

A Comparative Analysis of Denoising MRI Images using MRA Transforms

^[1] P.J. Mercy, ^[2] Dr. Heren Chellam. G

^[1] Research Scholar, Register Number: 18221172162015, Rani Anna Govt. College for Women, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012, Tamil Nadu, India.

^[2] Assistant Professor, Department of Computer Science, Rani Anna Govt. College for Women, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012, Tamil Nadu, India.

Abstract - A very challenging and important factor in biomedical images is the noise. Image Denoising is a methodology used to remove the noises present in the images. There has been always a search for efficient denoising methods in recent days. Different kinds of noises like Additive Gaussian Noise, Impulse Noise, Random Noise and Poisson Noise may affect the medical images. In this proposed method, a new adaptive threshold based Discrete Wavelet Transform is compared against the traditional Discrete Wavelet, Contour let and Curve let transform for denoising images. Wavelets use a smaller number of coefficient to represent an image and hence the processing time to denoise the image has proved to be lesser than the other transforms. The evaluations of data have been done by Peak Signal to Noise Ratio, Mean Square Error and the Structural Similarity.

Index Terms—Gaussian Noise, Peak Signal to Noise Ratio, Discrete Wavelet Transform, Curve let Transform, Contour let Transform, Structural Similarity, Random Noise, Impulse Noise, Poisson Noise, Denoise.

1.0 INTRODUCTION

Medical images consist of various types of complex noises in its acquisition and also transmission processes. The most important noise that affects the medical images are visual noise. This gives a snowy or grainy textured appearance for these affected images. It can either be dependent on the image or it may be image independent. None of the imaging techniques now is free of noises. Daubechies and Haar wavelets are some examples of the different types of wavelets that has been used for medical images.

The Wavelet Transform has performed considerably in the denoising area over the past decade. The Wavelets divides the image into various frequency components and examines each of the components separately using a Mathematical tool. The frequency determines the basic components of the wavelets. Small waves also called as Mother Wavelets are also used in the implementation of Wavelet Transform [3]. The mother wavelet is found to be a window function that moves with time. Different types of window functions can be generated by using the mother wavelets. This performs the decomposition of the signal $x(t)$ into $y(t)$ which is a weighted set of wavelets that are being scaled.

Contour let transforms one another transform that captures the smoothness of the image's contour that has elongated shapes in different varieties of directions. It has been used efficiently in denoising medical images. The contour let transform captures the smooth contours of the images by a two-fold filter bank. The first filter bank comprises of a Laplacian filter bank that captures the image discontinuities. The second filter bank is a Directional filter bank that utilizes the point discontinuities

into linear structures.

Curvelet transform is also another multiscale transform where the frame elements are indexed by location parameters and scale [5]. The curves in this transform has been represented by singularities and edge more efficiently than the other transforms. The other transforms use more number of coefficients to represent a curve whereas a curvelet transform does not use a larger number of coefficients. The wavelets require $\frac{1}{N}$ wavelets but a curvelet uses only $\frac{1}{\sqrt{N}}$ curvelets, thus reducing the complexity in space and time[5].

The human visual system is characterized by the receptive fields of the visual cortex that will be localized, oriented and band passed. Thus, it indicates that the human visual system has to be tuned based on the essential information that has to be catered in an image. For any image that has to be represented effectively, the properties that has to be looked into are local, directional, multiresolution expansions and anisotropy [1].

Localization: The basic components of an image has to be localized in both domains – spatial and frequency domains.

Directionality: The image that is denoised has to cater in all the variety of directions than considering only a few directions.

Multiresolution: The images that are denoised has to be approximated from coarse to fine resolutions.

Critical Sampling: A small amount of redundancy exists in the denoised image.

Anisotropy: The smoothness of the images are captured by using a variety of shapes that may be elongated and also that may have different aspect ratios.

The properties mentioned above makes the representation of the image look clearer and better denoised.

2.0 LITERATURE SURVEY

The problem of denoising the medical images include the multiresolution transforms. When the thresholding technique, is used the coefficients are being thresholded by comparing against a standard threshold; Whenever the coefficient becomes smaller than the threshold, then it can be set to zero, otherwise it has to be kept or modified. Multiresolution methods have in literature been deeply related to image processing, computer vision, biological and scientific computing [8]. The curvelet allows a nonadaptive sparse representation of objects with edges. It has got increased generated interest in the applied mathematics community and signal processing over the past few years. KotherMohideen et al (2008) denoised natural images that has been corrupted by Gaussian noise using wavelet multi resolution transform [9]. Florian Luisier & Thierry Blu (2008) have suggested a algorithm for denoising which is characterized as a LinearExpansion of Thresholds (LET) [9]. Bhat et al (2010) used optimal threshold to denoise natural images that has been degraded by Additive White Gaussian Noise. PSNR is used as a measurement [10]. The images that were used are natural images. Florian Luisier et al has proposed a Poisson-Gaussian Unbiased Risk Estimate with Linear Expansion of Thresholds (PURE-LET) to design and optimize a variety of Wavelet Transform classes using thresholding algorithms for denoising images corrupted by mixed Poisson-Gaussian noise [11]. BhawnaGoyal et al has cited how to denoise medical images effectively [12]. For X-ray and CT-Scan images a better denoising method for Poisson noise has been proposed by Dang et al[13]. In the case of processing medical imaging systems, denoising has been found to be one of the important image processing tasks. Hence an Automatic noise removal will improve the quality of diagnosis and requires careful treatment of obtained imagery. Taking advantage of the mentioned properties of all the multiresolution transform and their underlying structure, a denoising algorithm that processes the sub-bands of the image separating them into regions [14]. By estimating the characteristics and their adjacent coefficients in

the same scale, a dependency of the coefficients on the heavy-tailed nature and the intrascale statistical dependency is being calculated [15]. Malini et al has considered six Multi resolution transforms applied to colour images and has made a comparative analysis of them [16]. To analyze the image line or curve singularities, an idea is that is to be consider on partitioning of the image, is the ridgelet transform that can be applied to the subimages. Curvelet transform was proposed by the authors to handle image boundaries by mirror extension [17]. The connection between the continuous directional wavelet and the discrete filter bank and is established. Experiments were conducted on directional feature extractions, for image denoising and nonlinear approximation by the authors[18]. The denoising methods proposed by Ryan Moore et al demonstrate that the contourlets and curvelets are found to be an alternative for the Wavelets and Fourier Tansforms during signal processing. The results indicate that the contourlet and curvelet methods outperform the Wavelets and Fourier transforms [19]. Ultrasound images have been found to be affected by Speckle noise and hence the Multiresoultion transforms has been found to be used in these analyses as a preprocessing step. The wavelet, Contourlet and Curvelet has been used by the authors to remove the multiplicative speckle noise found in the image acquisition [20].

3.0 METHODOLOGY

Images in the database were denoised using wavelet, contourlet and curvelet transform. Various types of noises that include random noise, speckle noise, salt and pepper noise and Gaussian noise was added to the images. Various thresholding was applied to the images and for each thresholding the images were denoised as shown in Fig 1.

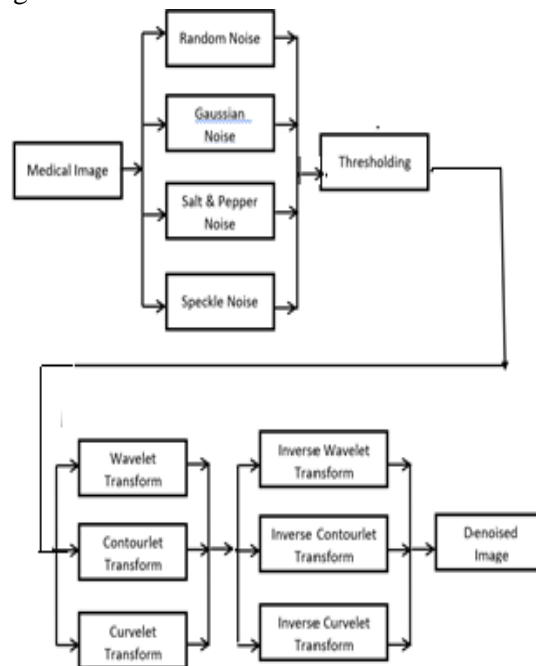


Fig 1. The Proposed methodology for Denoising the image

A comparative study for all the images were done using the Peak Signal to Noise Ratio, Mean Square Error and Structural Similarity and the best denoising method was selected.

3.1 Wavelet Transform

The wavelets examine the signals in both the time and frequency domain leading to efficient signal denoising. It has multiscale and multidimensional properties like Localization, Multiresolution and

Critical Sampling makes it excel efficiently in denoising. Wavelet methods in the decomposition stage, decompose the images into three high frequency *Sub-bands* and one smooth low frequency *sub-bands* as shown in Fig 2.

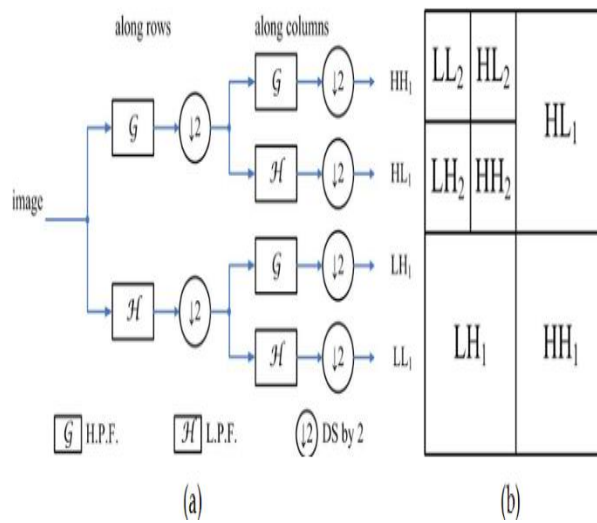


Fig. 2. Discrete Wavelet Transform a) Filter Banks b) Multiresolution analysis

With this method, the image resolutions seem to be the same before and after denoising. The result of a wavelet transform incorporates a pyramidal decomposition. The group of filters that are used to divide up the signal into different spectral components is called *Sub-band Coding*. The coefficients determine the type of filters that are being used.

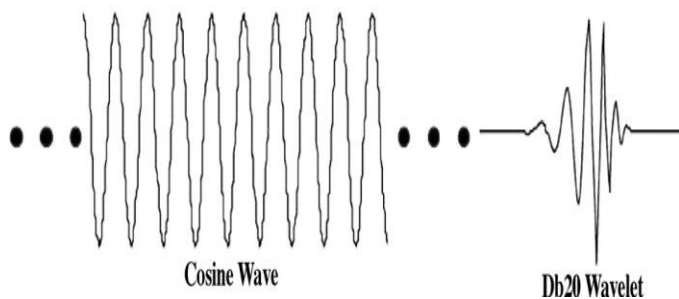


Fig. 3 A sinusoidal wave and its Discrete Wavelet Transform

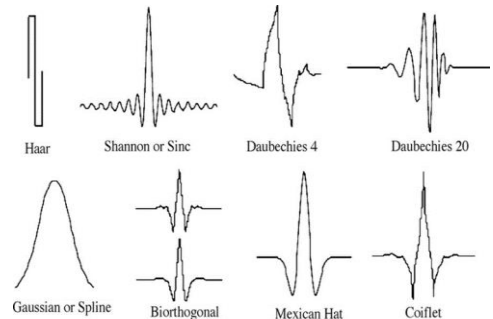


Fig. 4 An example of wavelets for CWT and DWT applications

It would be better if what is wavelet is being described. A waveform with limited duration and an average value of zero is called a wavelet. They have different shapes and sizes as shown in Fig. 3. Wavelets have a beginning and end, unlike the sinusoids that extend from minus infinity to plus infinity. A sinusoidal wave is having a constant frequency/stationary signals that are smooth and predictable. The wavelets on the other side are irregular with limited duration and they are often found to be non-symmetrical as shown in Fig 4. If any pulses or transient events start and end within a signal, then wavelets seem to be a better choice.

There are many different probing functions in wavelet that are divided into different orders/families. They are always composed of enlarged or compressed versions of the basic function of the wavelets

as well as its translations. Wavelet families and wavelets such as Morlet, Daubechies and Coiflet etc. are shown in Fig. 3.

The synthesis filter banks or the quadrature mirror filter determine mathematically the reconstruction of the original signal with addition to the approximations and details. Multilevel decomposition and reconstruction can be done to have an effective denoising method. The synthesis of filter banks and its reconstructions are shown in Fig 5.

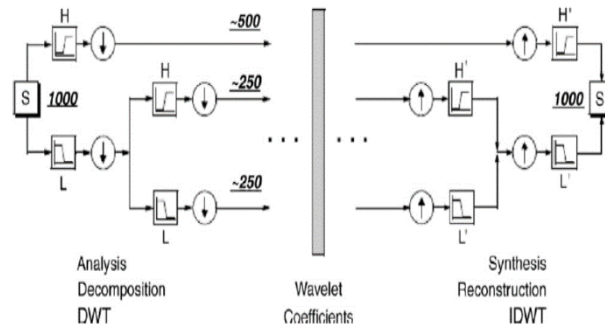
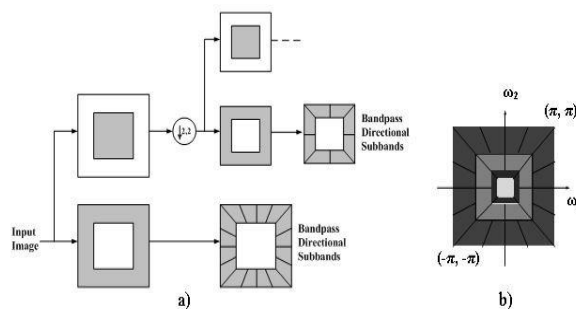


Fig. 5 – Multilevel decomposition using quadrature mirror filter structure

The forward algorithm and the reverse algorithm both combine the filtering operations. This always halves the data and enhances the computational speedup in processing. Hence the denoising of data is done effectively.

3.2 Contourlet Transform

The contourlet transform is an efficient directional multiresolution representation of an image with fine details of the structure. This transform possesses localization properties, multiresolution properties along with the directional and anisotropic properties. It is constructed using the filter banks and Laplacian Pyramids [4].



**Fig. 6. The Contourlet Transform a) Structure of the Laplacian together with the directional filter bank
 b) frequency partitioning by the contourlet transform**

The filter banks of the pyramid decompose the l level binary tree thereby yielding it to produce 2^l Sub-band triangle frequencies[1]. Fig 6 indicates the multiscale decomposition and directional decomposition of the Contourlet transform. The multiscale decomposition allows low frequencies to pass through in some of the Sub-bands. Thus, the directional filter bank is not able to produce a scarce display of images. The images are initially passed through the Laplacian pyramids and then they are operated through the directional filter banks as shown in Fig 7. Now the directional information is

available about the image. The low frequency domains are then subjected to the same steps by low pass filters.

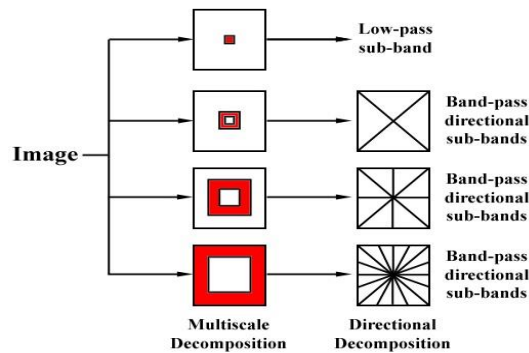
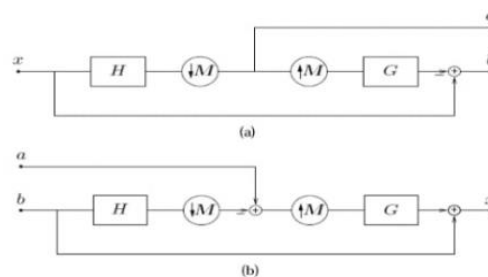


Fig. 7 The Contourlet Transform – Multiscale Decomposition and Directional Decomposition

The most important property of the Contourlet transform is its expansion on quadrangle nets. The number of polar squares is lesser than the amount of central frequency squares in the Contourlet Transform.



**Fig 8.Laplacian Pyramid a) One level decomposition
 b) The reconstruction scheme for the Laplacian Pyramid.**

The Laplacian Pyramid has been introduced by Burt and Adelson. The image produces a down sampled low pass version and a difference of the original and prediction resulting in an image that has been band passed. M indicates the sampling matrix, H – Low pass Filters and G – Synthesis filters. The image does not have scrambled frequencies. The reconstruction scheme is produced by adding the difference to the prediction from the Course signal to produce a better denoised image as predicted in Fig 8.

3.3 Curvelet Transform

Curvelet uses a multiscale Ridgelet Transform method. Its coefficient is produced by making the ridgelet transform to be performed on a single scale and then applied to the different *Sub-bands* of the frequency domains in the Wavelet domain. The following steps are performed for a Curvelet Transformation [2].

1. The image is sectioned into different *Sub-band* frequencies in the Wavelet domain.
2. A uniform mask is used in the frequency domain to divide into desirable scale squares.
3. Now the squares are normalized
4. The Ridgelet transformation is used in each of the normalized squares as shown in Fig 9.

The curves in the image are considered as straight lines with fine scales. To get an optimal rate of

convergence, the Curvelet uses the multiscale analysis and geometrical ideas using the simple thresholding methodologies. Whenever Multi-scale decomposition occurs, the point discontinuities are captured into linear structures [6]. The Curvelets have a variable width and also a variable length and hence a variable anisotropy.

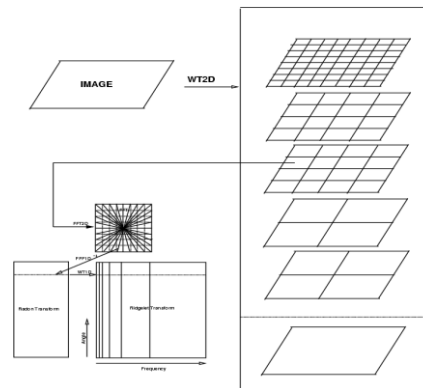
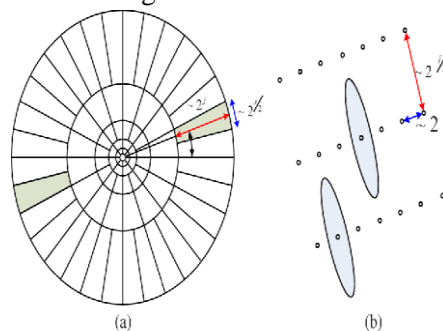


Fig. 9. The Curvelet Transform – Image decomposed into sub-bands

Fig 10 illustrates how the curvelet is represented in the frequency domain and spatial domain. Curvelets are well suited for representing smooth curves and it takes this advantage represents the higher resolution curvelets to be more elongated than the lower resolution curvelets.



**Fig 10. Curvelet tiling of space and frequency a) tiling in the frequency plane
 b) Spatial grid of the curvelet at a given scale and orientation [7]**

As the smooth curves are well represented by the curvelet coefficients, the curvelet is well suitable for denoising the images.

3.4 IMAGE DENOISING PERFORMANCE METRICS USED

The performance metrics are used to measure the quality between the noisy and denoised images. In this work Mean Square Error (MSE), Peak Signal to Noise Ratio (PSNR) and Structural Similarity(SSIM) metrics are used. PSNR is one of the metrics used to find the maximum possible power of a signal to the corrupted power. It is usually defined using the Mean Square Error (MSE).

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [I(i, j) - K(i, j)]^2 \quad (1)$$

where I is the monochrome image and K is the noisy image.

$$PSNR = 20 \cdot \log_{10}(MAX_I) - 10 \cdot \log_{10}(MSE) \quad (2)$$

where MAX_I is the maximum value of the pixel in the image.

$$SSIM(x, y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)} \quad (3)$$

where μ_x is the average value of x, μ_y is the average value of y, σ_x^2 is the variance of x and σ_y^2 is the variance of y, σ_{xy} is the covariance of x and y, and x and y are two windows of the same size.

4.0 RESULTS AND DISCUSSION

A dataset of 50 MRI images has been taken from different sources, both from the Internet and Real time images for the experimental analysis of denoising using the three multiresolution transforms and the implementation is done using MATLAB (R2017a).

To achieve the objective, experimentation is done by adding noise synthetically in the original images. This noisy image is then denoised using wavelet, contourlet and curvelet transform with different thresholds 2,3 and 4 respectively and quality metric for denoised image is calculated.

PSNR				
Noise	Threshold	Wavelet Transform	Contourlet Transform	Curvelet Transform
Random Noise	2	13.7863	17.9735	18.7704
	3	17.1024	18.5529	19.9770
	4	16.8597	15.2978	16.2853
Salt & Pepper Noise	2	7.0957	10.1732	14.1994
	3	7.5183	12.4733	14.7395
	4	7.9787	13.4142	15.1199
Gaussian Noise	2	9.2497	12.6770	14.7539
	3	11.5079	13.8157	14.1377
	4	12.9206	13.3737	13.4242
Speckle Noise	2	12.4304	15.0861	17.0210
	3	13.8708	15.6646	15.9815
	4	14.7358	14.8996	14.9834

Table 1 – PSNR Value for the different noises and the different MRA transforms

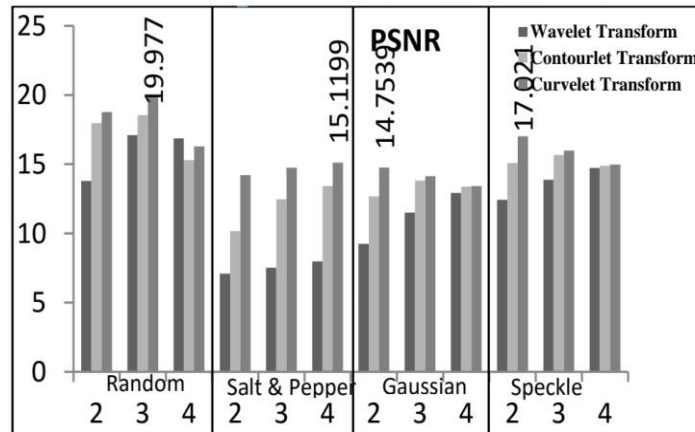


Fig. 11 – Graph depicting the PSNR Value for the different noises and the different MRA transforms

The metrics to quantify the results has been PSNR, MSE and SSIMs. The thresholds of the image have been varied for a variable threshold of 2,3,4. Now when Random noise is added to the image and the image when it is subjected to the wavelet, contourlet and curvelet transform, the curvelet transforms at threshold = 3 provides a better PSNR value of 19.977. A best PSNR value of 15.1199 is produced for the Salt & Pepper noise by the Curvelet transform at threshold = 4. Now when Gaussian noise is being added a suitable PSNR value of 14.7539 is produced by the curvelet transform at threshold = 2. When another noise such as Speckle noise is added to the image, a PSNR value of 15.9815 is produced by the curvelet transform at a threshold value of 3. Hence if an adaptive thresholding technique is used, the Curvelet will outperform all the other transforms as shown in

Table 1. The graph shown in Figure 11 depicts the PSNR in DB for the Random noise, Salt & Pepper Noise, Gaussian Noise and Speckle Noise. From the graph it is seen that Curvelet transform outperforms the wavelet and Contourlet transforms.

MSE				
Noise	Threshold	Wavelet Transform	Contourlet Transform	Curvelet Transform
Random Noise	2	0.00170	0.00066	0.00060
	3	0.00060	0.00062	0.00046
	4	0.00085	0.00123	0.00074
Salt & Pepper Noise	2	0.0074	0.0038	0.0016
	3	0.0068	0.0023	0.0012
	4	0.0061	0.0018	0.0011
Gaussian Noise	2	0.00458	0.00214	0.00132
	3	0.00276	0.00162	0.00155
	4	0.00198	0.00184	0.00183
Speckle Noise	2	0.00238	0.00123	0.00106
	3	0.00164	0.00110	0.00082
	4	0.00133	0.00135	0.00134

Table 2 – MSE Value for the different noises and the different MRA transforms

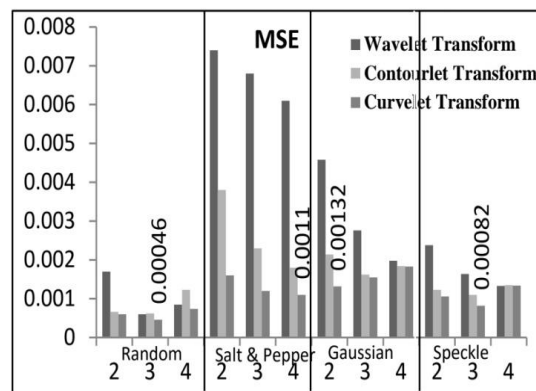


Fig. 12 – Graph depicting the MSE Value for the different noises and the different MRA transforms

Let us now analyze the denoised images using the Mean Square Error (MSE). It also shows that the curvelet at threshold 3 has less Mean Square Error of 0.00046 and value of 0.0011 at threshold 4, for Salt & Pepper noise. A MSE value of 0.00132 is produced by the Gaussian noise at a threshold of 2 and a value of 0.00082 at a threshold of 3 in the Curvelet Transforms. Hence we can say that Curvelet denoises the images better than the Wavelet and Contourlet Transforms as predicted by Table 2. From the graph it is seen that Curvelet transform outperforms the wavelet and Contourlet transforms. The graph shown in Figure 12 depicts the MSE for the Random noise, Salt & Pepper Noise, Gaussian Noise and Speckle Noise.

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SSIM				
Noise	Threshold	Wavelet Transform	Contourlet Transform	Curvelet Transform
Random Noise	2	0.56838	0.79742	0.82133
	3	0.85110	0.88165	0.93879
	4	0.80583	0.75086	0.75948
Salt & Pepper Noise	2	0.5267	0.4536	0.6935
	3	0.5129	0.5458	0.7269
	4	0.5137	0.6211	0.7279
Gaussian Noise	2	0.3342	0.4983	0.6452
	3	0.4657	0.5981	0.6334
	4	0.5626	0.6015	0.6148
Speckle Noise	2	0.75905	0.77837	0.79434
	3	0.78413	0.77942	0.82852
	4	0.78792	0.75254	0.76269

Table 3 – SSIM Value for the different noises and the different MRA transforms

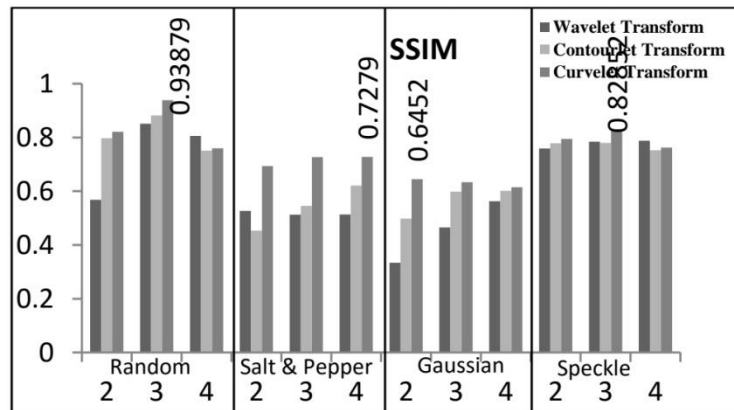


Fig. 13 – Graph depicting the SSIM Value for the different noises and the different MRA transforms

Table 3 depicts the SSIM value for the three transforms, for thresholds 2, 3, 4 and for noises Random noise, Salt & pepper noise, Gaussian noise and Speckle noise.

The graph shown in Figure 13 depicts the SSIM for the Random noise, Salt & Pepper Noise, Gaussian Noise and Speckle Noise. From the graph it is seen that Curvelet transform outperforms the wavelet and Contourlet transforms. When the structural similarity is considered the Curvelet outperforms the other transforms at threshold = 3 with a value of 0.93879. But when Salt & Pepper Noise is added to the image the curvelet produces a better result of 0.7279 at a threshold of 3. When a Gaussian noise is added, at a threshold of 2, the curvelet transform produces a result of 0.6452. Finally a value of 0.82852 is produced by the Curvelet transform at a threshold of 3. The comparison of the three transform for a MRI image is shown in Figure 14.

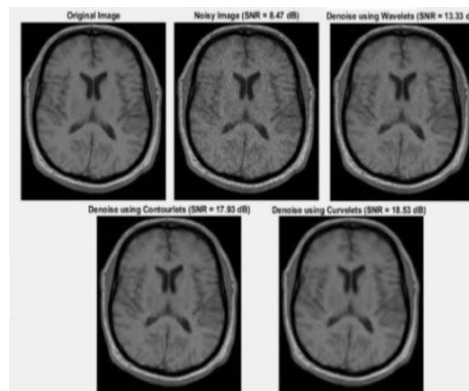


Fig. 14 – Result depicting the PSNR Value for the Random noise and the different MRA transforms

Using the three metrics, PSNR, MSE and SSIM, it is found that the Curvelet is computationally efficient and performs significantly superior in performance than Wavelet Transform and Contourlet Transforms.

5.0 CONCLUSION

MRI images corrupted by different types of noises such as random noise, salt and pepper noise, Gaussian noise and speckle noise were denoised using wavelet, contourlet and curvelet transform with variable threshold of 2,3 and 4. The paper evaluates performance of medical image denoising by calculation of PSNR, MSE and SSIM. According to results shown in Table 1 and Table 3 that curvelet transform increases the images' PSNR and SSIM. Table 2 shows that the MSE for curvelet transform has less MSE as compare to the other methods. It has been found that for any noise that is present in the image, the Curvelet Transform outperforms than the Wavelet Transform and Contourlet Transforms. An adaptive thresholding for the images depending upon the noise that is present in the image gives best results. Hence while denoising the image, the adaptive thresholding is being used, the results had proven to be much better than the achieved results. In all the cases the Curvelet has proved to be the best one.

Since MRI images include more curves, the Curvelet Transform has proved itself superior to the other transforms. Our system can be further improved with some necessary modifications for processing multiple frames of the MRI images using the Curvelet Transforms. Multiple frames can be replaced also with multiple slices.

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