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s – Topology On n – Dimensional Minkowski Space

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Abstract

The present paper focuses on the characterization of compact sets of Minkowski space with a non - Euclidean s – topology. We discuss the characterizations of closed set of n – dimensional Minkowski space with s – topology, and comparision of s – topology with other topologies has been carried out, in detail.

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1. INTRODUCTION

Non – Euclidean topologies on 4 – dimensional Minkowski space were first introduced by Zeeman [8] in 1967. These topologies include fine, space topology [5], time topology [6], t – topology [6] and s – topology [6]. Studying the homeomorphism group of 4 – dimensional Minkowski space with fine topology, Zeeman in his paper [8] mentioned that it is Hausdorff, connected, locally connected space that is normal, not locally compact and not first countable. His results were interesting both topologically and physically, because its homeomorphism group was the group generated by the Lorentz group, translations and dilatation which was exactly the one physicists would want it to be.

Further, Nanda and Panda [7] introduced the notion of non – Euclidean topology, namely, order topology, and obtained that it is a non – compact, non – Hausdorff, locally connected, connected, path connected, simply connected space.

In 2007, Dossena [3] proved that n – dimensional Minkowski space, $n > 1$, with the fine topology is separable, Hausdorff, non - normal, non - locally compact, non - Lindelof and non -first countable. Quite recently, in 2009, Agarwal and Shrivastava [2] obtained a characterization for compact sets of Minkowski space with t – topology besides studying its topological properties. It may be noted that t – topology on 4 – dimensional Minkowski space is same as that of the well - known path topology on strongly casual spacetime proposed by Hawking et. al in 1976 [4].

In this paper, we study the concept of s – topology on n – dimensional Minkowski space in detail. Further we investigate generalized open subsets of Minkowski space endowed with the Euclidean topology and s- topology, respectively. We discuss the characterization of closed set of M^S and comparison of s– topology with other topologies on M has been carried out, in detail.

2. NOTATION AND PRELIMINARIES

Let $\wedge$ denote an indexing set while $\mathbb{R}$, $\mathbb{N}$ denote the set of real and natural numbers respectively. For a subset S of A, A\S denotes the complement of S in A. For $s, t \in \mathbb{R}^n$, let $d_E(s,t)$ be the Euclidean distance between s and t. For $\epsilon > 0$, $N_\epsilon^E(a)$ denotes the ϵ– Euclidean neighbourhood about a given by the set $\{b \in \mathbb{R}^n, d_E(a,b) < \epsilon\}$. For $a, b \in \mathbb{R}^n$, let $[a,b]$ denote the line segment joining a and b.

Definition 2.1. The n – dimensional Minkowksi space denoted by M, is the n – dimensional vector space $\mathbb{R}^n$ with a bilinear form $g : \mathbb{R}^n \mapsto \mathbb{R}$, satisfying the following properties:

(i) for all $a, b \in \mathbb{R}^n, g(a,b) = g(b,a)$, that is the bilinear form is symmetric.
(ii) for all $b \in \mathbb{R}^n, g(a,b) = 0$, that is the bilinear form is non – degenerate and
(iii) There exists a basis $\{e_0, e_1, \ldots e_{n-1}\}$ for $\mathbb{R}^n$ with

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$$g(e_i, e_j) = \begin{Bmatrix} 1 \; if \; i = j = 0 \\ -1 \; if \; i = j = 1,2,\dots,n-1 \\ 0 \; if \; i \neq j \end{Bmatrix},$$

the bilinear form g is called the Lorentz inner product. Elements of M referred to as events. If $x = \sum_{i=0}^{n-1} x_i e_i$ is an event, then the coordinate x_0 is called the time component and the coordinates $x_1, \dots, x_{n-1}$ are called the spatial component of x relative to the basis $\{e_0, e_1, \dots e_{n-1}\}$. In terms of components, the Lorentz inner product $g(x, y)$ of two events $x = \sum_{i=0}^{n-1} x_i e_i$ and $y = \sum_{i=0}^{n-1} y_i e_i$ is defined by $x_0 y_0 \sum_{i=0}^{n-1} x_i y_i$. Lorentz inner product induces an indefinite characteristic quadratic form Q on M given by $\mathrm{Q}(x) = g(x, x)$. Thus $\mathrm{Q}(x) = x_0^2 - \sum_{i=0}^{n-1} x_i^2$. The group of all linear operators T on M which leave the quadratic form Q invariant, that is $\mathrm{Q}(x) = \mathrm{Q}(T(x))$, for all $x \in$ M, is called the Lorentz group.

Definition 2.2. A event $a \in$ M is called space-like, light-like (also called null) or time-like vector according as $\mathrm{Q}(a)$ is negative, zero or positive. The sets $C^S(a) = \{b \in M: b = a \; or \; \mathrm{Q}(b - a) > 0\}$ are likewise, respectively called the spacecone, lightcone (or null cone), and time cone at a.

Definition 2.3. For a given $a, b \in M$, the set $\{a + t(b - a): t \in \mathbb{R}\}$ is called a spacelike straight line or lightray or time-like straight line joining a and b according as $\mathrm{Q}(b - a)$ is negative or zero or positive.

Definition 2.4. The Euclidean topology on the n - dimensional Minkowski space M is the topology generated by the basis $B = \{ N_\epsilon^E(a): \epsilon > 0, a \in M\}$. M with the Euclidean topology will be denoted by M^E.

Definition 2.5. The s - topology on the n - dimensional Minkowski space M is defined by specifying the local base of neighbourhoods at each $x \in M$ given by the collection $\aleph(a) = \{ N_\epsilon^S(a) : \epsilon > 0\}$, where $N_\epsilon^S(a) = N_\epsilon^E(a) \cap C^S(a)$ and $N_\epsilon^E(a)$ is the s – neighbourhood of radius ϵ. M endowed with s – topology is denoted by M^S.

Definition 2.6. The collection $\aleph(a) = \{ N_\epsilon^t(a) : \epsilon > 0\}$, where $N_\epsilon^t(a) = N_\epsilon^E(a) \cap C^T(a)$ forms a local base for the family of neighbourhoods of $a \in M$. The topology generated by these neighbourhood systems is called t - topology.

Definition 2.7. The fine topology (resp. A - topology) on M is the finest topology on M which induces one - dimensional Euclidean topology on every time - like line (respectively, time-like line and light-like line) and three - dimensional Euclidean topology on every space-like hyperplane.

Definition 2.8. The time (respectively space) topology on M with respect to which induced topology on every time like (respectively, space-like hyperplane) is Euclidean.

3. COMPARISION OF s – TOPOLOGY WITH OTHER TOPOLOGIES

In this section, a necessary and sufficient condition for a set to be open in M^S, has been obtained and also characterization of closed set of M^S has been found and we compare s - topology with other topologies.

Theorem 3.1. Let M^S be the n – dimensional Minkowski space with s – topology and G be the the non - empty subset of M. Then G is open in M^S if and only if G ∩ σ, G ∩ τ, G ∩ λ open in $\sigma^E, \tau^E, \lambda^E$ respectively.

Proof. If G is open in M^S then by definition of s – topology on M, G ∩ σ, G ∩ τ, G ∩ λ are open in $\sigma^E, \tau^E, \lambda^E$ respectively. Conversely let T be the topology generated by the basis $B = \{\mathrm{G} \subseteq M$: G ∩ σ, G ∩ τ, G ∩ λ are open in $\sigma^E, \tau^E, \lambda^E$ respectively$\}$. Clearly s – topology is coarser than T. Let H ∈ T. Then H ∩ σ, H ∩ τ, H ∩ λ are open in $\sigma^E, \tau^E, \lambda^E$ respectively, because H is the union of elements of B. Hence G is open in s – topology.

Theorem 3.2. Let M^S be the n – dimensional Minkowski space with s – topology and F be the the non - empty subset of M. Then F is closed in M^S if and only if F ∩ σ, F ∩ τ, F ∩ λ closed in $\sigma^E, \tau^E, \lambda^E$ respectively.

***Proof*.** Let F be closed in M^S. Then M\F is open in M^S. (M \F) ∩ σ, (M \F) ∩ τ, (M \F) ∩ λ are open in $\sigma^E, \tau^E, \lambda^E$ respectively. This implies that {σ \ (F ∩ σ)}, {τ \ (F ∩ τ)}, {λ \ (F ∩ λ) are open in $\sigma^E, \tau^E, \lambda^E$ respectively. Hence (F ∩ σ), (F ∩ τ), (F ∩ λ) are closed in $\sigma^E, \tau^E, \lambda^E$ respectively. Conversely, let (F ∩ σ), (F ∩ τ), (F ∩ λ) are closed in $\sigma^E, \tau^E, \lambda^E$ respectively. This implies that {σ \ (F ∩ σ)}, {τ \ (F ∩ τ)}, {λ \ (F ∩ λ) are open in $\sigma^E, \tau^E, \lambda^E$ respectively. Further, (M \F) ∩ σ, (M \F) ∩ τ, (M \F) ∩ λ are open in $\sigma^E, \tau^E, \lambda^E$ respectively. This implies that M\F is open in M^S. Hence F is closed in M^S.

Theorem 3.3. Let M^S be the n – dimensional Minkowski space with s – topology. Then

(i) $C^T(0) - \{0\}$ is open in M^S.
(ii) $C^S(0) - \{0\}$ is open in M^S.

Proof.

(i) Since $C^T(0) - \{0\}$ is open in M^E and s – topology is finer than Euclidean topology we have $C^T(0) - \{0\}$ is open in M^S.
(ii) Similar to (i).

Theorem 3.4. Let M^S be the n – dimensional Minkowski space with s – topology. Then singletons are not open in M^S.

Proof. Let $x \in M$. Then for any $\epsilon > 0$ there exists no open $N_\epsilon^E(x)$ such that $N_\epsilon^E(x) \subseteq \{x\}$ because $N_\epsilon^E(x)$ has infinitely many points. Hence $\{x\}$ is not open in σ^E, x is not open in M^S. Hence singletons are not open in M^S.

Theorem 3.5. Let M^S be the n – dimensional Minkowski space with s – topology. Then

(i) $C^L(0)$ is not open in M^S.
(ii) $C^T(0)$ is not open in M^S.
(iii) $C^S(0)$ is not open in M^S.

Proof.

(i) Let λ be the light ray passing through origin. Since $C^L(0) \cap \lambda = \{0\}$ and singletons are not open in $\lambda^E, C^L(0) \cap \lambda$ is not open in λ^E.
(ii) Proof of (ii) and (iii) are similar.

Theorem 3.6. Let M^Sbe the n – dimensional Minkowski space with s – topology. Then $C^L(0)$ is closed in M^S.

Proof. Let $X = C^L(0)$. Then $X^c = M - C^L(0) = \{C^S(0) \cup C^T(0)\} - \{0\}$. This implies $X^c = \{(C^S(0) - \{0\}) \cup (C^T(0) - \{0\})\}$. By theorem 3.3, $(C^S(0) - \{0\})$ and $(C^T(0) - \{0\})$ are open in M^S. Hence X^C is open in M^S. This implies that $X = C^L(0)$ is closed in M^S.

Theorem 3.7. Let M^S be the n – dimensional Minkowski space with s – topology. Then

(i) $C^L(0) - \{0\}$ is not closed in M^S.
(ii) $C^T(0) - \{0\}$ is not closed in M^S.
(iii) $C^S(0) - \{0\}$ is not closed in M^S.

Proof.

(i) Let λ be the light ray passing through origin respectively. Since $(C^L(0) - \{0\}) \cap \lambda = \lambda - \{0\}$ which is not closed in λ^E. This implies that $C^L(0) - \{0\}$ is closed in λ^E. Hence $C^L(0) - \{0\}$ is not closed in M^S.

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(ii) Proof of (ii) and (iii) are similar.

Remark 3.8. Since M^S is connected, the sets ϕ and M are clopen in M^S.

Comparison of s - topology with other topologies.

In this section, comparison of s - topology with other topologies on M has been carried out.

Theorem 3.9. Let M be the n - dimensional Minkowski space. Then s - topology on M is not comparable with

(i) t - topology
(ii) A - topology
(iii) f - topology on M.

Proof. Since $C^T(0)$ is open in M^t. By theorem 3.5 (ii), $C^T(0)$ is not open in M^S. This implies that t – topology is not coarser than s - topology. Further, let $\{s_n\}$ be the sequence of distinct spacelike straight lines passing through a point z. Let $z_n \in s_n$ such that $d(z_n, z) \mapsto 0$ and Z = $\{z_n\}$. Then the set Z is closed in M^S because (Z ∩ σ), (Z ∩ τ), (Z ∩ λ) are finite sets, hence closed in $\sigma^E, \tau^E, \lambda^E$ respectively. This implies that Z^c is open in M^S. On the other hand Z^c is not open in the t – topology. This shows that s - topology is not coarser than t - topology. Similar argument for A and f – topology.

Theorem 3.10. Let M be the n - dimensional Minkowski space. Then s – topology is coarser than

(i) space topology
(ii) time topology
(iii) fine topology on M

Proof. The s - topology on M induces three - dimensional Euclidean topology on every space-ike hyper-plane and space topology is the finest such topology. Hence s - topology coarser than space topology on M. Similar argument for time and space topology.

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