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Support Vector Machine: Detecting Linear Separable Class By Boolean Logic Gates

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Abstract

Artificial Neural Network is used as promising tool to resolve various real world problems. Most often the patterns are derived from real world measurement and cluster together based on some properties in certain regions. Pattern recognition can interpret as classification of data based on certain knowledge. The task of classification is to assign a test object to one of two classes. This paper focuses on linear and non-linear separability using classification of dataset. Generally linear separable problems are simple to solve than non-linear separable problem. We considered simple boolean function AND and OR for classification. To carry out the experiment, Zoo data set is used. SVM yields excellent performance on classification problem. We measured accuracy score using SVM.

Keywords: *Boolean Functions, Convex Hull, Convex Set, Separability, Support Vector Machine (SVM)*

1. INTRODUCTION

In human bodies, the cells within nervous system which transmits information to other nerve cell, muscle or gland are called neuron. According to (Kumar, 2014) Neuron is information processing cell of the brain. Neurons are interconnected with synapses. It is a biological neural network. It integrates the processing elements, these processing elements process the information sent to them as external input with dynamic nature (Raj, K. S., Nishnath M., & Jeyakumar, G. , 2020). In computer science replication of biological neural network is tried by Artificial Neural Network (ANN). ANN receives input signals, analyses it and predict precise output. ANN are widely applicable in various areas such as image processing, zip code reader, DNA finger printing, medical diagnosis, stock market prediction and signature verification. In machine learning research area, linear separability is widely used.

There are various approaches to show linear separable and non-linear separable problem. Our study presents linearly separable and non-linearly separable problem using Boolean logic gates. This paper proves that AND and OR logic gates are linearly separable problem using Support Vector Machine (SVM). A SVM finds the best separating (maximal margin) hyper plane between two classes of training samples in the feature space which leads to maximal generalization (Mavroforakis, M. E., & Theodoridis, S.,Jan 2005, 2013).

This paper is organized into 8 sections. Section 2 includes basic terminologies related to separability problem. Section 3 presents review of related work done in separability problem, neural network, logic gates. Section 4 elaborates methods for testing linear separability using Boolean logic gates. Section 5 shows proposed research methodology. Section 6 discusses implementation of Boolean logic gates by conducting the experiment, Section 7 concludes the paper and finally Section 8 lists the references used to write the research paper.

2. BASIC TERMINOLOGIES

2.1 Pattern recognition

Human being is able to efficiently recognize patterns without any effort. According to that recognition the decision is taken. A person classifies data into known categories. To recognize the cry of the child is the best example of it.

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Application:Weather forecasting, two patterns i.e. “rain indicated” and “rain not indicated” are two classes.

2.2 Two class data classification problem

Simple binary threshold neurons and their network can classify data by adding a simple hyper plane between data sets. Suppose, Q is a set of data samples or patterns and $X = \{ X_1, \ldots X_Q\}, X_i \in R^n$. These patterns are drawn from two classes C_1 and C_2. These classes can be separated by adding an appropriate hyper plane

2.3 Convex sets

A convex set S contains all the points on all line segments with end points in S.Suppose $X, Y \in S \subset R^n$ *then S is convex iff* $\lambda X + (1-\lambda) Y \in S, 0 <= \lambda <= 1, \forall X, Y \in S$.
Let X_1 and X_2 are two pattern sets drawn from two classes C_1 and C_2. In Fig.1 convex (ellipse) and non-convex sets are shown.

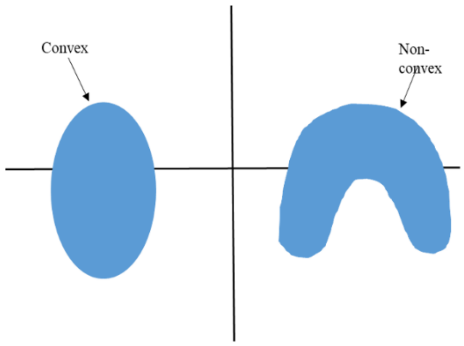

Figure 1 Convex and non-convex sets in R^2

2.4 Convex hulls

Let the pattern set Xi then its convex hull is *C(Xi)*. Here Xi is the smallest convex set in R^n which is known as convex hull. Every convex set Sα, such that $Xi \subset S\alpha \subset R^n$, where i is an index set. Fig.2 shows three types of Iris flowers data about petal length and petal width. The convex hulls of Iris sestosa and Iris versicolor are disjoint. So they are separable. But convex hulls of Iris versicolor and Iris virginica are not separable.

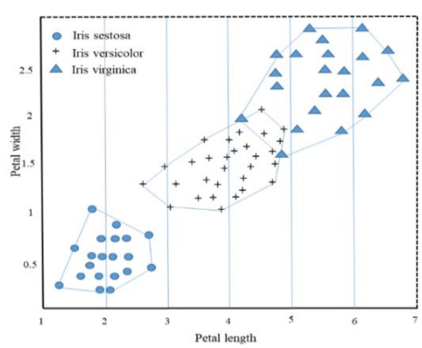

Figure 2 Convex hulls for Iris data

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2.5 Separability Problem

2.5.1 Linearly Separable

Suppose there are two pattern sets Xi and Xj, if the convex hulls of these patterns are disjoint or non-overlapping i.e. $C(Xi) \cap C(Xj) = \Phi$, it denotes that the pattern sets are linearly separable (Kumar, 2014). Here we can easily separate them by adding hyper plane between them. This leads to concept of linear separability. e.g. AND, OR, NAND, NOR logic gates. In Fig.3 convex hulls C1 and C2 are shown. These two convex hulls are disjoint.They can be separated by hyper plane.Thus, two convex hulls C_1 and C_2 of two pattern sets are linearly separable.

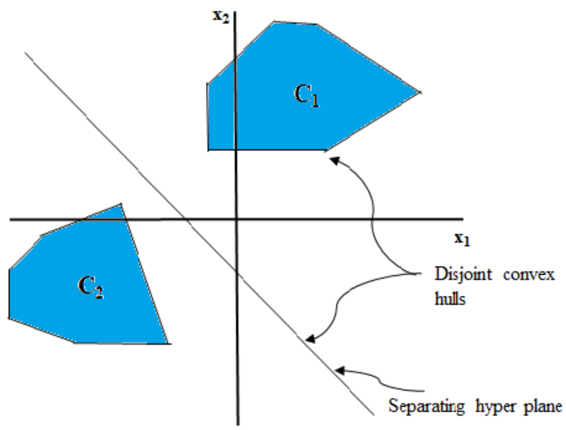

Figure 3 Linearly separable pattern classes

2.5.2 Non-linearly Separable

Suppose there are two pattern sets Xi and Xj , if the convex hulls of these patterns are joint or overlapping , then these pattern sets are not linearly separable. Here we cannot separate them by adding hyperplane between them. This leads to concept of Non-linearly separable problem. These pattern sets are not linearly separable but it can be non-linearly separable. e.g. XOR, XNOR logic gate.

In Fig.4 two convex hulls C_1 and C_2 are joint. So they are not linearly separable. They cannot separate by single straight line. But they can separate non-linearly by adding curve between them.

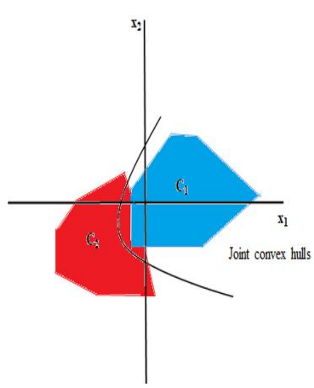

Figure 4 Non-linearly Separable

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3. RELATED WORK

1. Evgeny Bauman et al., "One-Class Semi-Supervised Learning: Detecting Linearly Separable Class by its Mean", in 2017 describes about semi-supervised classification method. Author has presented an algorithm which explains the class elements are linearly separable from other elements. Linear separability is identified and done maximal probability within in sets with the same mean. A new semi-supervised approach was proposed to one-class classification problem. Author has developed an algorithm which identifies linearly separable class using linear programming. Initially, linear SVM algorithm was ran 256-dimensional space and it had an error in the classification of only one element, class A is almost linearly separable in this space. Secondly, we applied the proposed algorithm. In this paper three applications cases were described: linear separability of the class, Gaussian distribution, and the case of linear separability in the transformed space of kernel functions. Furthermore, proposed algorithm was tested and analysed on USPS dataset. This dataset contains 9298-digit images of size 16x16 that is 256 pixels. Finally, performance of the algorithm was measured in terms of precision and recall factor.

2. Edgar Osuna et al., "Convex Hull in feature space for Support Vector Machine" in Conference Paper in Lecture Notes in Computer Science, November 2002 this study describes the working of SVM as well as a strong relationship between SVM and Convex Hulls. Author has proposed an algorithm which finds the extreme data points of Convex Hull in feature space. In SVM usually data classification is done in hyper-plane on decision surface. Author has shown an approach which finds extreme points in feature space using SVM's Kernel function. Application area for this method is incremental and parallel, needs some extension to non-linearly separable data set. Overall complexity is reduced by proposed method. Misclassified support vectors are rewritten as convex combinations of extreme points of the data set. One more important property of this method is dimensionality independence as convex hull extreme points are compared with another algorithm Quick-Hull.

3. Mirela Reljan-Delaney et al., "Solving the Linearly Inseparable XOR Problem with Spiking Neural Networks" in Computing Conference London UK, 2017 in this study author has coined a new term and emphasis on Spiking Neural Networks (SNN). In the area of artificial learning, SNN explores computational features and having strong competences. SNN has strong computational applicability and biologically accurate. This paper focus on approach which solves linearly inseparable problems by SNN. Author has conducted two experiments. First experiment is based on logic gates and second experiment is based on addition of receptive fields in order to filter the input. Author have shown that RFs as a powerful tool in the implementation of SNNs. SNN solves the XOR non-linearly separable problem. This methods also works well for scalable data and works on various data sets also. However, the network developed is not much elastic due to weights adjustment. Future plan is to create a much more flexible network, try to combine RFs with learning. This type of network will be capable to solve a wide range of frequencies and problems.

4. Taylor Simons et al., "A Review of Binarized Neural Networks " in MDPI Journal of Electronics 2019 in this research study author has reviewed Binary Neural Network. This type network takes binary values for activations of neurons. In BNN computations are done using bitwise operations that reduces execution time. BNN model size is small as compared to other neural network and accuracy rate is high. This network well for scalable dataset also. Because of efficiency of BNN, its model is good methodology for deep learning implementations. This paper covers methodology of BNN, architecture, development happen in this area, its review, hardware implementations of these networks and applications of BNN. Finally author concluded that even partial binarization have helped to make BNN accurate. In FPGAs and ASICs, BNN have been implemented. There are certain new tool like FINN that have made programming BNNs on FPGA accessible to more designers.

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5. Georgi I. Nalbantov et.al.,"Nearest Convex Hull Classification", in 2006,author proposed the new method of classification Nearest Convex Hull (NCH), which is used to estimate the class distance of the convex hull. To find out which was closest convex, NCH allots the test observations to the class. Overlapping of convex-hull was handled with support of slack variables and kernels. Here they used popular classifier method Support Vector Machine (SVM). NCH classifier method was a kind of instance-based large-margin classifier. They analysed the performance of NCH on various small- to middle-sized data sets. Then they compared the result of NCH with different techniques like SVM, Linear and Quadratic Discriminant Analysis. On all data set, NCH classifier accomplishes quite well. On three data sets SVM achieved finest accuracy. The other methods show relatively less promising and more unstable results. Here the advantage of NCH is that, it performs the task of binary to multi-class classification in a simple way. Other one is its claim robustness to outliers and have decent generalization qualities.

6. Kiran S. Raj et al., "Design of Binary Neurons with Supervised Learning for Linearly Separable Boolean Operations", in *International Conference On Computational Vision and Bio Inspired Computing ICCVBIC*, 2019, author designed and implemented artificial neuron which performs Boolean operations. They implemented this neuron, first without the learning capability and then with perceptron learning algorithm, neurons were added with learning capability. Their performances with expected result of Boolean operation were tested. They reveals that, for all type of Boolean operation artificial neurons can be simulated.

4. METHODS FOR TESTING SEPARABILITY

There are various approaches to show linearly separable and non-linearly separable problems. We have checked separability using boolean logic. A method based on Boolean logic:

4.1 Linearly Separable Problem

4.1.1 Boolean AND Function:

$$f_{\wedge}(x1, x2): B^2 \rightarrow \{0, 1\}$$

It maps from domain of 2^2 points i.e. B^2 into the range set (0, 1). Two pattern sets $X_0 = \{ (0, 0), (0, 1), (1, 0)\}$ *and* $x_1 = \{(1, 1)\}$.Tab.1 shows function of boolean AND gate.

In Fig.5 Corners of shaded triangle are shown by unfilled circles i.e. $C_0 = C(X_0)$. The corners are at *(0, 0), (0, 1), (1, 0)*. Another $C_1 = C(X_1)$ has only one point *(1, 1)*. Here C_0 and C_1 are disjoint and separated with straight line. We can draw many straight lines between filled and unfilled circles.

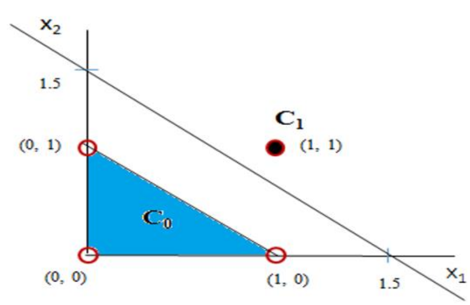

Figure 5 Geometry of Boolean AND gate

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4.1.2 Boolean OR Function:

$$f_{\vee}(x1, x2): B^2 \rightarrow \{0, 1\}$$

It maps from domain of 2^2 points i.e. B^2 into the range set (0, 1). Two pattern sets $X_0 = \{ (0, 0)\}$ *and* $x_1 = \{(0, 1), (1, 0), (1, 1)\}$.Tab.1 shows function of boolean OR gate.

In Fig. 6 Corners of shaded triangle are shown by filled circles i.e. $C_1 = C(X_1)$. The corners are at *(1, 1), (0, 1), (1, 0).* Another $C_0 = C(X_0)$ has only one point *(0, 0)*. Here C_0 and C_1 are disjoint and separated with straight line. We can draw many straight lines between filled and unfilled circles.

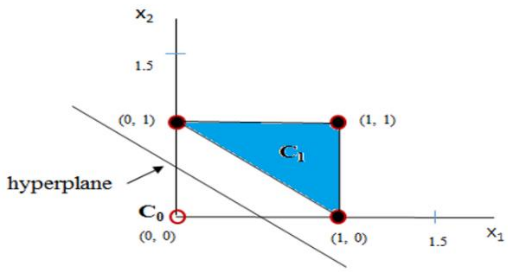

Figure 6 Geometry of Boolean OR gate

4.2 Non-linearly Separable Problem

4.2.1 Boolean XOR Function:

$$f_{\oplus}(x1, x2): B^2 \rightarrow \{0, 1\}$$

It maps from domain of 2^2 points i.e. B^2 into the range set (0, 1). Two pattern sets $X_0 = \{ (0, 0)\}$ and $X_0 = \{ (1, 1)\}$ appears on opposite sides and $x_1 = \{(0, 1), (1, 0)\}$ is at middle position. So we cannot separate X_0 and X_1 by adding single hyperplane. So XOR problem is not linearly separable. But it can be non-linearly separable.

Tab.1 shows function of boolean XOR gate. In Fig.7 Corners of shaded triangle are shown by filled circles i.e. $C_1 = C(X_1)$. The corners are at *(1, 1), (0, 1), (1, 0)*. Another $C_0 = C(X_0)$ has only one point *(0, 0)*. Here C_0 and C_1 are disjoint and separated with straight line. We can draw many straight lines between filled and unfilled circles.

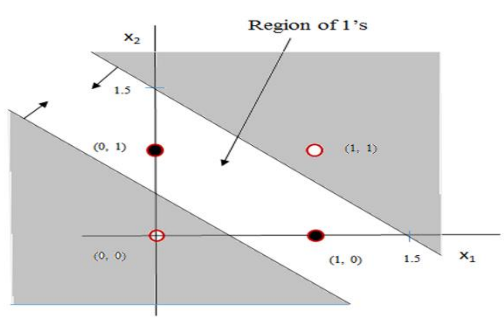

Figure 7 Geometry of Boolean XOR gate (Linearly not-separable)

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But XOR problem can be non-linearly separable. By taking the AND of OR and NAND gate. Logically it is shown in Tab.2. It is possible to separate the outputs. Fig.8 shows this situation by adding curve between the output of 1's and 0's.

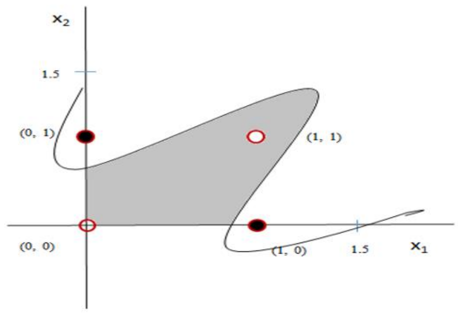

Figure 8 Geometry of Boolean XOR gate (Non-linearly separable)

5. PROPOSED RESEARCH METHODOLOGY

Consider a Zoo data set of Xi…Xn attributes which forms *k* different groups or classes. Let Xi, Xj denotes attributes in k^{th} class. From these attribute set of k^{th} class, we applied boolean AND, OR functions.

5.1 Linearly Separable Problem

To check linearly separability of AND and OR boolean logic gates, its functions are given below.

1. Boolean AND Function

$f_{\wedge}(x1, x2): B^2 \rightarrow \{0, 1\}$

2. Boolean OR Function

$f_{\vee}(x1, x2): B^2 \rightarrow \{0, 1\}$

If the convex hulls of these patterns are disjoint or non-overlapping i.e. $C(Xi) \cap C(Xj) = \Phi$ then these pattern sets are linearly separable (Kumar, 2014). To check linearly separable and non-linearly separable cases, Support Vector Machine (SVM) classification method is applied. Based on decision boundaries i.e. hyper plane, SVM classifies the data points. These data points fall on either side of the hyper plane can be classified into different classes. After analysis, performance result or accuracy score is calculated. Accuracy can be calculated by comparing actual test set of values and predicted values.

5.2 Non-linearly Separable Problem

Suppose there are two pattern sets Xi and Xj, if the convex hulls of these patterns are joint or overlapping, then these pattern sets are not linearly separable. Here we cannot separate them by adding hyper plane between them.

Table 1 Truth Table of Boolean Functions

INPUT		**AND**	**OR**	**XOR**
X1	**X2**	$f_{\wedge} = X_1.X_2$	$f_{\vee} = X_1 + X_2$	$f = X_1 \oplus X_2$ ----

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				$f_{\oplus}=X_1 . X_2 + \overline{X_1 . X_2}$
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

By taking the AND operation of OR and NAND we got XOR as shown in Tab.2.

Table 2 Truth Table For Non-Linearly Separable XOR Problem

INPUT		**OR**	**NAND**	**XOR**
X1	**X2**	$f_\vee = X_1+X_2$	$\overline{f_\wedge} = \overline{X_1.X_2}$	$f_\oplus = \overline{f_\vee . f_\wedge}$
0	0	0	1	0
0	1	1	1	1
1	0	1	1	1
1	1	1	0	0

5.3 Support Vector Machine (SVM)

Finding hyper plane in an N-dimensional space that classifies data distinctly is the objective of Support Vector Machine. Between the two classes of training samples in the feature space, which leads to maximal generalization, SVM finds the best hyper plane. In SVM classification, the target function is a hyper plane of the form $w'x+b = 0$, where w is a vector of coefficients and b in the intercept. The SVM hyper plane $w{*}'x+b{*} = 0$ (denoted as hSVM) is the one that separates the classes with the widest margin (Nalbantov, G., Groenen, P., & Bioch, J. ,2006).

Support vectors influence the position and orientation of the hyper plane. They are closer to the hyper plane. According to the position of data points on either side, hyper plane is placed. It signifies that the data is classified. Using these support vectors, the margin of the classifier is maximized.

6. EXPERIMENTAL SETUP

As we know that AND, OR logic gates are linearly separable, our study proves the same by carrying out experiment on Zoo dataset. We analysed accuracy score of SVM on ‘Zoo’ data set that is freely available in UCI repositories. This dataset is having multivariate attributes and 101 number of instances. Its Attributes are categorical, integer and boolean used for classification purpose. Among 18 attributes, one attribute is ‘type’ which is classified into 7 different output classes.

Here, we analysed our result through the implementation of python code with the help of matplotlib for plotting graph and scikit-learn (sklearn) to fit the SVM model. From Zoo dataset we selected two attributes *fins* and *tail*. We also selected two classes *class-4* and *class-6*. We performed bitwise AND operation on these two attributes. According to Tab.1 class wise conditions are applied. These results are

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visualized using scatter plot. These results are given to SVM model. The original dataset is randomly splitted into 80% and 20% as training and testing set respectively. Based on this training and testing data set SVC model is fitted. As a result, accuracy score is measured which gives 'best fit' means 100% accurate classification for Zoo data set. Same process is applied for the bitwise OR operation and we got the same result. Few sample records of Zoo dataset along with Boolean AND and Boolean OR operations are shown in Tab.3 and Tab.4 respectively.

Table 3 Comparison of Zoo dataset attributes along with classes for AND Gate

Animal name	**fins**	**tail**	**Type**	**Boolean AND**
bass	1	1	4	1
carp	1	1	4	1
catfish	1	1	4	1
chub	1	1	4	1
dogfish	1	1	4	1
flea	0	0	6	0
gnat	0	0	6	0
haddock	1	1	4	1
herring	1	1	4	1
honeybee	0	0	6	0

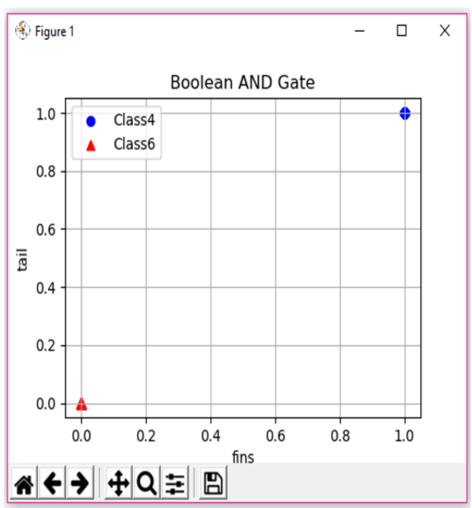

Figure 9 Linear Separability using AND Gate

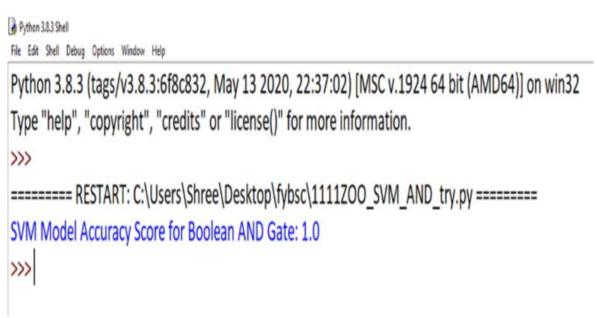

Figure 10 Accuracy Measurement Score of AND Gate

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Table 4 Comparison of Zoo dataset attributes along with classes for Boolean OR Gate

Animal name	fins	tail	type	Boolean OR
bass	1	1	4	1
carp	1	1	4	1
catfish	1	1	4	1
chub	1	1	4	1
dogfish	1	1	4	1
flea	0	0	6	0
gnat	0	0	6	0
haddock	1	1	4	1
herring	1	1	4	1
honeybee	0	0	6	0

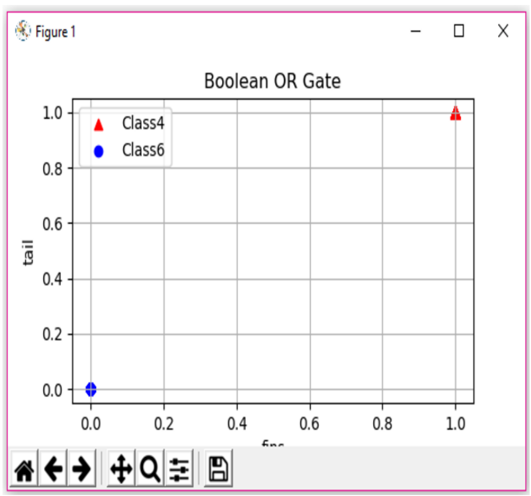

Figure 11 Linear Separability using OR Gate

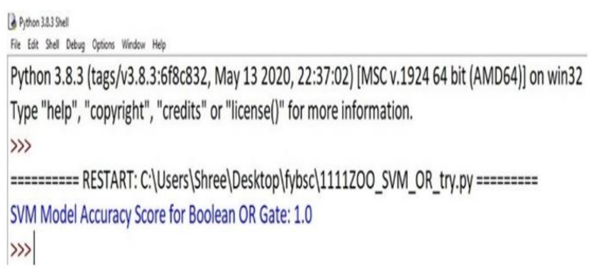

Figure 12 Accuracy Measurement Score of OR Gate

7. CONCLUSION

There are various approaches to show linearly separable and non-linearly separable problems. To check linearly separable and non-linearly separable cases, Support Vector Machine (SVM) classification method is applied. Based on decision boundaries i.e. hyper plane, SVM classifies the data points. In this paper we proposed boolean function using logic gates for linear separability of Zoo data set. AND and OR boolean functions are implemented in python programming. Results are visualized using scatter plot.

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These results are given to SVM model to measure accuracy score. Proposed method reveals higher classification accuracy.

This research study can be extended further to include simulation of all the Boolean functions NAND, NOR, XOR and XNOR using Support Vector Machine.

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