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A new oscillation criteria of first order nonlinear advanced differential equation with several deviating arguments.

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Abstract

In this paper, we established a new oscillation criteria for first order nonlinear advanced differential equation with several non monotone arguments. A new oscillation condition involving limsup and liminf is obtained. An example illustrating the result is also given.

Keywords: *non monotone, nondecreasing, several deviating arguments, Grönwall inequality.*

1.Introduction

Consider the first order nonlinear advanced differential equation of the form

$$u'(t)-\sum_{i=1}^{m} p_i(t)g_i(u(\sigma_i(t)))=0 \qquad t\geq t_0>0. \tag{1.1}$$

Througout this paper, we assume the following hypotheses hold :

(H1) $p_i(t),\sigma_i(t)\in C([t_{0,}\infty),R)$, $\sigma_i(t)$ is non-monotone or nondecreasing.

(H2) $\sigma_i(t)\geq t$ for $t\geq t_0$ and $\lim_{t\to\infty}\sigma_i(t)=\infty$ for $1\leq i\leq m$.

(H3) $g_i\in C(R,R)$ and $ug_i(u)>0$ for $u\neq 0$ for $1\leq i\leq m$.

By a solution u(t) of (1.1) we mean an absolutely continuous function on $\left[\sigma_i(T),\infty\right)$ for some $T\geq t_0$ and satisfying (1.1) for atmost all $t\geq T$. A solution of (1.1) is called oscillatory if it has arbitrarily large zeros. Otherwise it is called non oscillatory.

In the special case for $m=1$, (1.1) reduces to

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$$u'(t) - p(t)g(u(\sigma(t))) = 0 \qquad t \geq t_0 > 0\,. \tag{1.2}$$

where the functions p, σ are real valued functions, $\sigma(t) \geq t$ for $t \geq t_0$ and $\lim_{t\to\infty} \sigma(t) = \infty$.

Recently, there has been a considerable interest in the study of the oscillatory behaviour of the following special form of (1.1)

$$u'(t) - p(t)u(\sigma(t)) = 0\,, \qquad t \geq t_0 \tag{1.3}$$

In 1983, Fukagai and Kusano [7] proved that if

$$\lim_{t\to\infty} \int_t^{\sigma(t)} p(s)ds > \frac{1}{e}\ ,$$

then all solution of (1.3) are oscillatory, while if

$$\int_t^{\sigma(t)} p(s)ds \leq \frac{1}{e} \text{ for all sufficiently large t,}$$

then (1.3) has a non-oscillatory solution.

In 1990, Zhou[14] proved that if $\sigma(t) \leq \sigma_0$, $1 \leq i \leq m$ and

$$\liminf_{t\to\infty} \sum_{i=1}^{m} p_i(t)(\sigma_i(t) - t) > \frac{1}{e}\,,$$

then all solution of (1.3) are oscillate.

In 2011, Braverman and Karpuz,[3] proved that the following linear differential equation

$$u'(t) + p(t)u(\sigma(t)) = 0\,, \qquad t \geq t_0 \tag{1.4}$$

where p is a function of non-negative real numbers and $\sigma(t)$ is a non-monotone of positive real numbers such that $\sigma(t) < t$ for $t \geq t_0$ and $\lim_{t\to\infty} \sigma(t) = \infty$. They proved that if

$$\limsup_{t\to\infty} \int_{\delta(t)}^{t} p(s)\exp\left\{\int_{\sigma(s)}^{\delta(t)} p(u)du\right\}ds > 1$$

where $\delta(t) = \sup_{s\leq t} \sigma(s)$, $t \geq 0$, then all solution of (1.4) are oscillate.

The objective of this paper is to find a new condition for all solutions of (1.1) to be oscillatory when the arguments are not necessarily monotone.

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2. Oscillation results

In this section, we present a new oscillation criteria for the equation (1.1) under the assumption that $\sigma_i(t),\ 1 \le i \le m$ are not necessarily monotone. Set

$$\delta_i(t) := \inf_{s \ge t} \sigma_i(s), \qquad t \ge t_0 \tag{2.1}$$

Clearly, $\delta_i(t)$ are nondecreasing and $\sigma_i(t) \ge \delta_i(t),\ 1 \le i \le m$ for all $t \ge t_0$.

Assume that the function g in (1.1) satisfies the following condition

$$\limsup_{|u| \to 0} \frac{u}{g_i(u)} = L_i,\ \ 0 \le L_i < \infty, \qquad \text{for } 1 \le i \le m. \tag{2.2}$$

Lemma 2.1(Grönwall inequality)

If

$$u'(t) - p(t)u(t) \ge 0, \qquad t \ge t_0, \tag{2.3}$$

where $p(t) \ge 0$ and $u(t) \ge 0$, then we have

$$u(s) \ge u(t)\exp\{\int_t^s p(u)du\}, \qquad s \ge t \ge t_0. \tag{2.4}$$

Lemma 2.2[5]

Assume that (1.1) holds and

$$\liminf_{t \to \infty} \int_t^{\sigma(t)} p(s)ds = m > 0$$

then we have

$$\liminf_{t \to \infty} \int_t^{\sigma_i(t)} \sum_{j=1}^m p_j(s)ds = \liminf_{t \to \infty} \int_t^{\delta_i(t)} \sum_{j=1}^m p_j(s)ds = m, \tag{2.5}$$

where $\delta_i(t) := \inf_{s \ge t} \sigma_i(s)$, $t \ge 0$.

Theorem 2.1

Assume that the hypotheses (H_2), (H_3) and the condition (2.2) hold. If $\sigma_i(t)$ are non-monotone or non decreasing and if

$$\liminf_{t \to \infty} \int_t^{\delta(t)} \sum_{i=1}^m p_i(s)\exp\left\{\int_{\delta_i(t)}^{\sigma_i(t)} \sum_{j=1}^m p_j(u)du\right\}ds > \frac{L'}{e}, \tag{2.6}$$

where $L' = \max_{1 \le i \le m} L_i$ and $\delta(t) = \min_{1 \le i \le m} \delta_i(t)$, then all solutions of (1.1) oscillate.

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Proof:

Assume the sake of contradiction, that there exists a non oscillatory solution $u(t)$ of (1.1). Since $-u(t)$ is also a solution of (1.1) whenever $u(t)$ is a solution of (1.1), we can confine our discussion only to the case where the solution $u(t)$ of (1.1) is eventually positive. Then, there exists $t_1 > t_0$ such that $u(t) > 0$, $u(\sigma_i(t)) > 0$ and $u(\delta_i(t)) > 0$ for all $t \geq t_1$.

Thus, from (1.1) we have

$$u'(t) \geq \sum_{i=1}^{m} p_i(t) g_i(u(\sigma_i(t))) \geq 0 \text{ for all } t \geq t_1 \tag{2.7}$$

and therefore $u(t)$ is an eventually nondecreasing function.

Case(i)

Suppose $L_i > 0$ for $1 \leq i \leq m$, in view of (2.2) we can choose $t_2 > t_1$, so large such that

$$g_i(u(t)) \geq \frac{1}{2L_i} u(t) \geq \frac{1}{2L'} u(t) \text{ for all } t \geq t_2. \tag{2.8}$$

By (1.1), we have

$$\frac{u'(t)}{u(t)} - \sum_{i=1}^{m} p_i(t) \frac{g_i(u(\sigma_i(t)))}{u(t)} = 0 \text{ for all } t \geq t_2. \tag{2.9}$$

Integrating (2.9) from t to $\delta(t)$, we get

$$ln \frac{u(\delta(t))}{u(t)} - \int_t^{\delta(t)} \sum_{i=1}^{m} p_i(s) \frac{g_i(u(\sigma_i(s)))}{u(s)} ds = 0 \quad \text{for all } t \geq t_2. \tag{2.10}$$

Using (2.8) in (2.10) we get

$$ln \frac{u(\delta(t))}{u(t)} - \frac{1}{2L'} \int_t^{\delta(t)} \sum_{i=1}^{m} p_i(s) \frac{u(\sigma_i(s))}{u(s)} ds \geq 0 \quad \text{for all } t \geq t_2. \tag{2.11}$$

By Grönwall inequality , we have

$$ln \frac{u(\delta(t))}{u(t)} - \frac{1}{2L'} \int_t^{\delta(t)} \sum_{i=1}^{m} p_i(s) \frac{u(\delta_i(t))}{u(s)} \exp\left\{ \int_{\delta_i(t)}^{\sigma_i(s)} \sum_{j=1}^{m} p_j(u) du \right\} ds \geq 0 \text{ for all } t \geq t_2. \tag{2.12}$$

Using $t \leq s \leq \delta(t) \leq \delta_i(t)$ and the monotonicity of $u(t)$ we have $\frac{u(\delta_i(t))}{u(s)} \geq 1$.

Therefore (2.12) becomes

$$ln \frac{u(\delta(t))}{u(t)} - \frac{1}{2L'} \int_t^{\delta(t)} \sum_{i=1}^{m} p_i(s) \exp\left\{ \int_{\delta_i(t)}^{\sigma_i(s)} \sum_{j=1}^{m} p_j(u) du \right\} ds \geq 0. \tag{2.13}$$

From (2.6), there exists a constant $d > 0$ such that

$$\int_t^{\delta(t)} \sum_{i=1}^{m} p_i(s) \exp\left\{ \int_{\delta_i(t)}^{\sigma_i(s)} \sum_{j=1}^{m} p_j(u) du \right\} ds = 8L'd > \frac{L'}{e} \text{ for all } t \geq t_2 \geq t_1. \tag{2.14}$$

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Therefore

$$\frac{1}{2L'}\int_{t}^{\delta(t)}\sum_{i=1}^{m}p_i(s)\exp\left\{\int_{\delta_i(t)}^{\sigma_i(s)}\sum_{j=1}^{m}p_j(u)du\right\}ds = 4d \quad . \tag{2.15}$$

Combining (2.13) and (2.15) we get

$$ln\frac{u(\delta(t))}{u(t)} - 4d \geq 0 \text{ for all } t \geq t_3 > t_2,$$

That is

$$\frac{u(\delta(t))}{u(t)} \geq e^{4d} \geq 4ed > 1 \; . \tag{2.16}$$

Repeating the above procedure, it follows by induction that for any positive integer k,

$$\frac{u(\delta(t))}{u(t)} \geq (4ed)^k \to \infty \text{ as } k \to \infty, \tag{2.17}$$

since $4ed > 1$.

By Lemma (2.2), we have

$$\liminf_{t\to\infty}\int_{\delta_i(t)}^{\sigma_i(s)}\sum_{j=1}^{m}p_j(u)du = \liminf_{t\to\infty}\int_{\delta_i(t)}^{\delta_i(s)}\sum_{j=1}^{m}p_j(s)ds, \tag{2.18}$$

where $\delta_i(t) = \inf_{t\leq s}\sigma_i(s)$, $t > 0$.

Also from (2.6) and (2.18), it follows that there exists a constant $d > 0$ such that

$$\int_{t}^{\delta(t)}\sum_{i=1}^{m}p_i(s)\exp\left\{\int_{\delta_i(t)}^{\sigma_i(s)}\sum_{j=1}^{m}p_j(u)du\right\}ds = d > \frac{L'}{e}.$$

From (2.6) there exists a real number $t^* \in (t, \delta(t))$ such that

$$\int_{t}^{t^*}\sum_{i=1}^{m}p_i(s)\exp\left\{\int_{\delta_i(t)}^{\sigma_i(s)}\sum_{j=1}^{m}p_j(u)du\right\}ds > \frac{L'}{2e} \tag{2.19}$$

and

$$\int_{t^*}^{\delta(t)}\sum_{i=1}^{m}p_i(s)\exp\left\{\int_{\delta_i(t)}^{\sigma_i(s)}\sum_{j=1}^{m}p_j(u)du\right\}ds > \frac{L'}{2e}. \tag{2.20}$$

By (1.1) we have

$$u'(t) \geq \sum_{i=1}^{m}p_i(s)g_i(u(\sigma_i(s))) \geq 0 \text{ for all } t \geq t_3 . \tag{2.21}$$

Integrating (2.7) from t to t^*, we get

$$u(t^*) - u(t) \geq \int_{t}^{t^*}\sum_{i=1}^{m}p_i(s)g_i(u(\sigma_i(s)))ds$$

or

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$$u(t^*) \geq \int_t^{t^*} \sum_{i=1}^m p_i(s) g_i(u(\sigma_i(s)))ds$$

Using (2.8) in the last inequality we get

$$u(t^*) \geq \frac{1}{2L'} \int_t^{t^*} \sum_{i=1}^m p_i(s) u(\sigma_i(s))ds$$

Now using Grönwall inequality we get

$$u(t^*) \geq \frac{1}{2L'} \int_t^{t^*} \sum_{i=1}^m p_i(s) u(\delta_i(t)) \exp\left\{ \int_{\delta_i(t)}^{\sigma_i(s)} \sum_{j=1}^m p_j(u)du \right\} ds$$

or

$$u(t^*) \geq \frac{1}{2L'} u(\delta(t)) \int_t^{t^*} \sum_{i=1}^m p_i(s) \exp\left\{ \int_{\delta_i(t)}^{\sigma_i(s)} \sum_{j=1}^m p_j(u)du \right\} ds \tag{2.22}$$

Now using (2.19) the last inequality becomes

$$u(t^*) \geq \frac{u(\delta(t))}{4e} \text{ for all } t \geq t_3. \tag{2.23}$$

Similarly integrating (2.21) from t^* to $\delta(t)$ and also using (2.20), we obtain

$$u(\delta(t)) \geq \frac{u(\delta(t^*))}{4e} \text{ for all } t \geq t_3. \tag{2.24}$$

Combining (2.23) and (2.24), we get

$$u(t^*) \geq \frac{u(\delta(t))}{4e} \geq \frac{u(\delta(t^*))}{16e^2}.$$

That is

$$\frac{u(\delta(t^*))}{u(t^*)} \leq 16e^2 < \infty.$$

which is a contradiction to (2.17).

Case(ii)

Suppose $L = 0$

Assume that

$$\limsup_{|u| \to 0} \frac{u}{g_i(u)} = L_i = 0, \quad 0 \leq L_i < \infty. \tag{2.25}$$

Since $\dfrac{u(t)}{g_i(u(t))} > 0$, there exists $t_4 \geq t_3$ such that

$$\frac{u}{g_i(u)} < \varepsilon \text{ or } \frac{g_i(u)}{u} > \frac{1}{\varepsilon}, \quad t \geq t_4, \tag{2.26}$$

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where $\varepsilon > 0$ is an arbitrary real number. Thus, from (1.1) and (2.26), we have

$u'(t) > \frac{1}{\varepsilon}\sum_{i=1}^{m} p_i(t)u(\sigma_i(t))$.

Integrating (2.21) from t to $\delta(t)$ and using (2.26), we get

$$u(\delta(t)) - u(t) > \frac{1}{\varepsilon}\int_{t}^{\delta(t)}\sum_{i=1}^{m} p_i(s)u(\sigma_i(s))ds,$$

That is

$$u(\delta(t)) > \frac{1}{\varepsilon}\int_{t}^{\delta(t)}\sum_{i=1}^{m} p_i(s)u(\delta_i(t))\exp\left\{\int_{\delta_i(t)}^{\sigma_i(s)}\sum_{j=1}^{m} p_j(u)du\right\}ds$$

or

$$u(\delta(t)) > \frac{u(\delta(t))}{\varepsilon}\int_{t}^{\delta(t)}\sum_{i=1}^{m} p_i(s)\exp\left\{\int_{\delta_i(t)}^{\sigma_i(s)}\sum_{j=1}^{m} p_j(u)du\right\}ds$$

or

$$1 > \frac{L'}{e\varepsilon}$$

or

$$\varepsilon > \frac{L'}{e}.$$

which is a contradiction to $\lim_{|u|\to 0}\frac{u(t)}{g_i(u(t))} = 0$.

The proof is completed.

Theorem 2.2

Assume that the assumptions (H_2), (H_3) and the condition (2.2) hold, if

$$\limsup_{t\to\infty}\int_{t}^{\delta_i(t)}\sum_{i=1}^{m} p_i(s)\exp\left\{\sum_{j=1}^{m} p_j(u)du\right\}ds > L', \tag{2.27}$$

where $\sigma_i(t)$ is non-monotone or nondecreasing and $\delta(t)$ is defined as in (2.1), then all the solutions of (1.1) oscillate.

Proof:

Assume for the sake of contradiction, that there exists a non oscillatory solution $u(t)$ of (1.1). Since $-u(t)$ is also a solution of (1.1), whenever $u(t)$ is a solution of (1.1) therefore it is enough to prove the theorem for positive solutions of (1.1). Then, there exists $t_1 \ge t_0$ such that $u(t) > 0, u(\sigma_i(t)) > 0$ and $u(\delta_i(t)) > 0, 1 \le i \le m$, for all $t \ge t_1$ .Then, from (1.1) we have

$$u'(t) \ge \sum_{i=1}^{m} p_i(t)g_i(u(\sigma_i(t))) \ge 0 \text{ for all } t \ge t_1.$$

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and therefore $u(t)$ is nondecreasing for all $t \geq t_2$.

Again using (2.2), we have a constant $\xi > 1$ such that

$g_i(u(t)) \geq \frac{1}{\xi L_i} u(t) \geq \frac{1}{\xi L'} u(t)$ for all $t \geq t_2$.

Therefore

$$u'(t) \geq \sum_{i=1}^{m} p_i(t) g(u(\sigma_i(t))) \geq \frac{1}{\xi L'} \sum_{i=1}^{m} p_i(t) u(\sigma_i(t)) \text{ for all } t \geq t_2. \tag{2.28}$$

Integrating (2.28) from t to $\delta(t)$ and using the monotonicity of $u(t)$, we have

$$u(\delta(t)) - u(t) \geq \frac{1}{\xi L'} \int_t^{\delta(t)} \sum_{i=1}^{m} p_i(t) u(\sigma_i(s)) ds. \tag{2.29}$$

Using Lemma (2.2) in (2.29) and using Grönwall inequality, we have .

$$u(\delta(t)) \geq \frac{u(\delta(t))}{\xi L'} \int_t^{\delta(t)} \sum_{i=1}^{m} p_i(s) \exp\left\{ \sum_{j=1}^{m} \int_{\delta_i(t)}^{\sigma_i(s)} p_j(u) du \right\} ds. \tag{2.30}$$

That is

$$u(\delta(t)\left(1 - \frac{1}{\xi L'} \int_t^{\delta(t)} \sum_{i=1}^{m} p_i(s) \exp\left\{ \sum_{j=1}^{m} \int_{\delta_i(t)}^{\sigma_i(s)} p_j(u) du \right\} ds \right) \geq 0,$$

or

$$\int_t^{\delta(t)} \sum_{i=1}^{m} p_i(s) \exp\left\{ \sum_{j=1}^{m} \int_{\delta_i(t)}^{\sigma_i(s)} p_j(u) du \right\} ds \leq \xi L'.$$

Taking lim supremum, we have

$$\limsup_{t \to \infty} \int_t^{\delta(t)} \sum_{i=1}^{m} p_i(s) \exp\left\{ \sum_{j=1}^{m} \int_{\delta_i(t)}^{\sigma_i(s)} p_j(u) du \right\} ds \leq \xi L' \text{ for all } t \geq t_2. \tag{2.31}$$

From (2.27) we have

$$\limsup_{t \to \infty} \int_t^{\delta(t)} \sum_{i=1}^{m} p_i(s) \exp\left\{ \sum_{j=1}^{m} \int_{\delta_i(t)}^{\sigma_i(s)} p_j(u) du \right\} ds = M > L'.$$

Then

$$L' < \frac{M + L'}{2} < M.$$

By choosing $\xi = \frac{M + L'}{2L'} > 1$ we have

$$\limsup_{t \to \infty} \int_t^{\delta(t)} \sum_{i=1}^{m} p_i(s) \exp\left\{ \sum_{j=1}^{m} \int_{\delta_i(t)}^{\sigma_i(s)} p_j(u) du \right\} ds = M \leq \xi L' = \frac{M + L'}{2}.$$

which is a contradiction to $\frac{M + L'}{2} < M$ and the proof is completed.

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3 Example

Example 3.1.

Consider the equation

$$u'(t)-\frac{1}{25}u(\sigma_1(t))ln(|12+u(\sigma_1(t))|)-\frac{2}{25}u(\sigma_2(t))ln(|15+u(\sigma_1(t))|)=0, \quad t>0,$$

where

$$\sigma_1(t)=\begin{cases} 5t-20k+1, & if\ t\in[5k,5k+1] \\ -2t+15k+8, & if\ t\in[5k+1,5k+2] \\ 4t-15k-4, & if\ t\in[5k+2,5k+3] \\ -2t+15k+14, & if\ t\in[5k+3,5k+4] \\ 5k+6, & if\ t\in[5k+4,5k+5] \end{cases}$$

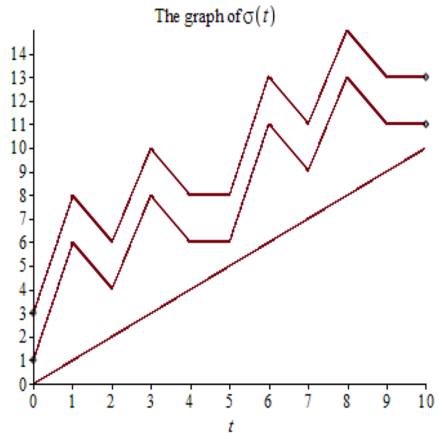

Figure 1.The graph of $\sigma_1(t)$, $\sigma_2(t)$

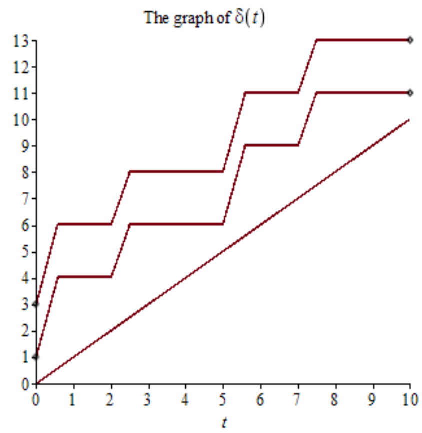

Figure 2.The graph of $\delta_1(t)$, $\delta_2(t)$

By (2.1), we have

$$\delta_1(t)=\inf_{s\ge t}\sigma_1(s)=\begin{cases} 5t-20k+1, & if\ t\in[5k,5k+3/5] \\ 5k+4, & if\ t\in[5k+3/5,5k+2] \\ 4t-15k-4, & if\ t\in[5k+2,5k+5/2] \\ 5k+6, & if\ t\in[5k+5/2,5k+5] \end{cases}$$

and $\delta_2(t)=\inf_{s\ge t}\sigma_2(s)=\delta_1(t)+2$, $k\in N_0$ and N_0 is the set of non negative integers.

Therefore

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$\delta(t) = \min_{1\le i\le 2}\{\delta_i(t)\} = \delta_1(t)$.

If we put $p_1 = \frac{1}{25}$, $p_2 = \frac{2}{25}$, $g_1(u) = uln(|12 + u(\sigma_1(t))|)$ and $g_2(u) = uln(|15 + u(\sigma_2(t))|)$.

Then we have

$$L_1 = \limsup_{|u|\to 0} \frac{u}{g_1(u)} = \limsup_{|u|\to 0} \frac{u}{uln(12 + |u(\sigma_1(t))|)} = \frac{1}{ln12}$$

$$L_2 = \limsup_{|u|\to 0} \frac{u}{g_2(u)} = \limsup_{|u|\to 0} \frac{u}{uln(15 + |u(\sigma_2(t))|)} = \frac{1}{ln15}$$

$$L' = \max\{L_1, L_2\} = L_1 = \frac{1}{ln12}.$$

Now at $t = 5k + 2.5$, $k \in N_0$ we have

$$\int_t^{\delta(t)} \sum_{i=1}^{2} p_i(s)\exp\left\{\sum_{j=1}^{2}\int_{\delta_i(t)}^{\sigma_i(s)} p_j(u)du\right\}ds$$
$$= \int_t^{\delta(t)} p_1(s)\exp\left\{\int_{\delta_1(t)}^{\sigma_1(s)} (p_1(u) + p_2(u))du\right\}ds + \int_t^{\delta(t)} p_2(s)\exp\left\{\int_{\delta_2(t)}^{\sigma_2(s)} (p_1(u) + p_2(u))du\right\}ds$$

$$\int_{5k+2.5}^{5k+6} \frac{1}{25}\exp\left\{\int_{5k+6}^{4s-15k-4} \frac{3}{e}du\right\}ds + \int_{5k+2.5}^{5k+6} \frac{2}{25}\exp\left\{\int_{5k+8}^{4s-15k-2} \frac{3}{e}du\right\}ds$$
$$= \int_{5k+2.5}^{5k+6} \frac{1}{25}\exp\left\{\frac{3}{e}(4s - 20k - 10)\right\}ds + \int_{5k+2.5}^{5k+6} \frac{2}{25}\exp\left\{\frac{3}{e}(4s - 20k - 10)\right\}ds$$
$$= \int_{5k+2.5}^{5k+6} \frac{3}{25}\exp\left\{\frac{3}{25}(4s - 20k - 10)\right\}ds$$
$$= \frac{1}{4}[\exp\frac{42}{25} - 1]$$
$$= 1.091338899928 > 1$$

$$\liminf_{t\to\infty} \int_t^{\delta(t)} \sum_{i=1}^{m} p_i(s)\exp\left\{\int_{\delta_i(t)}^{\sigma_i(t)} \sum_{j=1}^{m} q_j(u)du\right\}ds > 1 > \frac{L'}{25} = \frac{1}{25ln12}$$

$$\limsup_{t\to\infty} \int_t^{\delta(t)} \sum_{i=1}^{m} p_i(s)\exp\left\{\int_{\delta_i(t)}^{\sigma_i(t)} \sum_{j=1}^{m} q_j(u)du\right\}ds > 1 > L' = \frac{1}{ln12}$$

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That is, all conditions of Theorem2.1 and Theorem2.2 are satisfied. Therefore all solutions of (1.1) oscillate.

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