

Hybrid Nanofluid Flow Over A Stretching Surface With Nonlinear Radiation Effect: Numerical Study

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Abstract

This article explores impact of nonlinear thermal radiation on a two dimensional, steady, incompressible water based hybrid nanofluid flow past a permeable stretching sheet .We have used zero mass flux boundary condition revised model in this study for controlling nanoparticle volume fractions passively instead of actively at the boundary which makes this study different and unique with respect to existing literature. Silver and Ferrous Oxide particles are used in this investigation. The set of partial differential equation are transformed into system of ordinary differential equations with the help appropriate similarity variables and solved by shooting technique. Influence of flow parameters on velocity, temperature and concentration profiles are discussed and visualized through graphs. Important engineering quantities like skin friction coefficient, Nusselt number are tabulated. Results show that hybrid nanofluid enhances heat transfer rate as compared to nanofluid.

Key words: Hybrid nanofluid, nonlinear thermal radiation, MHD

1.Introduction

Hybrid nanofluid is a new kind of nanofluid in which more than one nano sized particle will be suspended into base fluid. Even though heat transfer rate can be improved with multiple nanoparticle suspension, there are some issues to be resolved such as suitable nano particles combination and their volume fraction etc. In many industrial processes the qualitative output depends on how quickly heat transfer takes place. Regular fluid like water, Ethylene Glycol etc. do not have high heat transfer capacity as an alternate researcher started adding nanoparticles to base fluids to increase the thermal properties of the base fluids. As an extension to the Mono particle nanofluid researchers started including multiple types of nano particles into base fluid in order to enhance heat transfer rate. Hybrid nanofluids are widely used in many industrial processes [1] where heat transfer rate is crucial such as nuclear reactor, parabolic geometry based solar collectors, heat exchangers, polymer extrusion, cooling/heating chambers and so on. This attracted many researchers to switch over their studies from mono particle nanofluid to Hybrid nanofluid. Suriya Devi et al. [2] studied thermal behavior of $Cu - Al_2O_3$ with water as base fluid .Their results indicate that notable heat transfer improvement can be achieved with hybrid nanofluid instead of mono particle nanofluid .S P Anjali Devi et. al [3] explored hybrid nano fluid flow over permeable stretching sheet and concluded that hybrid nano fluids are better improvement over nanofluids. Cattaneo-Christov heat flux impact $TiO_2 - CuO/EG$ was investigated by Jamshed et al.[4] and proved that spherical shaped nano particles have highest rate heat transfer as compared to hexagon, platelet type nanoparticles. SS Ghadikoleai et al[5] studied squeezing flow with $C_2H_6O_2 - H_2O$ hybrid base fluid suspended with $Fe_3O_4 - Ag$ nano particles. SS Ghadikolaei et al [6] studied $TiO_2 - Cu$ hybrid nanofluid stagnation point flow over a stretching sheet. Recently Naveed Ahmed et al [7] examined $Fe_3O_4 - Ag$ impact on flow between two rigid plates. Manjunatha et al [8] used Runge kutta fourth order method to study variable viscosity effect on stretching sheet with hybrid nano particles. Besides these above said works numerous theoretical and experimental works can be observed in literature related to hybrid nanofluids.(See [1], [9]–[12]).

Knowledge about radiation plays vital role in heat transfer applications like furnace design, nuclear reactor safety, solar energy, extrusion process and so on. But most of the authors used Rosseland linear approximation in their studies which is not appropriate in situation where temperature difference is more. Linearised Rosseland approximation is basically effective Prandtl number with radiation parameter, but wherein nonlinear approximation includes one more parameter temperature ratio (θ_w). Probably Asterios Pantokroutarous [13] was the first one to introduce nonlinear radiation approximation while investigating natural convective flow over isothermal plate. Many authors studied nonlinear radiation impact. Prasanna et al [14] studied Sisko nanofluid flow with chemical reaction, linear radiation past a nonlinear stretching sheet. Ganesh and Rudraswamy [15] explored the viscoelastic 3D nanofluid flow with combined influence of viscous dissipation, joule heating in the presence of nonlinear thermal radiation. Syed Tauseef Mohyud-Din et al [16] investigated the nonlinear radiation effects in a time dependent squeezed flow of a Casson model between two parallel disks. Nonlinear thermal convection in a Casson fluid flow over a horizontal plate with convective boundary condition is examined by Sachin Shaw et al [17]. Hamad [18] investigated analytically magnetic field impact for electrically conducting nanofluid flow over a linear stretching sheet. Fang [19] has investigated analytically a power-law model based viscous fluid flow over a moving surface and obtained closed form solution. Besides these studies some other works are reported in the literatures of MHD flows with metal and metal-oxide nanoparticle suspension with water, ethylene glycol as base fluids. [20]–[23]

More appropriate and realistic boundary condition on two component nanofluid model is given by Kuznetsov and Nield [24] in which volume fraction of nanoparticle on the boundary is controlled passively rather than actively. Later on some authors implemented this revised model in stretching sheet problem. Later on many authors utilized this boundary condition in their investigations, Rana and Lokendra [25] implemented this revised model to explain the heat transfer behavior of boundary layer flow over stretching sheet. Sandeep et al. [26] analyzed the influence of thermal radiation, chemical reaction on MHD flow of a nanofluid past a permeable stretching-shrinking sheet. Puneet rana et al [27] applied Kuznestov-Neild revised model to study an electrically conducting alumina-water based nanofluid flow past a horizontal shrinking cylinder slip conditions at the surface. Recently John and Ioan pop et al [28] verified stability of nanofluid flow past a permeable stretching-shrinking sheet over the heated surface. Sandeep and Ali j chamka [29] numerically explored revised model with ferrous nanoparticles magneto hydrodynamic flow. Looking at the existing literatures on nanofluid flow most of the authors discussed with regular fluid and some authors implemented with mono particle based nanofluid. But no work has been found in hybrid nanofluid with this revised model hopefully this paper will fill that gap. The main aim of this paper is to investigate numerically heat transfer enhancement of $\text{Fe}_3\text{O}_4 - \text{Ag}$ composites with Kuznestov –Neild Revised model.

2. Mathematical Model

We have considered a steady, incompressible, two dimensional, magneto hydro dynamic hybrid nanofluid flow over a permeable stretching sheet in porous medium. The sheet is stretched along the x axis with a linear velocity $u_w(x) = cx$ where $c > 0$. Uniform magnetic field B_0 is applied in the direction of y-axis. The stretching sheet velocity is assumed to be constant. The temperature near the wall is T_w and T_∞ , C_∞ indicates temperature, concentration away from the sheet.

The governing equations of fluid flow with the above assumption can be expressed in the following form.

$$u_x + u_y = 0 \quad (1)$$

$$\rho_{hnf}\{uu_x + vu_y\} = \mu_{hnf}u_{yy} - \sigma_{hnf}B_0^2u - \nu u \quad (2)$$

$$(\rho c_p)_{hnf}\{uT_x + vT_y\} = k_{hnf}T_{yy} + \tau \left[D_B C_y T_y + \frac{D_T}{T_\infty} (T_y)^2 \right] - (q_r)_y \quad (3)$$

$$\{uC_x + vC_y\} = D_B C_{yy} + \frac{D_T}{T_\infty} T_{yy} \quad (4)$$

The imposed boundary conditions on governing equations (1)-(4) are

$$u = U_w, v = 0, T = T_w, D_B C_y + \frac{D_T}{T_\infty} T_y = 0 \text{ at } y = 0,$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty. \quad (5)$$

Here u, v indicates horizontal and vertical velocity components respectively, T being temperature and C is the nano particles volume fraction, $\tau = \frac{(\rho c)_p}{(\rho c)_f}$ here $(\rho c)_p$ is the notation used to indicate effective heat capacitance of the nanoparticles, $(\rho c)_f$ is the heat capacity of the base fluid, ν is the kinematic viscosity, D_B and D_T are Brownian, thermophoretic diffusion coefficients, k is permeability parameter. By using Roseland nonlinear thermal radiation approximation, the radiative heat flux q_r is written as

$$q_r = \frac{-4\sigma^* \partial T^4}{3k^* \partial y} = \frac{-16\sigma^* T_\infty^3}{3k^*} \frac{\partial T}{\partial y} \text{ where } \sigma^*, k^* \text{ are Stefan boltzman constant, mean absorption coefficient respectively..}$$

Now equation (3) can be simplified as

$$(\rho c_p)_{hnf} \{uT_x + vT_y\} = \frac{\partial}{\partial y} (k_{hnf} + \frac{-16\sigma^*}{3k^*} T_\infty^3 T_y) + \tau \left[D_B C_y T_y + \frac{D_T}{T_\infty} (T_y)^2 \right] \quad (6)$$

Where $T = T_\infty [1 + (\theta_w - 1)]$ with temperature ratio $\theta_w = \frac{T_w}{T_\infty}$

Introducing similarity transformations $\psi = \sqrt{c\nu} f(\eta), \eta = \sqrt{\frac{c}{\nu}} y, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_\infty}$

Using $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ continuity equation (1) satisfied and

the flow governing equation (2)-(4) reduces to the following form

$$\ddot{f} + \left[1 - \phi_2 \left\{ (1 - \phi_1) + \phi_1 \left(\frac{\rho_{s1}}{\rho_f} \right) \right\} + \phi_2 \left(\frac{\rho_{s2}}{\rho_f} \right) \right] (f\ddot{f} - \dot{f}^2) - (1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5} Mf - k_p \dot{f} = 0 \quad (7)$$

$$\left[\frac{k_{hnf} + R(1 + (\theta_w - 1)\dot{\theta})^3}{Pr} \right] \ddot{\theta} + \left[1 - \phi_2 \left\{ (1 - \phi_1) + \phi_1 \left(\frac{(\rho c_p)_{s1}}{(\rho c_p)_f} \right) \right\} + \phi_2 \left(\frac{(\rho c_p)_{s2}}{(\rho c_p)_f} \right) \right] f\dot{\theta} + Nb\dot{\theta}\dot{\phi} + Nt\dot{\theta}^2 = 0 \quad (8)$$

$$\ddot{\phi} + Le f\dot{\phi} + \frac{Nt}{Nb} \ddot{\theta} = 0 \quad (9)$$

Non-dimensional forms of Boundary conditions (5) are

$$f(0) = S, f(0) = 1, \theta(0) = 1, Nt\dot{\theta}(0) + Nb\dot{\phi}(0) = 0$$

$$\dot{f}(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \phi(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (10)$$

$$\text{Here } f_w = \frac{v_w}{\sqrt{\alpha\nu}}, Nt = \frac{\tau D_T (T_w - T_\infty)}{\alpha T_\infty}, Nb = \frac{\tau D_B C_\infty}{\alpha}, R = \frac{16\sigma^* T_\infty^3}{(\rho c_p)_{nf} 3\alpha k^*}, M = \frac{\sigma B_0^2}{\alpha \rho_f}, k_p = \frac{\nu}{ka}$$

where $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, Nb is Brownian motion parameter, Nt is thermoporosis parameter, M is magnetic field parameter and kp is porosity parameter. Generally in fluid dynamics investigation two engineering quantities namely skin friction coefficient and Nusselt number plays key role which are defined as

$$c_f = \frac{\tau_w}{\rho_f u_w^2} \text{ and } Nu_x = \frac{x q_w}{k_f (T_w - T_\infty)} \text{ here } \tau_w, q_w \text{ indicates wall shear stress, wall heat flux respectively.}$$

$$\text{i.e. } \tau_w = \mu_{hnf} \left(\frac{\partial u}{\partial y} \right)_{y=0} \text{ and } q_w = k_{hnf} \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (14)$$

Introducing similarity transformations, equation (14) can be reduced into the following form

$$C_f Re_x^{-\frac{1}{2}} = - \frac{\mu_f}{(1-\phi_1)^{2.5} (1-\phi_2)^{2.5}} f''(0),$$

$$Nu_x Re_x^{-\frac{1}{2}} = -(1 + R\theta_w^3) \frac{k_{hnf}}{k_f} \theta'(0)$$

Table 1 : Thermo physical properties of particles and base fluid (see [7] , [29])

Type of particle	$\rho (kg/m^3)$	$C_p (J/kgK)$	$K (W/mK)$	$\sigma (S/m)$
Silver (Ag)	10500	235	429	6.3×10^7
Iron Oxide (Fe_3O_4)	5180	670	9.7	0.74×10^6
Water (H_2O)	997.1	4179	0.613	0.05

Table 2: Thermal conductivity, dynamic viscosity theoretical relations (see [2],[3],[5])

Property	Nanofluid	Hybrid Nanofluid
Viscosity (μ)	$\frac{\mu_f}{(1-\phi)^{2.5}}$	$\frac{\mu_f}{(1-\phi_1)^{2.5} (1-\phi_2)^{2.5}}$
Density (ρ)	$(1-\phi_1)\rho_f + \phi_1\rho_s$	$(1-\phi_2)\{(1-\phi_1)\rho_f + \phi_1\rho_{s1}\} + \phi_2\rho_{s2}$
Heat Capacity (ρc_p)	$(1-\phi_1)(\rho c_p)_f + \phi_1(\rho c_p)_s$	$(1-\phi_2)\{(1-\phi_1)(\rho c_p)_f + \phi_1(\rho c_p)_s\} + \phi_2(\rho c_p)_{s2}$
Thermal conductivity (k)	$\frac{k_{nf}}{k_f} = \frac{k_s + (m-1)k_f - (m-1)\phi(k_f - k_s)}{k_s + (m-1)k_f + (m-1)\phi(k_f - k_s)}$	$\frac{k_{hnf}}{k_f} = \frac{k_{s2} + (m-1)k_{bf} - (m-1)\phi_2(k_{bf} - k_{s2})}{k_{s2} + (m-1)k_{bf} + (m-1)\phi_2(k_{bf} - k_{s2})}$
		Where $\frac{k_{bf}}{k_f} = \frac{k_{s1} + (m-1)k_f - (m-1)\phi_1(k_f - k_{s1})}{k_{s1} + (m-1)k_f + (m-1)\phi_1(k_f - k_{s1})}$

Table 3: Skin friction and Nusselt number Computations for different flow parameters

M	R	k_p	θ_w	Nt	S	Le	$C_f Re_x^{-\frac{1}{2}}$		$Nu Re_x^{-\frac{1}{2}}$	
							Ag	$Ag - Fe_3O_4$	Ag	$Ag - Fe_3O_4$
1							2403261	2.541782	4.575318	8.454510
2							2.710918	2.866371	4.555426	8.359681
3							2.978632	3.148863	4.538869	8.284223
	0.4						-	-	4.575318	8.544501
	0.6						-	-	4.498102	9.319465
	0.8						-	-	4.470518	10.09472
		0.1					2.403261	2.541782	4.575331	8.454508
		0.2					2.435960	2.576027	4.573160	8.440781
		0.3					2.468063	2.609674	4.571050	8.433929
			1.0				2.403261	2.541782	4.327794	8.025643
			1.1				2.403261	2.541782	4.575318	8.454508
			1.2				2.403261	2.541782	4.887174	8.927783
				0.5			-	-	4.575318	8.454508
				1.0			-	-	3.607678	7.306595
				1.5			-	-	2.885303	6.327749
					1.0		2.403261	2.541782	4.575318	8.454508
					1.5		2.865599	3.032804	6.422416	11.48175
					2.0		3.368535	3.566938	8.320328	14.62156
						1	-	-	4.585318	8.454508
						2	-	-	4.011000	7.921537
						5	-	-	3.220484	7.310900

3. Results and Discussion

In this work, we analyzed thermal behavior of hybrid nanofluid with silver-ferrous oxide nanoparticles with the help numerical method which is implemented in MATLAB. More precisely impact of non linear thermal radiation impact is discussed on velocity ,temperature, concentration distributions for stretching sheet with suction $S>0$. In order to solve the differential equations (7)- (9) we have to reduce them into system of seven first order differential equations and then suitable guesses are chosen for missing boundary conditions .A finite value for η_∞ is selected such that all boundary conditions are satisfied with desired level of accuracy. We have considered $\eta_\infty = 6$ in all our computations through out this paper with step size $\Delta h=0.001$. Influence of various

flow parameters are illustrated through graph and tables. In this investigation we considered Silver nano particle as $\phi_1 = 0.03$ and it is fixed through the computation and $0 \leq \phi_2 \leq 0.02$.

Figures 1-3 shows the influence of magnetic parameter (M) on velocity, temperature and concentration profile. From figure-1 it is evident that rising magnetic field strength will creates resistive drag force called Lorentz force which reduces fluid flow. From figure-2, figure-3 we observe that whenever magnetic field is applied then suspended nanoparticles form a chain like structures which enhances heat transfer and concentration. From Table-3 it is clear that velocity of flow reduces due to magnetic field hence skin friction decreases. The influence of radiation parameter (R) on temperature, concentration profiles is shown in Figs. 4 and 5 for both nanofluid and hybrid nanofluid. From graphs we observed that when the values of radiation parameter (R) are raised, the thermal boundary layer thickness is enhanced in both mono, hybrid nanofluids. It happens because increment to R values means providing more heat to fluid which in turn causes enhancement of temperature and thickness of thermal boundary layer. Particularly non linear thermal radiation is more effective on temperature than linear radiation. Figure-6 explores effect of porous parameter on temperature distribution from graph it is clear that increasing porosity parameter is enhancing temperature it is because of resistance caused by porous medium. Influence of temperature ratio parameter (θ_w) on the temperature gradients for linear, nonlinear radiation cases is given in Figure-7. It is evident from those figures that the temperature associated thermal boundary layer thickness are enhanced by rising (θ_w). This happens due to the fact that fluid temperature at wall (T_w) is much higher than the free stream temperature (T_∞) for increasing values of (θ_w) which improves the thermal state of the fluid. The suction influence on various profiles is replicated in figures 8-10, increasing suction is basically tends to forcibly pushing the fluid into a vacant space so that changes takes place in boundary layer. Therefore higher the value of S leads to a reduction in velocity, temperature and concentration distributions. And also from Table-3 we can see that skin friction coefficient rises as S increases this happens because of the increased shear stress on surface which influences velocity gradient. Figures-11 represents impact of Brownian motion on concentration profiles and concentration boundary layer reduces with increased (Nb) parameter due to decreased mass transfer. Thermophoresis Nt influence is discussed in Figures-12 and 13 these plots shows that there is accelerated trend for increased values of Nt.

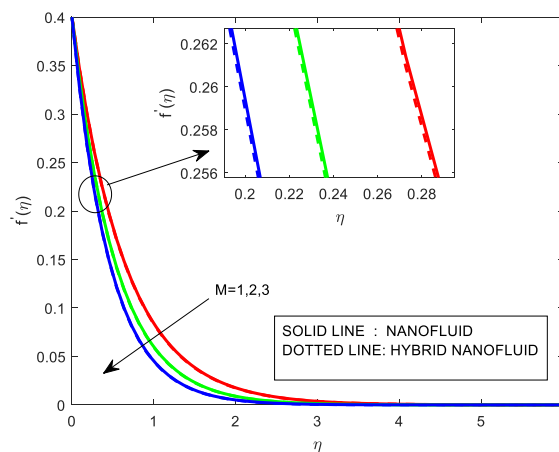


Fig 1: Velocity profile for various M values

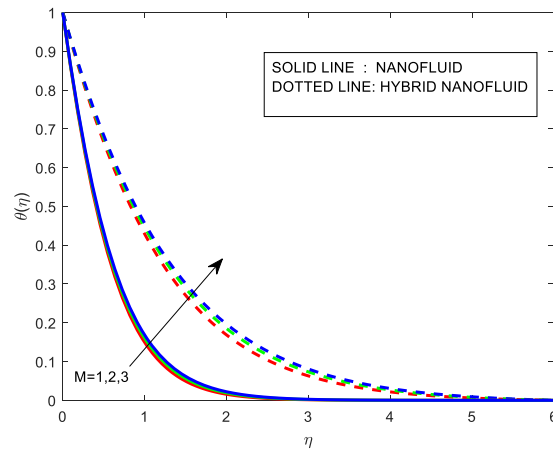


Fig 2: Temperature profile for various M values

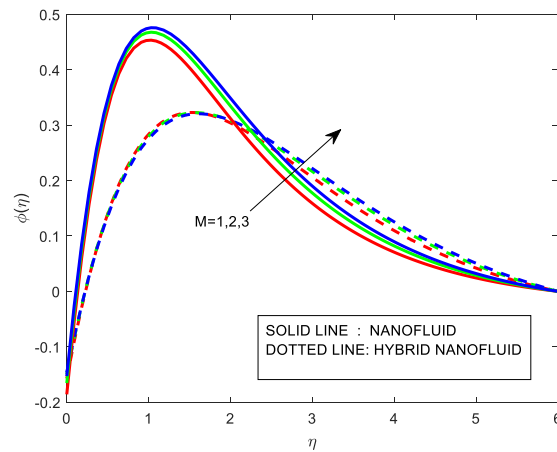


Fig 3: Concentration profile for various M values

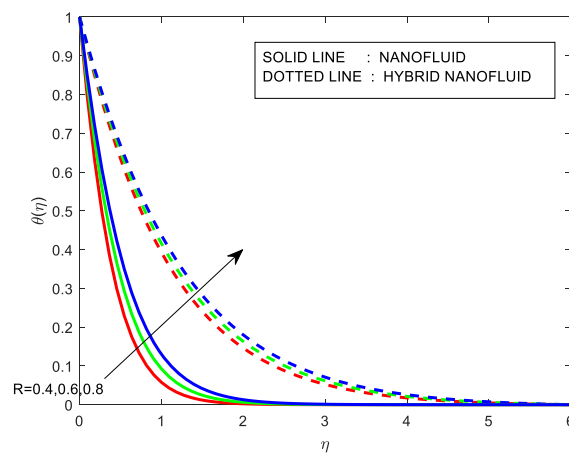


Fig 4: Temperature Profile for R values

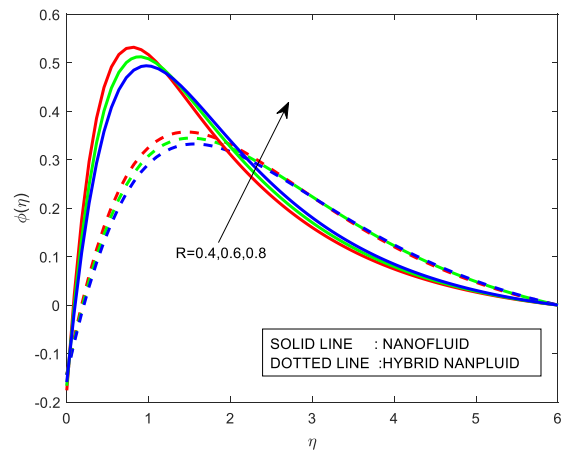


Fig 5: Concentration Profile for R values

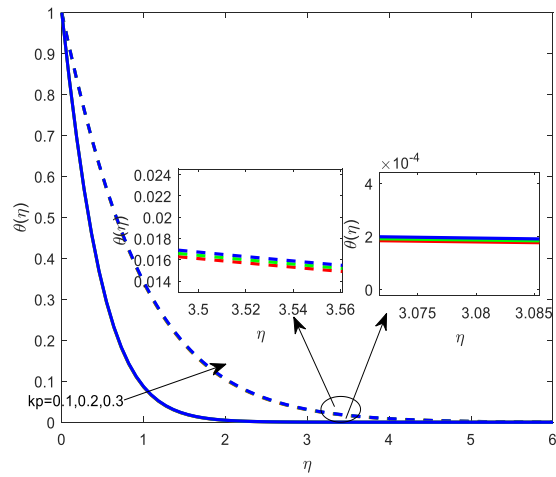


Fig 6: Concentration Profile for k_p values

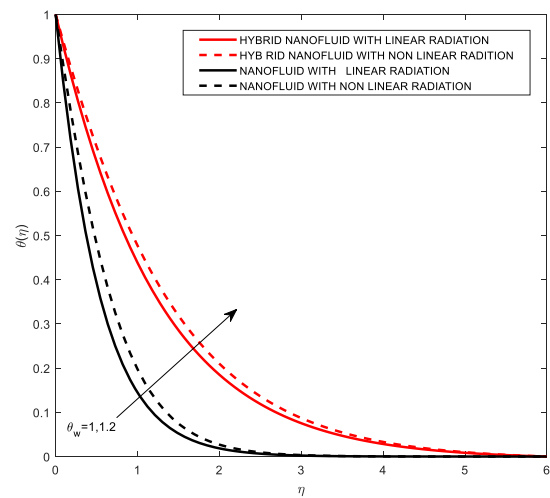


Fig 7: Temperature Ratio on Temperature profile

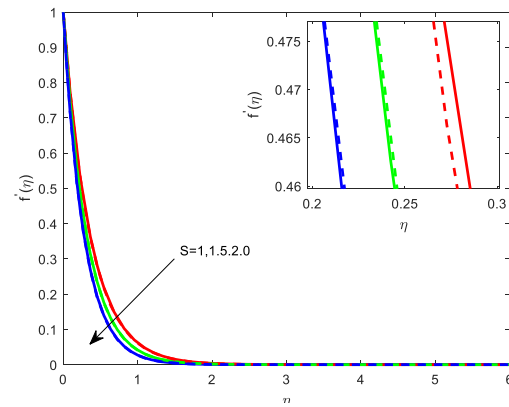


Fig 8: Suction parameter on velocity Profile

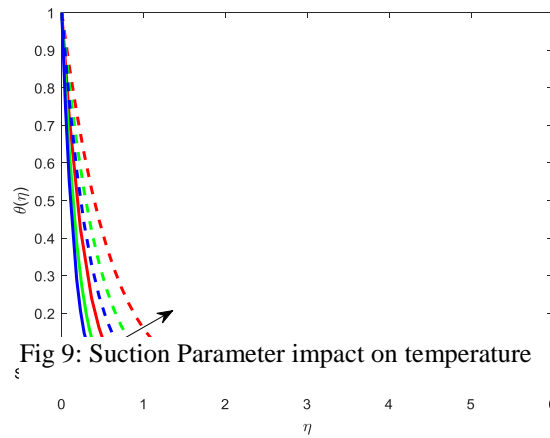


Fig 9: Suction Parameter impact on temperature

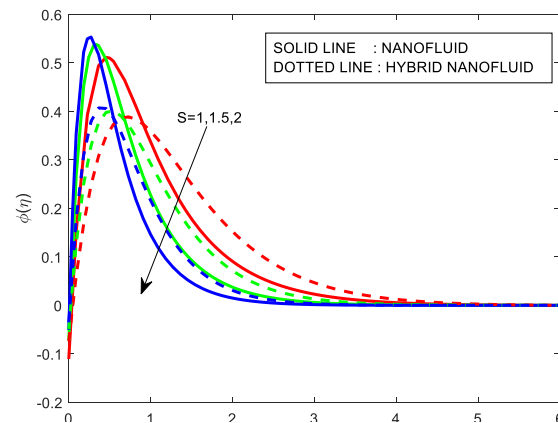


Fig 10: Suction parameter impact on concentration Profile

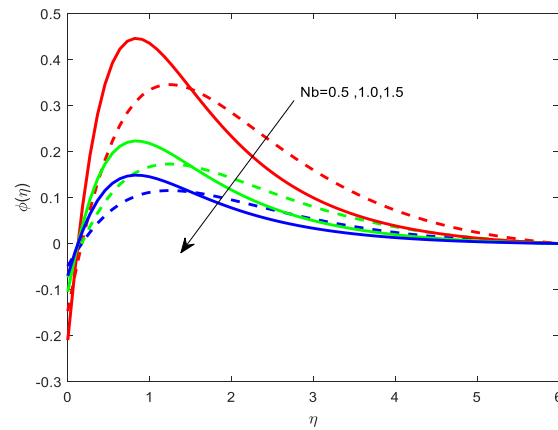


Fig 11: Brownian motion effect on Concentration

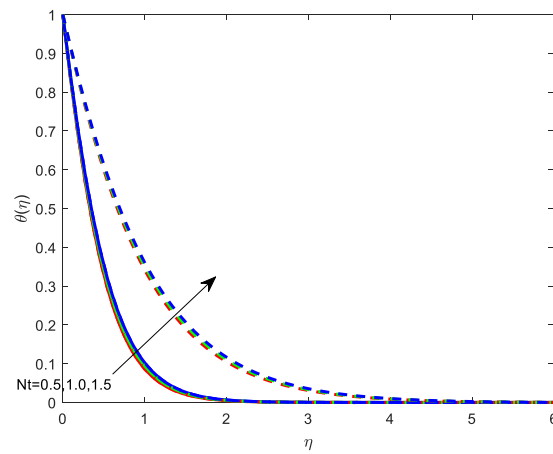


Fig 12: Thermophoresis parameter effect on Temperature

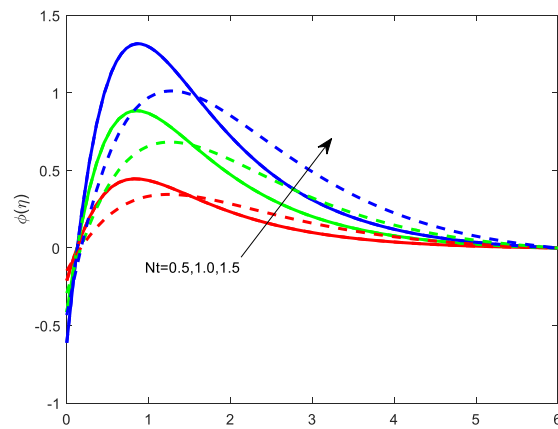


Fig 13: Thermophoresis effect on Concentration

5. Conclusions

Two dimensional boundary layer flow of hybrid nanofluid past a linear permeable stretching sheet has been executed under the influence of nonlinear thermal radiation. kuznestov-neild revised boundary condition is utilized in this study. The main observations in this study are summarized as follows:

- Skin friction coefficient C_f escalate for increseasing values of magnetic parmeter in both the cases of nanofluid and hybrid nanofluid.
- Local nusselt number increases due to hybridity and also with increment of volume fraction but decreases with raise of porosity parameter (kp) for nanofluid and hybrid nanofluid.
- Radiation parameter has diminishing impact on nusselt number hybrid nanofluid.
- Raise of Suction parameter values increases the skin friction and nusselt number for both nanofluid and hybrid nanofluid.
- Thermophoresis parameter decelerates the nusselt number for both nanofluid and hybrid nanofluid.

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