0, \quad (1)$$

where,  $m$  implies the mass of the dimer,  $l$  attest length of the dimer,  $k$  controvert harmonic constant between the dimers affiliation to the same protofilament,  $E$  tags magnitude of the intrinsic electric field while  $q$  represents the charge within the dipole,  $\eta$ -viscosity coefficient,  $A$  and  $B$  describes the positive parameters of double well potential. The organization of the paper is as follows. In sec. II, we come into the soliton solutions originating from the symbolic computation, namely, extended rational sinh-cosh method. This method is convenient to acquire the dromion-like solution. Eventually, we conclude this work in sec. III.

## 2. Dromion solutions using extended rational sinh-cosh method

In the recent years, various analytical methods have been used to solve the nonlinear partial differential equations (PDEs). Hirota's bilinearization method [15], modified extended tanh method [16,17], Jacobi-elliptic function method [14], double exponential function method [15]. We describe the first step of the new extended rational methods for finding exact solutions of partial differential equations (PDEs)

$$F \left[ z, \frac{\partial z}{\partial t}, \frac{\partial z}{\partial x}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial t^2}, \dots \right] = 0, \quad (2)$$

where,  $z = z(x, t)$ , is an unknown function and  $F$  is a polynomial in  $z$  and its various partial derivatives. According to extended rational sinh-cosh method [18,19], we assume that the exact solution can be expressed in the following forms

$$z(\xi) = \frac{a_0 \sinh(\mu \xi)}{a_2 + a_1 \cosh(\mu \xi)}, \quad \cosh(\mu \xi) \neq -\frac{a_2}{a_1}, \quad (3)$$

where,  $a_0$ ,  $a_1$  and  $a_2$  are parameters to be found in terms of the other parameters. The nonzero constant  $\mu$  is the wave number. The derivatives of the predicted solutions are

$$\begin{aligned} z_\xi(\xi) &= \frac{a_0 \mu [\cosh(\mu \xi) a_2 + a_1]}{[a_2 + a_1 \cosh(\mu \xi)]^2}, \\ z_{\xi\xi}(\xi) &= \frac{-a_0 \sinh(\mu \xi) \mu^2 [-a_2^2 + a_2 a_1 \cosh(\mu \xi) + 2a_1^2]}{[a_2 + a_1 \cosh(\mu \xi)]^3}. \end{aligned} \quad (4)$$

We seek its traveling wave solution of the form

$$z(x, t) = z(\xi), \quad \xi = x - ct, \quad (5)$$

where  $c$  is the soliton speed. Substituting Eq. (5) into Eq. (1), we get the ordinary differential equation (ODE),

$$z_{\xi\xi}(\xi) - T_1 z_{\xi}(\xi) - T_2 z(\xi) + T_3 z^3(\xi) - T_4 = 0, \quad (6)$$

where,

$$T_1 = \left[ \frac{c\gamma}{mc^2 - kl^2} \right], \quad T_2 = \left[ \frac{A}{mc^2 - kl^2} \right], \quad T_3 = \left[ \frac{B}{mc^2 - kl^2} \right] \text{ and } T_4 = \left[ \frac{qE}{mc^2 - kl^2} \right].$$

Substituting the Eq. (4) into Eq. (6), we get,

$$\begin{aligned} & \frac{a_0 \sinh(\mu\xi) \mu^2}{(a_2 + a_1 \cosh(\mu\xi))} - \frac{3a_0 \cosh(\mu\xi) \mu^2}{(a_2 + a_1 \cosh(\mu\xi))^2 a_1 \sinh(\mu\xi)} + \frac{2a_0 \sinh(\mu\xi)^3}{(a_2 + a_1 \cosh(\mu\xi))^3 a_1^2 \mu^2} \\ & - T_1 \left( \frac{a_0 \cosh(\mu\xi) \mu}{(a_2 + a_1 \cosh(\mu\xi))} \right) - \frac{a_0 \sinh(\mu\xi)^2}{(a_2 + a_1 \cosh(\mu\xi))^2 a_1 \mu} - \frac{T_2 a_0 \sinh(\mu\xi)}{(a_2 + a_1 \cosh(\mu\xi))} \\ & + \frac{T_3 a_0^3 \sinh(\mu\xi)^3}{(a_2 + a_1 \cosh(\mu\xi))^3} - T_4 = 0, \end{aligned} \quad (7)$$

after simplifying the Eq. (7) we get,

$$\begin{aligned} & a_0 \sinh(\mu\xi) \mu^2 a_2^2 - a_0 \sinh(\mu\xi) \mu^2 a_2 a_1 \cosh(\mu\xi) - 2a_0 \sinh(\mu\xi) \mu^2 a_1^2 - T_1 a_0 \mu \cosh(\mu\xi) a_2^2 \\ & - T_1 a_0 \mu \cosh(\mu\xi)^2 a_2 a_1 - T_1 a_0 \mu a_2 a_1 - T_1 a_0 \mu a_1^2 \cosh(\mu\xi) - T_2 a_0 \sinh(\mu\xi) a_2^2 \\ & - 2T_2 a_0 \sinh(\mu\xi) a_2 a_1 \cosh(\mu\xi) - T_2 a_0 \sinh(\mu\xi) a_1^2 \cosh(\mu\xi)^2 + T_3 a_0^3 \sinh(\mu\xi) \cosh(\mu\xi)^2 \\ & - T_3 a_0^3 \sinh(\mu\xi) - T_4 a_2^3 - 3T_4 a_2^2 a_1 \cosh(\mu\xi) - 3T_4 a_2 a_1^2 \cosh(\mu\xi)^2 - T_4 a_1^3 \cosh(\mu\xi)^3 = 0, \end{aligned} \quad (8)$$

collecting the coefficients of  $\cosh(\square\square)$  and  $\sinh(\square\square)$ , we obtain

$$\begin{aligned} & a_0 \mu^2 a_2^2 - 2a_0 \mu^2 a_1^2 - T_2 a_0 a_2^2 - T_3 a_0^3 = 0, \\ & -T_1 a_0 \mu a_2^2 - T_1 a_0 \mu a_1^2 - 3T_4 a_1 a_2^2 = 0, \\ & -a_0 \mu^2 a_1 a_2 - 2T_2 a_0 a_1 a_2 = 0, \\ & -T_1 a_0 \mu a_1 a_2 - 3T_4 a_1^2 a_2 = 0, \\ & -T_2 a_0 a_1^2 + T_3 a_0^3 = 0, \\ & -T_4 a_1^3 = 0, \\ & -T_1 a_0 \mu a_1 a_2 - T_4 a_2^3 = 0, \end{aligned} \quad (9)$$

solving the above system of equations, we obtain  $a_1$ ,  $\square$  and  $a_2$ ,

$$a_1 = \sqrt{\frac{T_3}{T_2}} a_0, \quad \mu = \sqrt{-2T_2}, \quad a_2 = \sqrt{\frac{-T_1 T_3 \sqrt{-2T_2} a_0^3}{T_2}} \frac{1}{T_1 a_0 \sqrt{-2T_2} + 3T_4 \sqrt{\frac{T_3}{T_2}} a_0}, \quad (10)$$

substituting the above equation into Eq. (5), we obtain the solitary wave solution,

$$z(x, t) = \frac{a_0 \sinh(\mu\xi)}{\left( \sqrt{\frac{-T_1 T_3 \sqrt{-2T_2} a_0^3}{T_2}} + \sqrt{\frac{T_3}{T_2}} a_0 \cosh(\mu\xi) \right)}. \quad (11)$$

The above equation represents the dromion-like solutions of Eq. (1). We have plotted the Eq. (11), for microtubulin system and obtained the dromion-like profile. We fix the viscosity value  $\eta = 0.001 \times 10^{-28}$ , and by keeping the parameter values  $m = 10^{26}$  kg,  $E = 10^5 \text{ V m}^{-1}$ ,  $q = 6 \times 10^{-18} \text{ C}$ ,  $l = 8 \times 10^{-8}$  m,  $c = 2.1$  m/s,  $a_0 = 1$ ,  $A = 1$  and  $B = 10^2$ , we obtain the dromion-like forms which is shown in the Fig. (1). By increasing the value of  $\eta = 0.01 \times 10^{-28}$ , we obtain the dromion profile with decreasing amplitude which is depicted in the Fig. (1b). Further, increasing the value of  $\eta = 0.1 \times 10^{-28}$ ,  $1 \times 10^{-28}$ ,  $5 \times 10^{-28}$  and  $10 \times 10^{-28}$ , the amplitudes of the dromion-like forms are decreasing by increasing the value of viscosity and traveling along the tubulin dimers which is shown in the Fig. (1 c-f). This amplitude variations are pointed in the darker region and exist in the corresponding contour plots which is depicted in the Fig. (1 a-f).

### 3. Conclusions

We investigate the microtubule dynamics and look for the governed nonlinear excitations in the form of dromions and the viscosity & amplitude of the dromions are inversely proportional to each other. This dromions will move smoothly from one end of the protofilament to another end of the protofilament. The nonlinear behaviour of microtubule system can be understood by the evolutionary plots which are in the form of dromions.

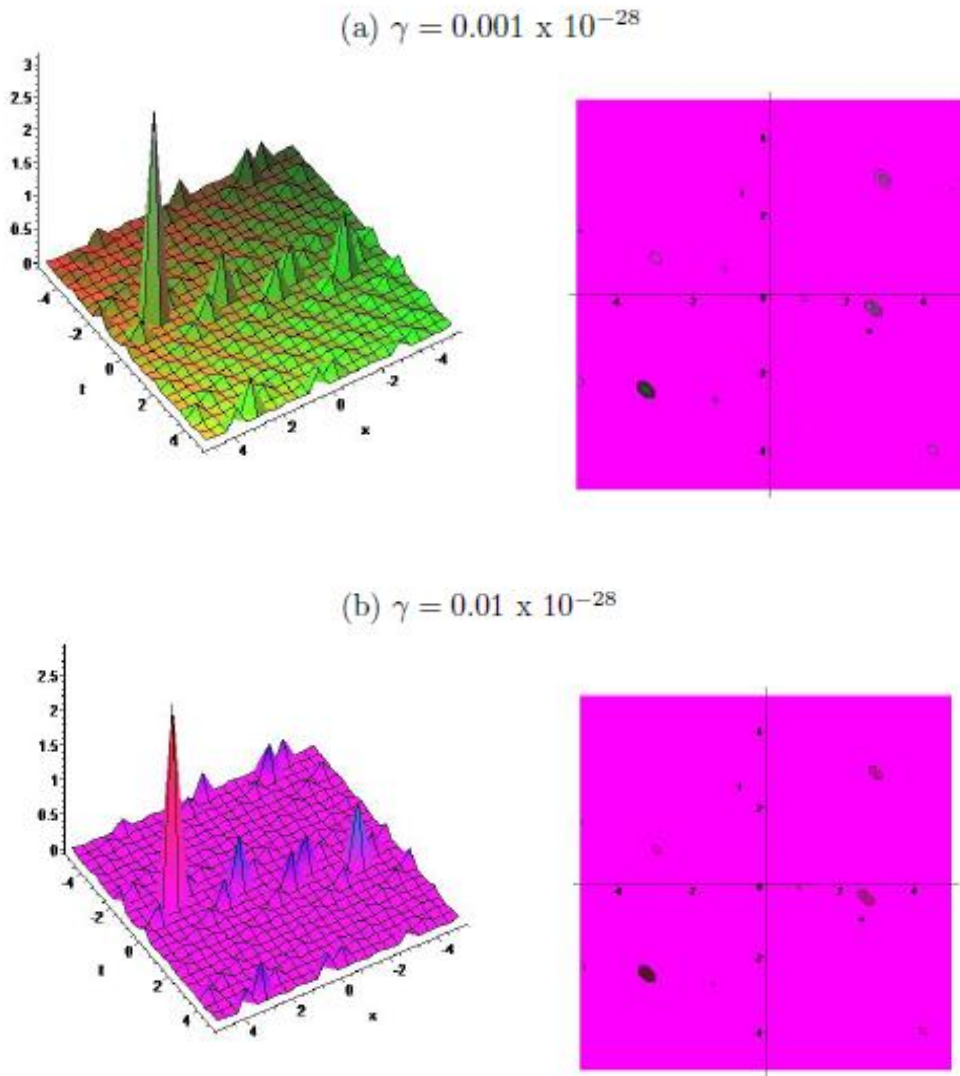
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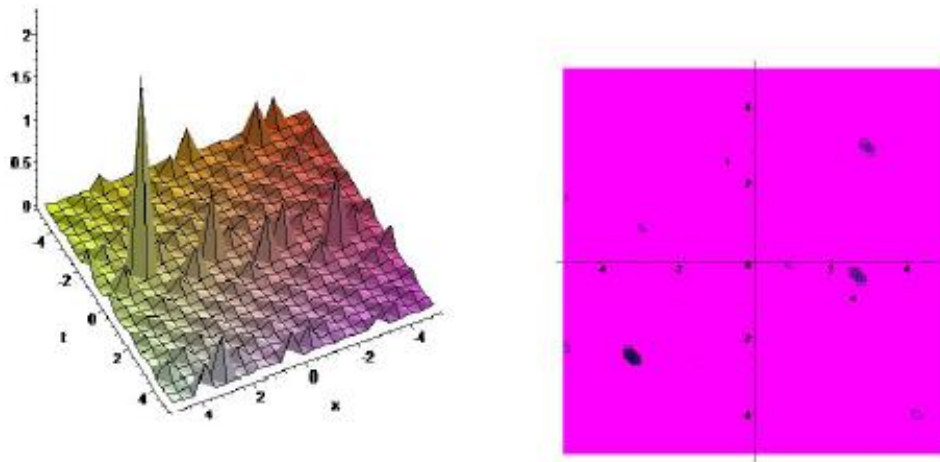
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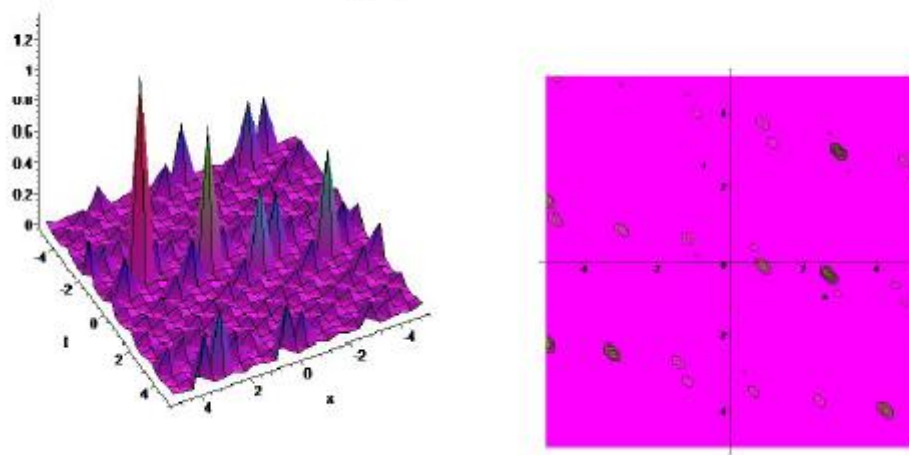
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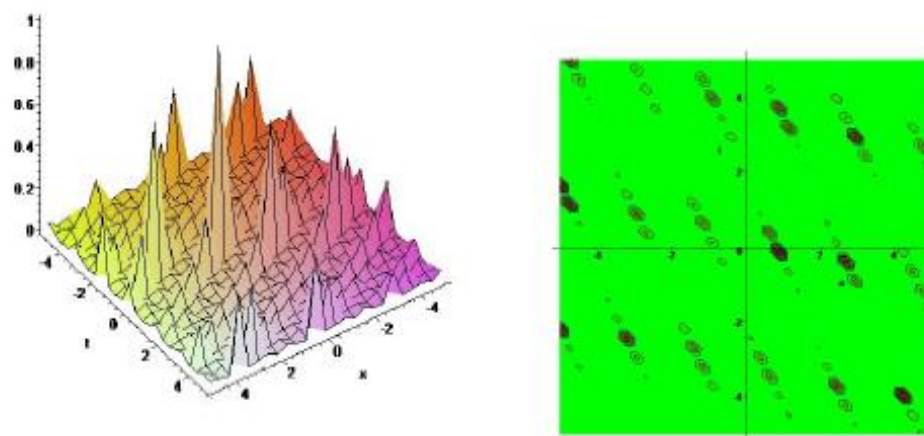
(c)  $\gamma = 0.1 \times 10^{-28}$

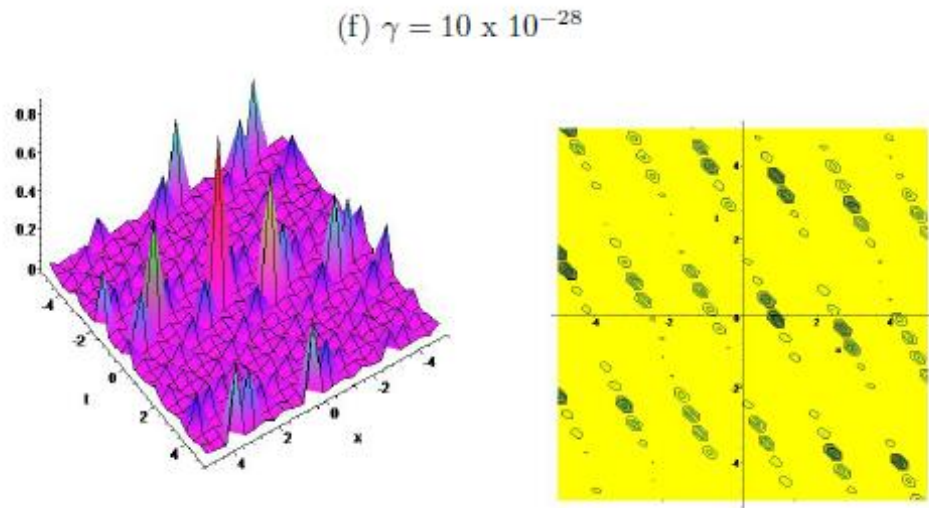


(d)  $\gamma = 1 \times 10^{-28}$



(e)  $\gamma = 5 \times 10^{-28}$





**Figure 1: Profile of dromions along the microtubules for Eq. (11).**